

Informed search algorithms

Chapter 4

Outline

- ◇ Best-first search
- ◇ A* search
- ◇ Heuristics
- ◇ Hill-climbing
- ◇ Simulated annealing

Review: Tree search

```
function TREE-SEARCH(problem, fringe) returns a solution, or failure
  fringe  $\leftarrow$  INSERT(MAKE-NODE(INITIAL-STATE[problem]), fringe)
  loop do
    if fringe is empty then return failure
    node  $\leftarrow$  REMOVE-FRONT(fringe)
    if GOAL-TEST[problem] applied to STATE(node) succeeds return node
    fringe  $\leftarrow$  INSERTALL(EXPAND(node, problem), fringe)
```

A strategy is defined by picking the *order of node expansion*

Best-first search

Idea: use an *evaluation function* for each node
– estimate of “desirability”

⇒ Expand most desirable unexpanded node

Implementation:

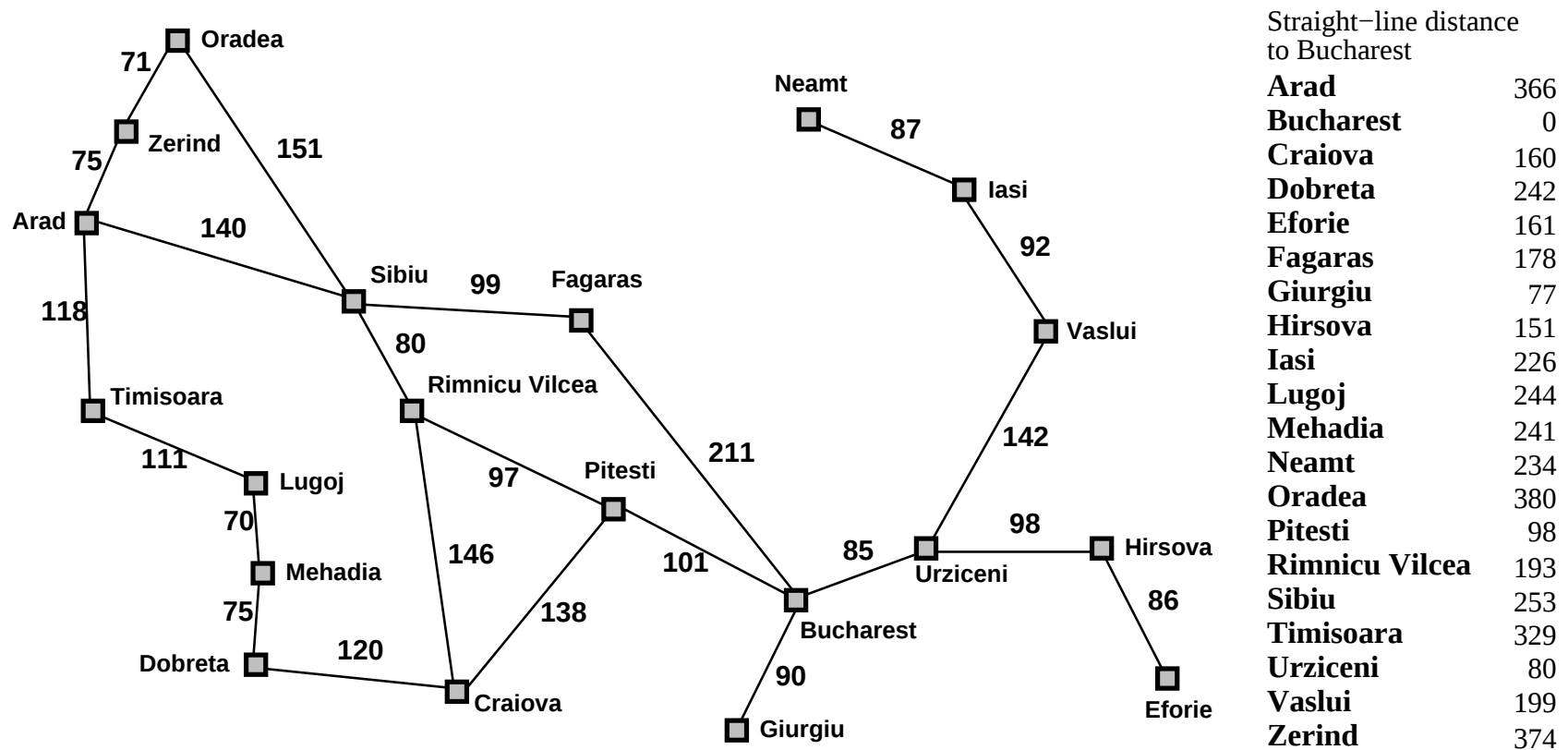
fringe is a queue sorted in decreasing order of desirability

Special cases:

greedy search

A* search

Romania with step costs in km



Greedy search

Evaluation function $h(n)$ (heuristic)

= estimate of cost from n to the closest goal

E.g., $h_{\text{SLD}}(n)$ = straight-line distance from n to Bucharest

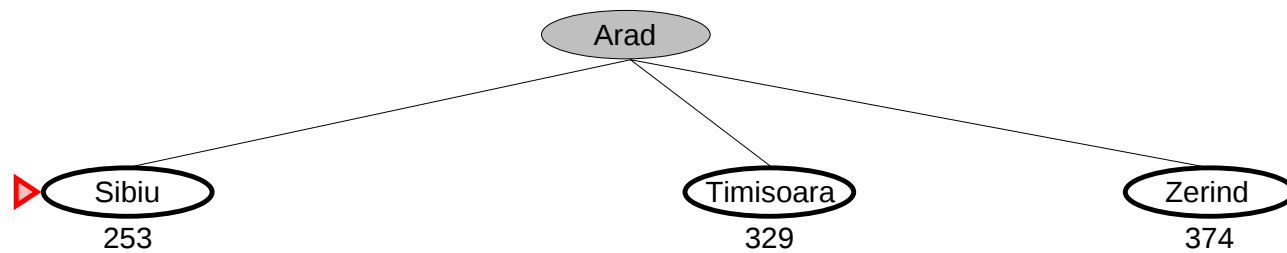
Greedy search expands the node that *appears* to be closest to goal

Greedy search example

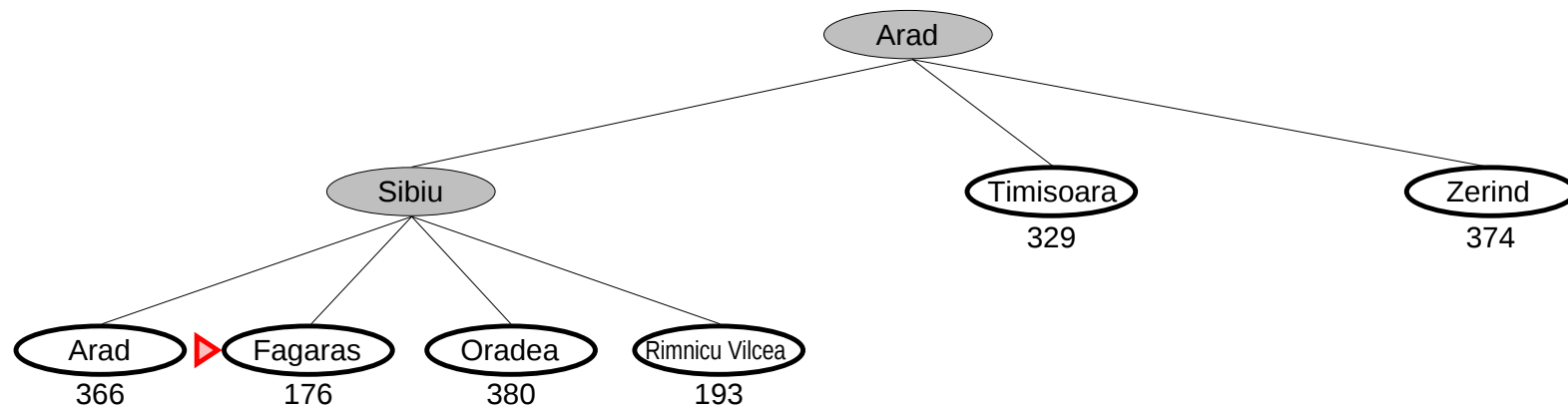


Arad
366

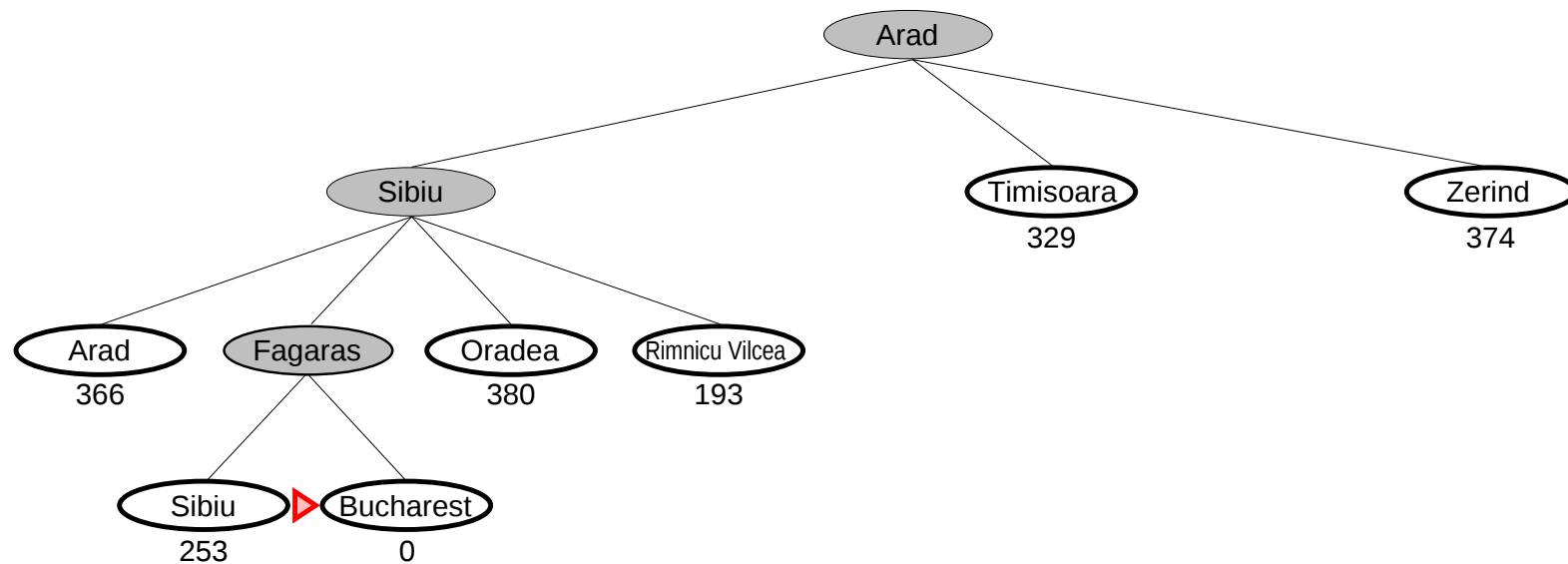
Greedy search example



Greedy search example



Greedy search example



Properties of greedy search

Complete??

Properties of greedy search

Complete?? No—can get stuck in loops, e.g., with Oradea as goal,
Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$

Complete in finite space with repeated-state checking

Time??

Properties of greedy search

Complete?? No—can get stuck in loops, e.g.,

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Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space??

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Space?? $O(b^m)$ —keeps all nodes in memory

Optimal??

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Complete in finite space with repeated-state checking

Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space?? $O(b^m)$ —keeps all nodes in memory

Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

$g(n)$ = cost so far to reach n

$h(n)$ = estimated cost to goal from n

$f(n)$ = estimated total cost of path through n to goal

A* search uses an *admissible* heuristic

i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the *true* cost from n .

(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G .)

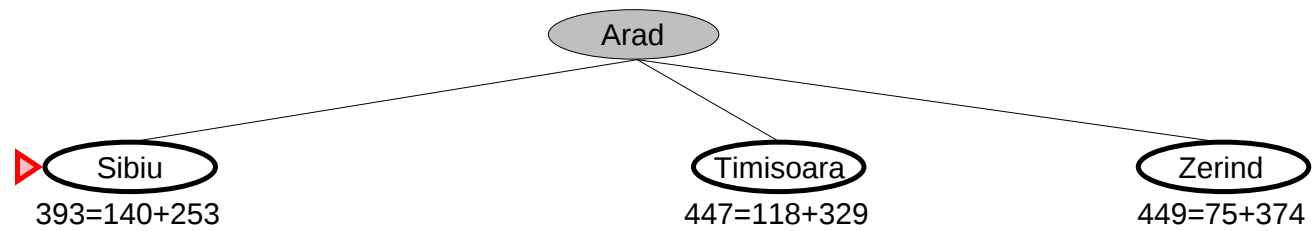
E.g., $h_{\text{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

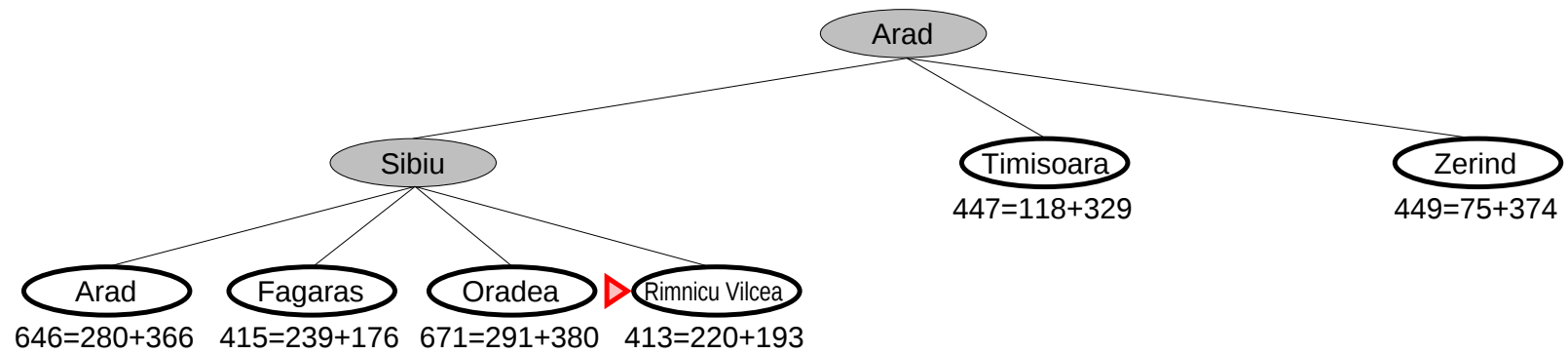
A* search example

▶ Arad
 $366 = 0 + 366$

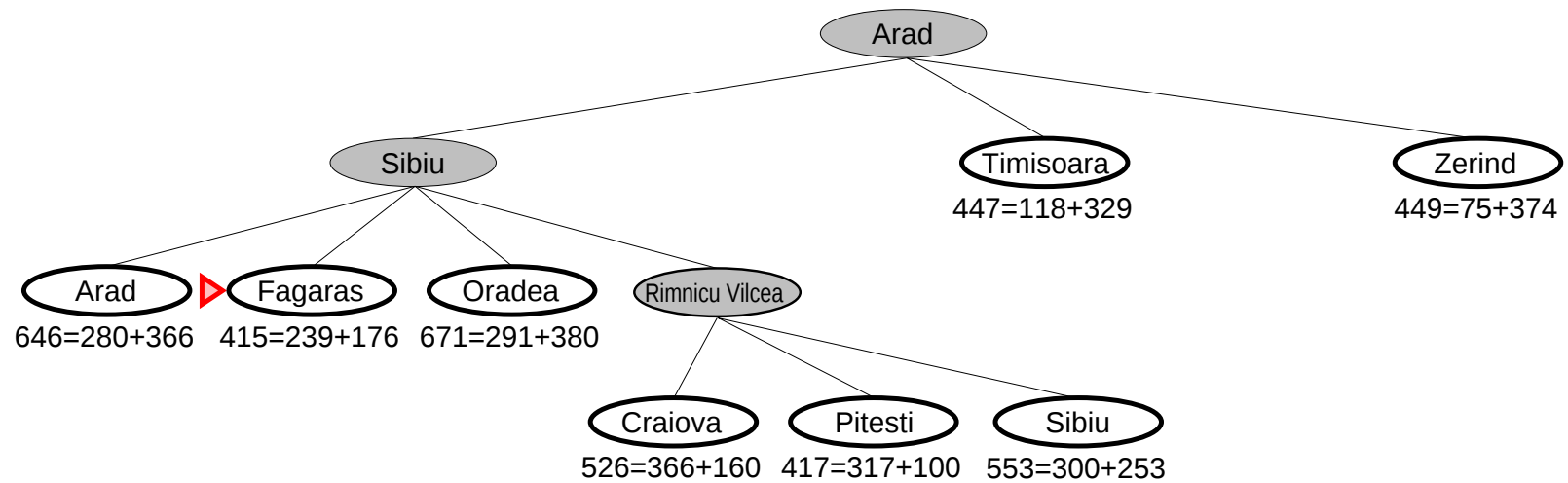
A* search example



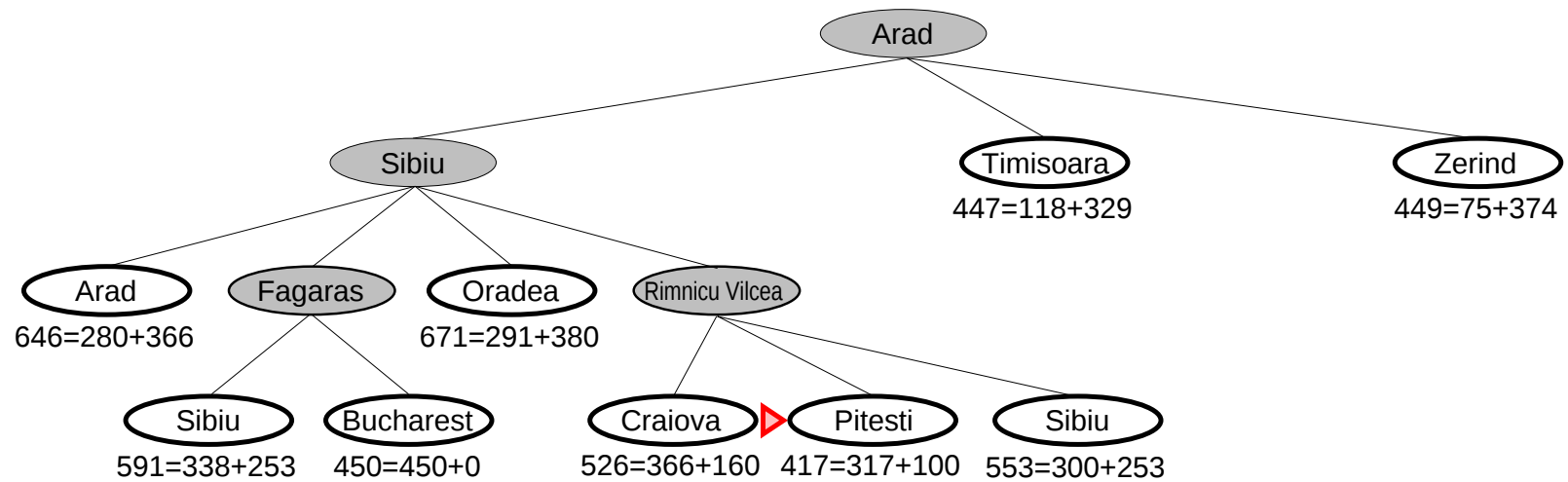
A* search example



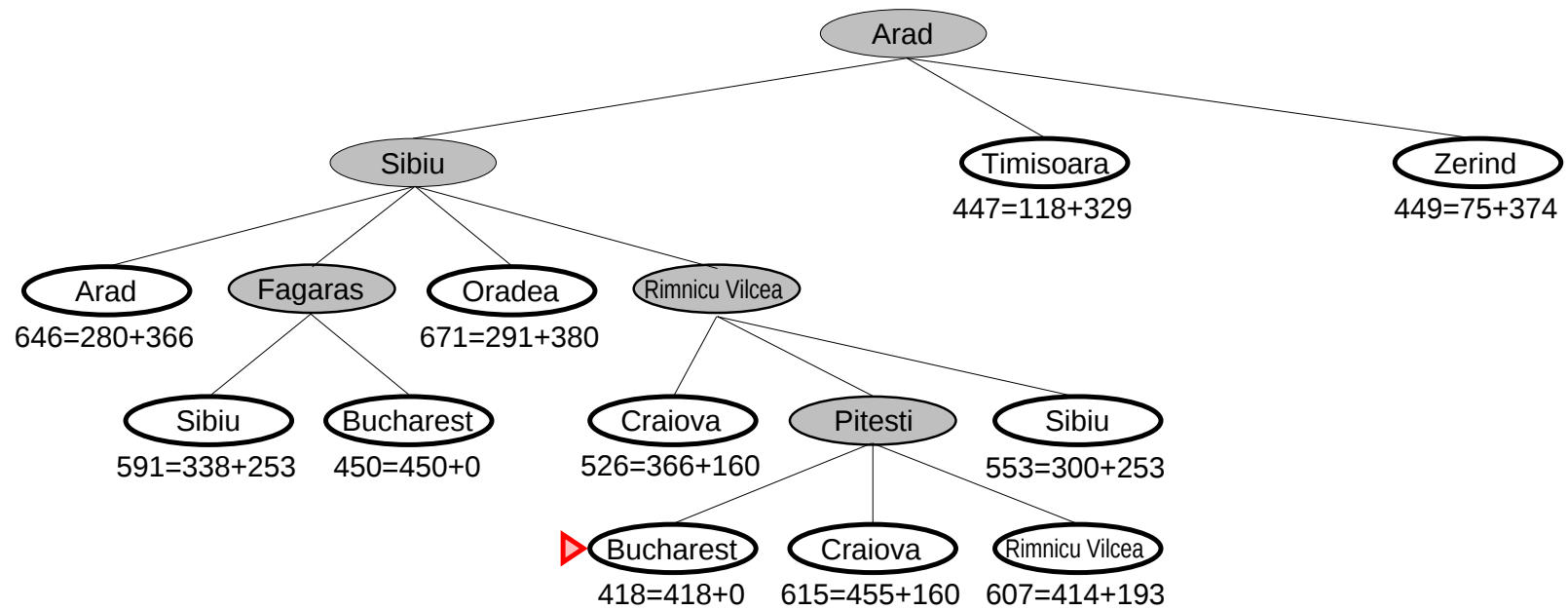
A* search example



A* search example

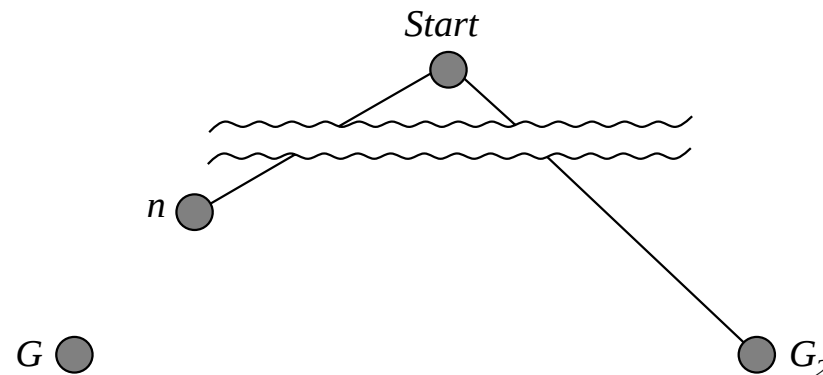


A* search example



Optimality of A^* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue. Let n be an unexpanded node on a shortest path to an optimal goal G_1 .



$$\begin{aligned}
 f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
 &> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
 &\geq f(n) && \text{since } h \text{ is admissible}
 \end{aligned}$$

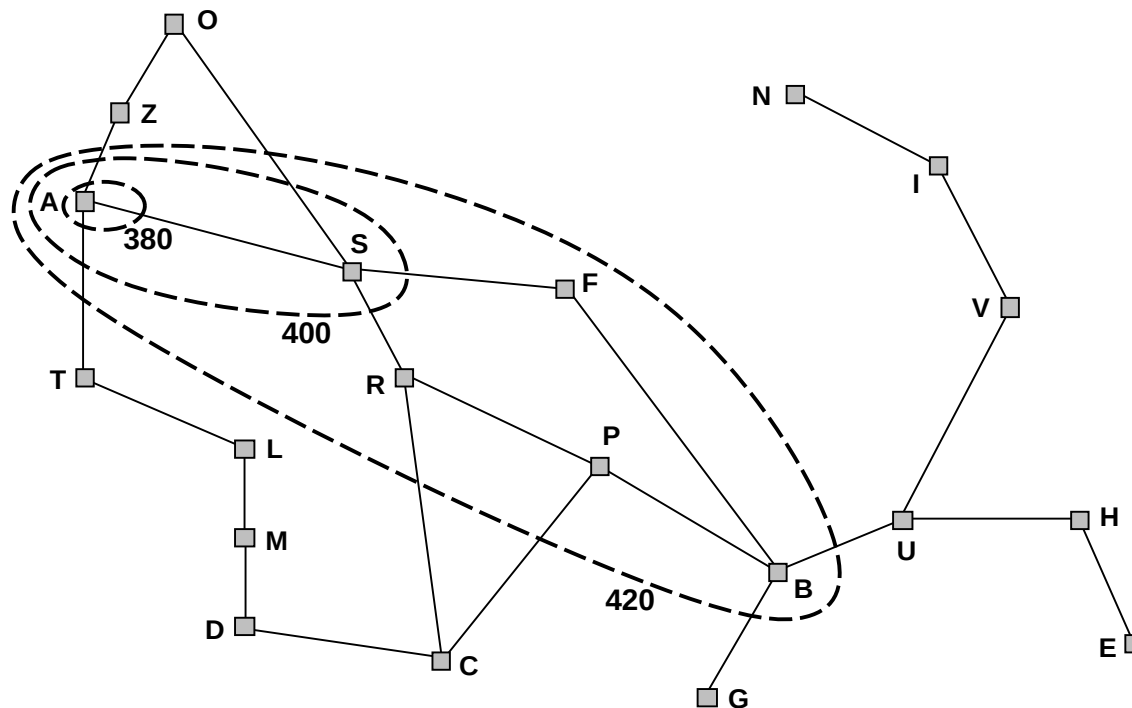
Since $f(G_2) > f(n)$, A^* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds “ f -contours” of nodes (cf. breadth-first adds layers)

Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A^*

Complete??

Properties of A^*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time??

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Time?? Exponential in [relative error in $h \times$ length of soln.]

Space??

Properties of A^*

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Time?? Exponential in [relative error in $h \times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal??

Properties of A^*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in $h \times$ length of soln.]

Space?? Keeps all nodes in memory

Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A^* expands all nodes with $f(n) < C^*$

A^* expands some nodes with $f(n) = C^*$

A^* expands no nodes with $f(n) > C^*$

Proof of lemma: Consistency

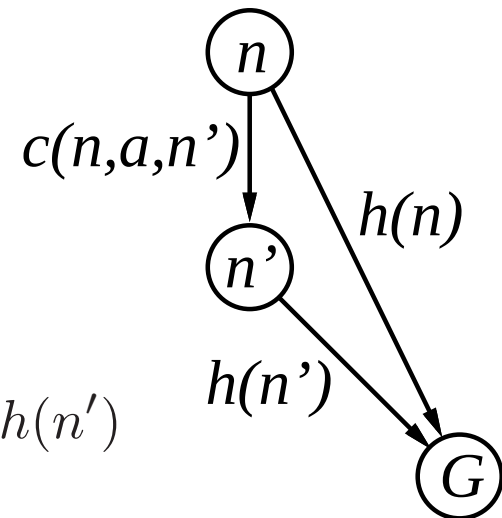
A heuristic is *consistent* if

$$h(n) \leq c(n, a, n') + h(n')$$

If h is consistent, we have

$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

I.e., $f(n)$ is nondecreasing along any path.



Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

$h_2(n)$ = total **Manhattan** distance

(i.e., no. of squares from desired location of each tile)

5	4	
6	1	8
7	3	2

Start State

1	2	3
8		4
7	6	5

Goal State

$h_1(S) = ??$

$h_2(S) = ??$

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles

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(i.e., no. of squares from desired location of each tile)

5	4	
6	1	8
7	3	2

Start State

1	2	3
8		4
7	6	5

Goal State

$h_1(S) = ??$ 7

$h_2(S) = ??$ $4+0+3+3+1+0+2+1 = 14$

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 *dominates* h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes

$A^*(h_1) = 539$ nodes

$A^*(h_2) = 113$ nodes

$d = 24$ IDS $\approx$ 54,000,000,000 nodes

$A^*(h_1) = 39,135$ nodes

$A^*(h_2) = 1,641$ nodes

Relaxed problems

Admissible heuristics can be derived from the *exact* solution cost of a *relaxed* version of the problem

If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*, then $h_1(n)$ gives the shortest solution

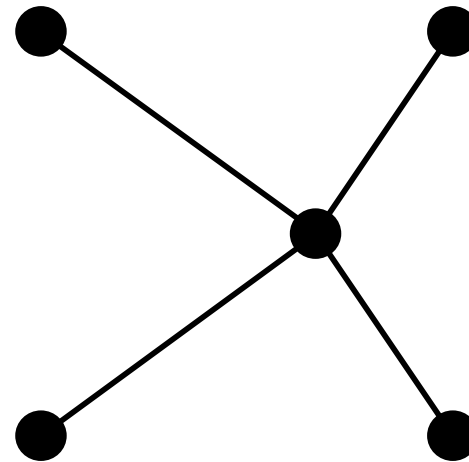
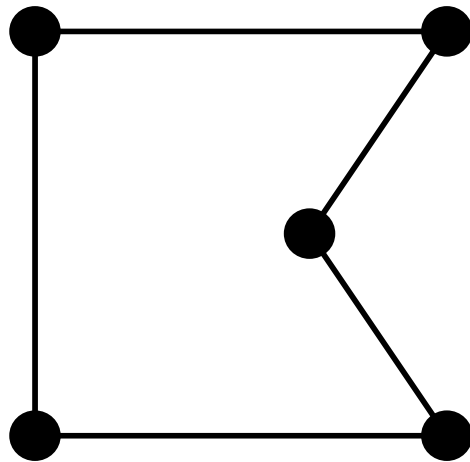
If the rules are relaxed so that a tile can move to *any adjacent square*, then $h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem is no greater than the optimal solution cost of the real problem

Relaxed problems contd.

Well-known example: [travelling salesperson problem](#) (TSP)

Find the shortest tour visiting all cities exactly once



[Minimum spanning tree](#) can be computed in $O(n^2)$
and is a lower bound on the shortest (open) tour

Iterative improvement algorithms

In many optimization problems, *path* is irrelevant;
the goal state itself is the solution

Then state space = set of “complete” configurations;

find *optimal* configuration, e.g., TSP

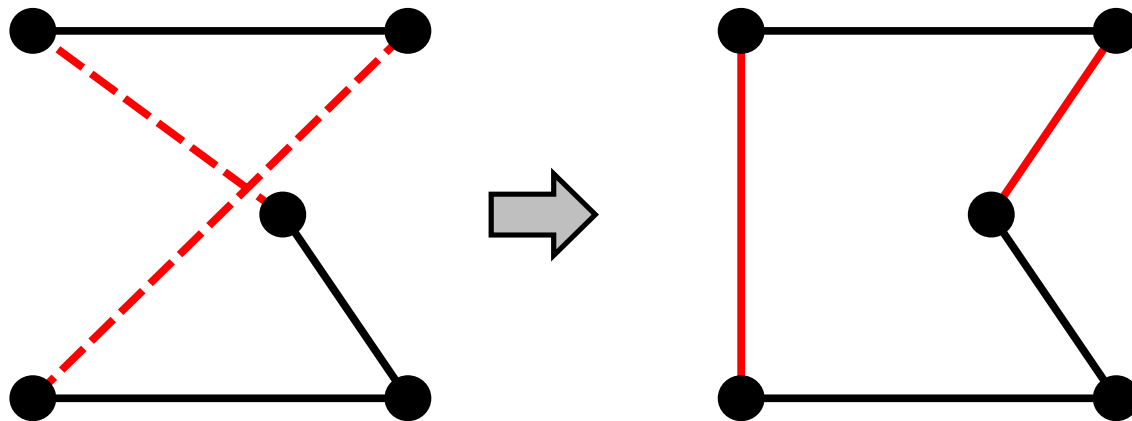
or, find configuration satisfying constraints, e.g., timetable

In such cases, can use *iterative improvement* algorithms;
keep a single “current” state, try to improve it

Constant space, suitable for online as well as offline search

Example: Travelling Salesperson Problem

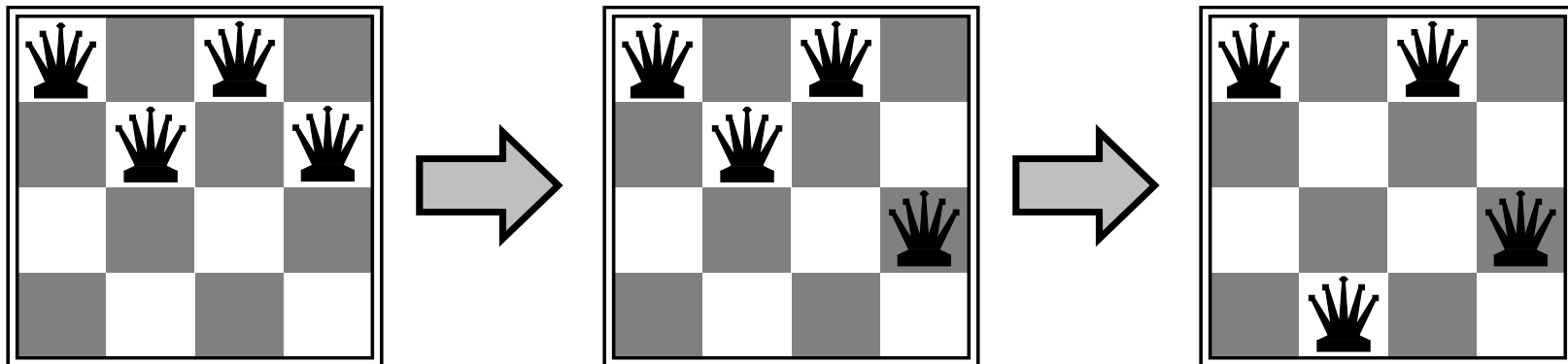
Start with any complete tour, perform pairwise exchanges



Example: n -queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

Move a queen to reduce number of conflicts



Hill-climbing (or gradient ascent/descent)

“Like climbing Everest in thick fog with amnesia”

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

inputs: *problem*, a problem

local variables: *current*, a node

neighbor, a node

current $\leftarrow$ MAKE-NODE(INITIAL-STATE[*problem*])

loop do

neighbor $\leftarrow$ a highest-valued successor of *current*

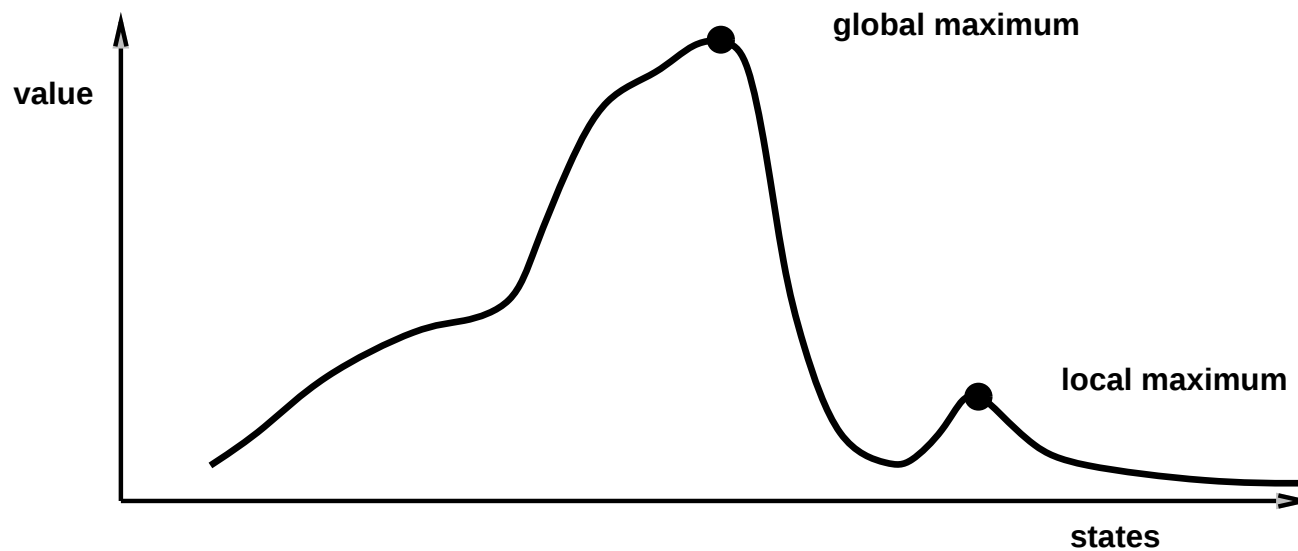
if VALUE[*neighbor*] < VALUE[*current*] **then return** STATE[*current*]

current $\leftarrow$ *neighbor*

end

Hill-climbing contd.

Problem: depending on initial state, can get stuck on local maxima



In continuous spaces, problems w/ choosing step size, slow convergence

Simulated annealing

Idea: escape local maxima by allowing some “bad” moves
but gradually decrease their size and frequency

function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state

inputs: *problem*, a problem

schedule, a mapping from time to “temperature”

local variables: *current*, a node

next, a node

T, a “temperature” controlling prob. of downward steps

current $\leftarrow$ MAKE-NODE(INITIAL-STATE[*problem*])

for *t* $\leftarrow$ 1 **to** ∞ **do**

T $\leftarrow$ *schedule*[*t*]

if *T* = 0 **then return** *current*

next $\leftarrow$ a randomly selected successor of *current*

$\Delta E \leftarrow$ VALUE[*next*] – VALUE[*current*]

if $\Delta E > 0$ **then** *current* $\leftarrow$ *next*

else *current* $\leftarrow$ *next* only with probability $e^{\Delta E / T}$

Properties of simulated annealing

At fixed “temperature” T , state occupation probability reaches Boltzman distribution

$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

T decreased slowly enough $\implies$ always reach best state

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.