§ 9.2 Rational Numbers and Decimals

We will show that every finite decimal can be written as a fraction

$0.5 = \frac{1}{2}$ $\quad 0.25 = \frac{1}{4}$ $\quad 0.125 = \frac{1}{8}$ $\quad 0.75 = \frac{3}{4}$

$0.2 = \frac{1}{5}$

Every finite decimal can be written as a decimal fraction, a fraction whose denominator is a power of 10

$$0.125 = 125 \text{ thousandths} = \frac{125}{\underbrace{1000}_{10^3}}$$

Obviously this is not in simplest terms. The denominator is $10^3 = 2^3 \times 5^3$ and $125 = 5^3$ so

$$\frac{125}{1000} = \frac{\cancel{5^3}\,1}{2^3 \times \cancel{5^3}} = \frac{1}{2^3}$$

From this example we see that the denominator of the simplified decimal fraction involves only powers of 2 and powers of 5

Any fraction with denominator consisting only of powers of 2 and powers of 5 is equivalent to a decimal fraction

eg Represent $\frac{7}{40}$ as a decimal.

Th^m : A fraction represents a finite decimal if and only if in simplest terms its denominator consists of powers of 2 and 5 respectively

Fractions that are not equivalent to finite decimals have infinite decimal expansions of a special type

Eg Compute $\frac{5}{9}$ using long division

Eg Compute $\frac{2}{7}$ using long division.

Eg Compute $\frac{3}{13}$ using long division

What do you observe? How could you explain why this happens?

All repeating decimals correspond to rational numbers

Eg Compute $\frac{7}{1125}$.

Some have digits before it starts repeating

Eg what number corresponds to $0.\overline{69}$

Eg What number corresponds to $0.00\overline{69}$

Eg $0.386969\overline{69}$ represents which fraction.

What does a decimal that does not eventually repeat represent?