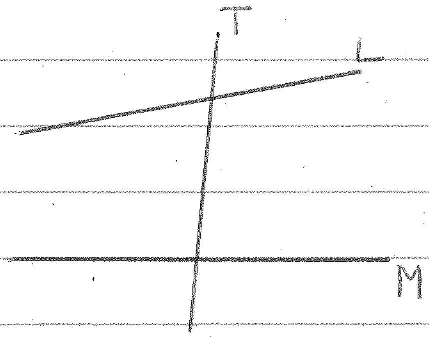
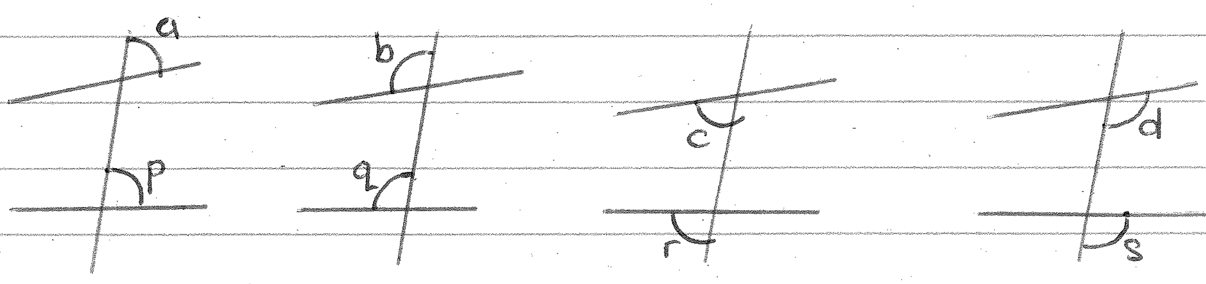
§ 3.2 Finding Angles Using Parallel Lines

A line T is called a transversal to a pair of lines L and M if T intersects both L & M and not in the same point



Transversal forms 8 angles The can be put into pairs of corresponding angles

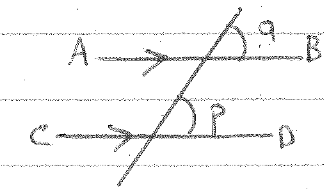


After examining a few examples we are lead to the following observations

If a transversal intersects two parallel lines then corresponding angles are equal, i.e.

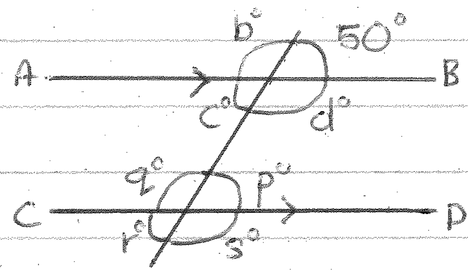
if $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ then

$a = p$



(Abbrev: corr. $\angle$s, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$)

Example:



$b + 50 = 180$

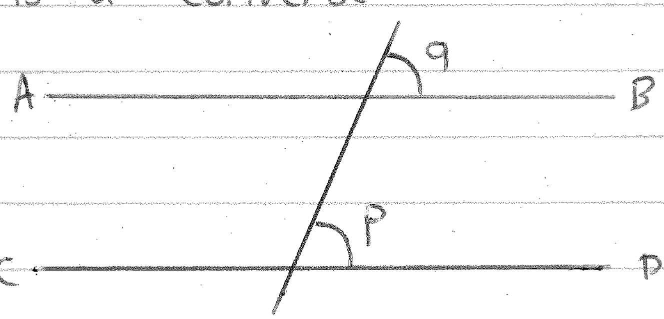
$\angle$s on a line.

$b = 130$

$\left.\begin{array}{l} c = 50 \\ d = 130 \end{array}\right\}$ vert. opp. $\angle$s

$\left.\begin{array}{l} p = 50° \\ q = 130° \\ r = 50° \\ s = 130° \end{array}\right\}$ corr. $\angle$s , $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

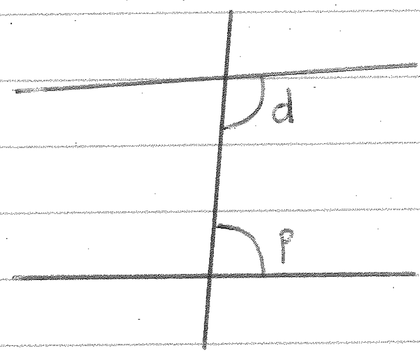
There is a converse



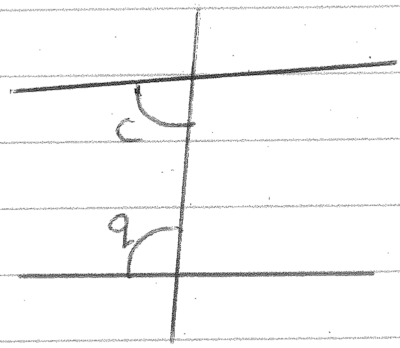
If $a = p$ then

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

corr. $\angle$s converse

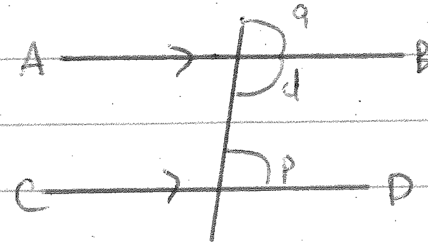


same side interior angles



same side interior angles.

If we have parallel lines



$a = 180 - d$ $\angle s$ on a line

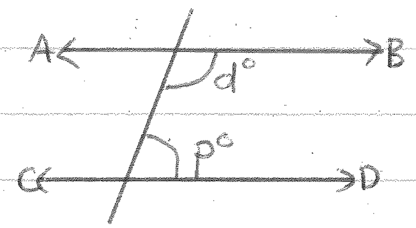
$p = 180 - d$ corr. $\angle s$ $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

$\therefore d + p = 180$

If a transversal intersects two parallel lines then the interior angles on the same side of the transversals are supplementary

int $\angle s$, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

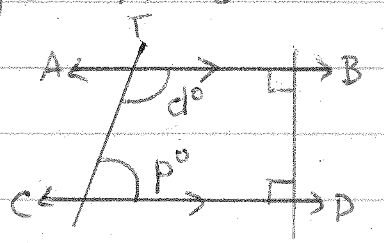
There is a converse to the statement too



If $d + p = 180$ then $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

int ∠s conv.

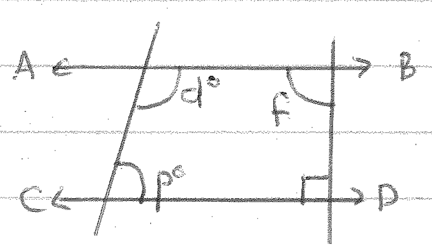
I will give you proofs of the two interior angle facts



Using our elementary definition of parallel lines we draw a second transversal

$$d + p + 90 + 90 = 360 \quad \angle s \text{ in a quad.}$$

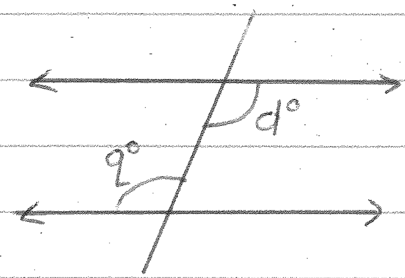
$$\therefore \; d + p = 180^\circ$$



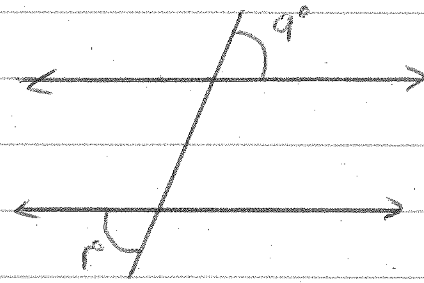
Suppose we are given
$d + p = 180$
then

$$d + p + f + 90 = 360$$
$$180 + f + 90 = 360$$
$$\therefore \; f = 90 \quad \text{so } \overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$$

Alternate Angles



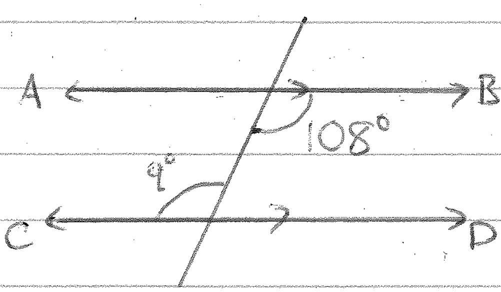
Alternate Interior Angles



Alternate Exterior Angles

Combining the vertically opposite angles are equal fact with the corresponding angle fact we get :

Eg



Find q ?

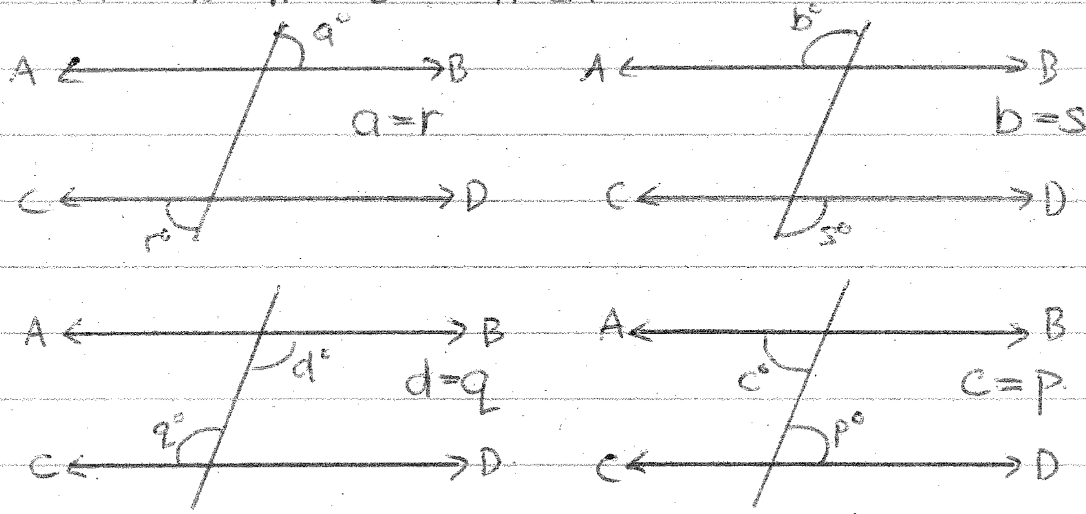
Introduce b°

$b = 108$ vert. opp. $\angle$s

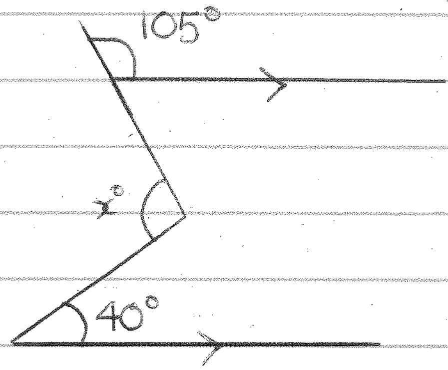
$q = 108$ corr. $\angle$s, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

If a transversal intersects two parallel lines then alternate angles are equal.

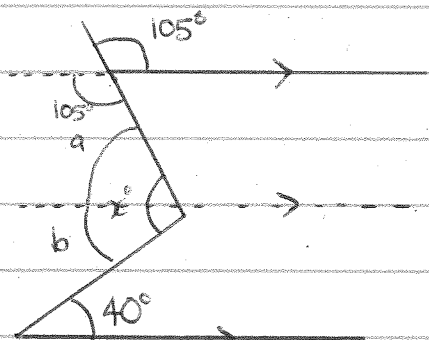
If AB $\parallel$ CD then



$\angle$ alt. $\angle$s, $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



Insert a new parallel line through the vertex at which the unknown angle lies



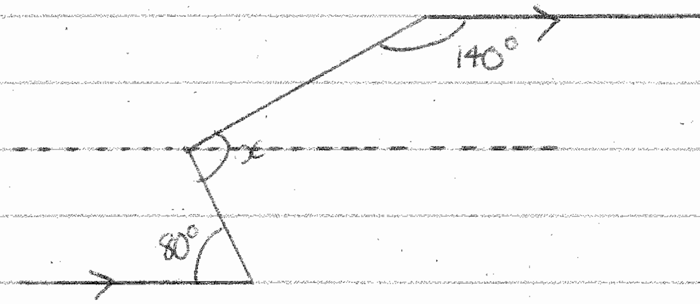
$b = 40^\circ$

alternate $\angle$s are equal.

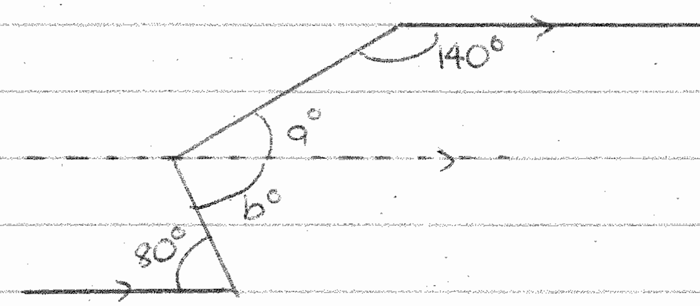
$a + 105 = 180$

$a = 180 - 105 = 75$

$x = a + b = 40 + 75 = 115$



Add a new parallel line through the vertex

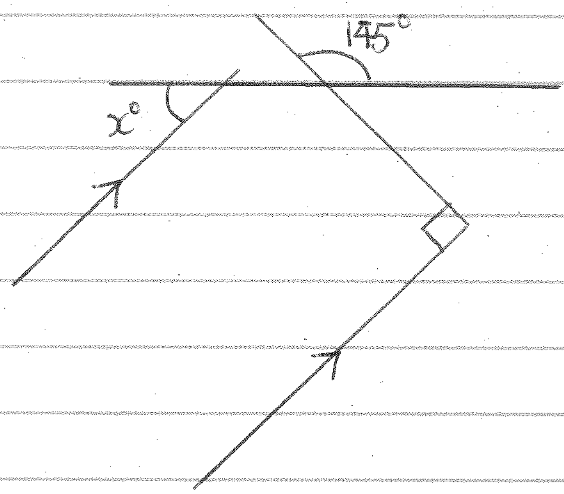


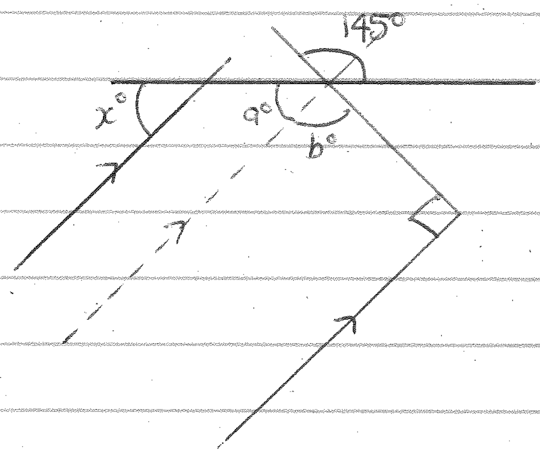
$a + 140 = 180$ same side int. ∠s

$a = 40$

$b = 80$ alt ∠s.

$a + b = 120$





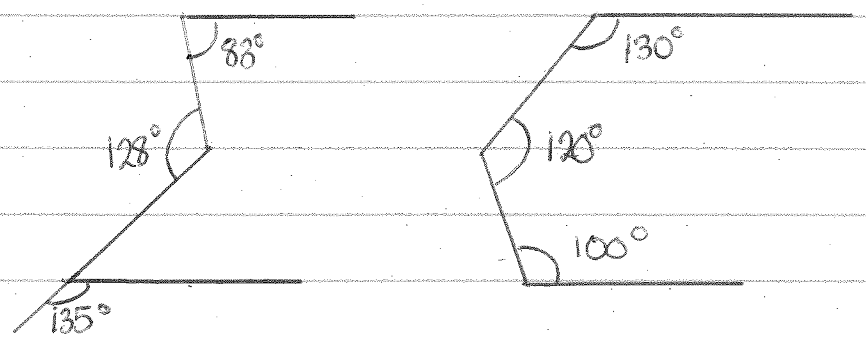
$a + b = 145$ vert opp ∠s

$b + 90 = 180$ same side int. ∠s

$b = 90$

$a = 145 - 90 = 55$

$x = 55°$ corr. ∠s



Label the parallel lines in the above diagram.