Calculus I

Loyola University Chicago – Math 161.004 – Fall 2014

Workshop Write-ups: Guidelines & Hints

Your workshop write-ups will be graded on correctness of the mathematical content and *the manner in which it is presented*. The framework I have set up for the assignments and their grading should allow for perfect scores each time (good luck):

- Two problems will be chosen each **Wednesday,** to be typeset carefully using LaTeX.[1]
- Each problem will be completed (collaboratively) by three student volunteers.
- Rough drafts (*informally*) due by *midnight Tuesday*—to be posted to our class "project" on cloud.sagemath.com—after which time your cohort and I will make comments, questions, and suggestions that could improve the solutions' content or exposition.
- Final solutions are due **noon** on **Friday** (9 days after first assigned).

Typesetting help. I have provided a sample LaTeX document in the "latex demos" folder in our cloud project (`cloud.sagemath.com`) that you should use as a template. Finally, don't hesitate to ask the Piazza forum for help if you cannot figure out how to do something you'd like to do.

Since you may never have been graded on how you present your ideas in math, here are some guidelines.

Look at one of the examples in the book. The authors begin by writing **a statement of the problem.** They use **complete English sentences.** They explain those steps which are **not obvious** to a calculus student (and don't explain the steps that are). If there is a graph, they **label it,** and they **discuss** what can be deduced from the graph in the context of the example. At the end, they **state the conclusion.**

Don't be discouraged if your initial write-ups receive low grades because of poor exposition. Writing solutions with explanations is probably not something you have done in a math class before. You will improve. Because mathematics is used to solve problems *and explain the solutions to others*, writing clear solutions is a good habit to develop. Creating a good write-up forces you to think more carefully about how you did the problem—and therefore helps you learn calculus.

Precision, part I. Here is a joke:

> A mathematician, a physicist, and an engineer are visiting Scotland for the first time and riding through the countryside by train. "Oh!" the engineer says upon seeing a sheep on a hill, "the sheep in Scotland are black!" The physicists chimes in, "no, no, that sheep is black." "Well," the mathematician adds, "*that side* of that sheep is black."

It may not be very funny to you, but it is because you don't know many mathematicians yet. Like physicists and engineers, we develop tools and ideas to solve problems. Unlike these individuals, we like to know things *for certain* and only claim things we certainly know. Precision is our bread-and-butter. We take care to be very precise—both with our language and with our reasoning. This is reflected in our grammar. Consider the following example:

$\pi = 3.14159$ FALSE! $\qquad \pi \approx 3.14159$ TRUE.

Take Away: *Don't say things that aren't true when solving a mathematical problem.*

[1] There are many *Introduction to LaTeX* guides on the web. In the past, I have turned to one by Tobias Oetiker farely often. (Ask me if you want a copy.) These days, I just google a phrase like "latex square root" or "latex include graphics" to more quickly find what I need. If you want to draw a figure natively (using LaTeX code), then google "tikz examples."

Sentences (Symbols, part I). When reducing a mathematical expression, use the "=" sign only when two things are equal, and the " $\implies$ " sign only when the second statement is a direct consequence of the first:

Good Uses	**Bad Uses**
$n^2 + n^2$ $= 2n^2$	$n^2 + n^2$ $2n^2$
$7x = 3 \implies x = \frac{3}{7}$	$7x = 3 = x = \frac{3}{7}$

Here are two english-language sentence analogs of the bad uses above:

Apples and apples, two apples.   Alexis is the name of the cat is named sophie.

Take Away: *Practice good mathematical grammar; know what the elements of your sentence mean.*

Symbols, part II. Mathematicians invented algebra so we wouldn't have to write sentences like this:

> If thou art diligent and wise, O stranger, compute the **number of cattle of the Sun**, who once upon a time grazed on the fields of the Thrinacian isle of Sicily, divided into four herds of different colours, one **milk white,** another a **glossy black,** a third **yellow** and the last **dappled.** In each herd were bulls, mighty in number according to these proportions: Understand, stranger, that **the white bulls were equal to a half and a third of the black together with the whole of the yellow,** while **the black were equal to the fourth part of the dappled and a fifth, together with, once more, the whole of the yellow.** ...But come, understand also all these conditions regarding the cattle of the Sun. **When the white bulls mingled their number with the black, they stood firm, equal in depth and breadth,** and the plains of Thrinacia, stretching far in all ways, were filled with their multitude. ...—Archimedes, 200 B.C.
>
> Full text available www.math.nyu.edu/~crorres/Archimedes/Cattle/Statement.html

Instead we write something like,

> Let W, B, Y, D be the number of cattle coloured white, black, yellow, and dappled, respectively. Let w, b, y, d be the number of bull in each herd. The numbers satisfy:
>
> $$\begin{aligned} w &= \tfrac{5}{6}b + y \\ b &= \tfrac{9}{20}d + y \\ &\vdots \\ w + b &= N^2 \\ &\vdots \end{aligned}$$
>
> where N is some positive integer.   **Challenge Problem #1:** find N.

Take Away: *If you have a quantity you're considering, don't be afraid to give it a variable name if it will help with the clarity of your exposition.* (With that said, PLEASE don't leave variables undefined, it is sloppy and confusing...though rarely as confusing as the passage above.)

Consistency. Regarding symbol choices, if you name the hypotenuse of a triangle C at some point, do not later on refer to it as c (or x!).

Take Away: *Give every quantity one name (symbol) and one name only.*

Catching Your Breath. The language of mathematics packs a lot of information into a small space. This makes reading a half-page of equations fairly difficult. The conscientious writer recognizes this.

Take Away: *Pause from time to time and write a sentence.* That sentence should **tell the reader where he is heading NEXT,** not what he just slogged through.

Clarity, part I. Note the use of Y and y in the cattle problem above. These are related quantities so they are given related names. Also, they are *codes* in the sense that, as best as possible, the symbols chosen reflect something essential about what they represent ("yellow" things). Similarly, a formula for the volume of a box in terms of its height should begin "$V(h) = \ldots$," not "$f(x) = \ldots$."

Take Away: *Reserve similar symbols for related quantities and choose your symbols well.*

Clarity, part II How would you feel listening to the news if every time someone mentioned Vladimir Putin, they suffixed it with the phrase "the nefarious KGB agent turned president of Russia who once seduced George W. Bush using only his eyes?" Pretty insulting right? We all know this, get to the point! Similarly, when using some (well-labeled) figure to setup a related-rates problem, don't write

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{h}{y}\,.$$

Omit the middle step; we all know this and saying it just detracts from the readability of your presentation. Another example: instead of writing

> we have
>
> $$\begin{aligned} 32x^7(x+2)^3 &= 6\frac{(x+2)(x-2)}{x^2} && \text{clear denominator} \\ 32x^9(x+2)^3 &= 6(x+2)(x-2) && \text{cancel the 2} \\ 16x^9(x+2)^3 &= 3(x+2)(x-2) && \text{cancel a factor of } (x+2) \\ 16x^9(x+2)^2 &= 3(x-2)\,, \end{aligned}$$

simply write

> we have
>
> $$\begin{aligned} 32x^7(x+2)^3 &= 6\frac{(x+2)(x-2)}{x^2} \\ 16x^9(x+2)^2 &= 3(x-2)\,. \end{aligned}$$

Or, if you really feel like the reader won't follow you, write

> we have
>
> $$32x^7(x+2)^3 = 6\frac{(x+2)(x-2)}{x^2}$$
>
> or, after clearing denominator and canceling like terms,
>
> $$16x^9(x+2)^2 = 3(x-2)\,.$$

Take Away: *Clutter does not engender clarity.*

Challenge Problem #2: I threw away a potential solution just now. Catch my mistake.