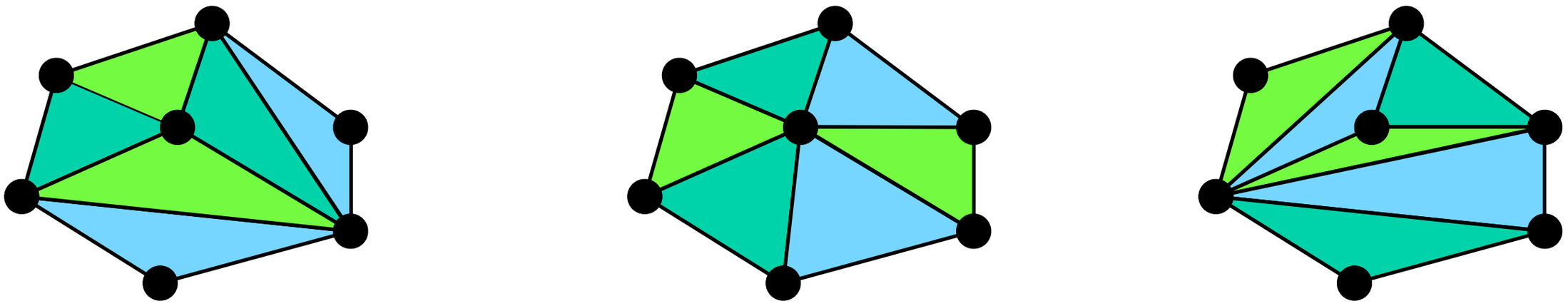
Triangulation

Triangulations

A *triangulation* of set of points in the plane is a *partition* of the convex hull to triangles whose vertices are the points, and do not contain other points.

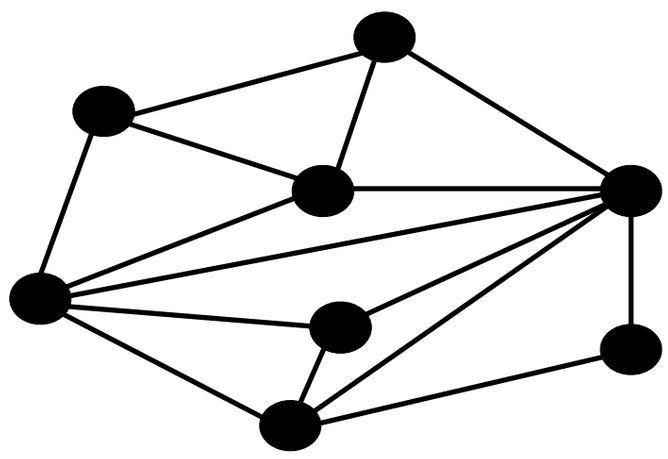
There are an exponential number of triangulations of a point set.



An O(n^3) Triangulation Algorithm

Repeat until impossible:

- Select two sites.
- If the edge connecting them does not intersect previous edges, keep it.



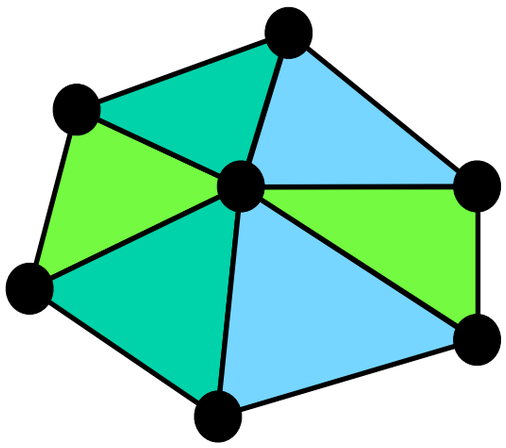
Do you get consistent results?

Which ones are better?

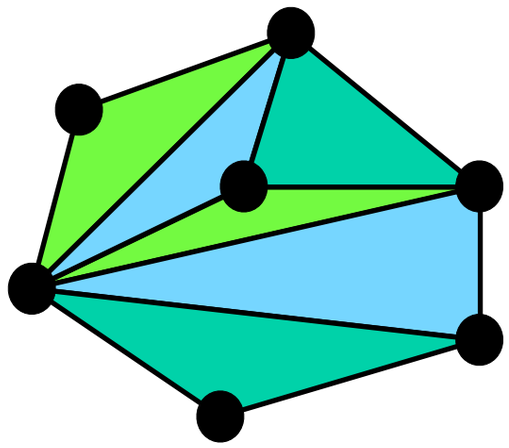
"Quality" Triangulations

Let $\alpha(T_i) = (\alpha_{i1}, \alpha_{i2}, .., \alpha_{i3})$ be the vector of angles in the triangulation T in increasing order:

- A triangulation T_1 is "better" than T_2 if the smallest angle of T_1 is larger than the smallest angle of T_2
- Delaunay triangulation is the "best" (maximizes the smallest angles)



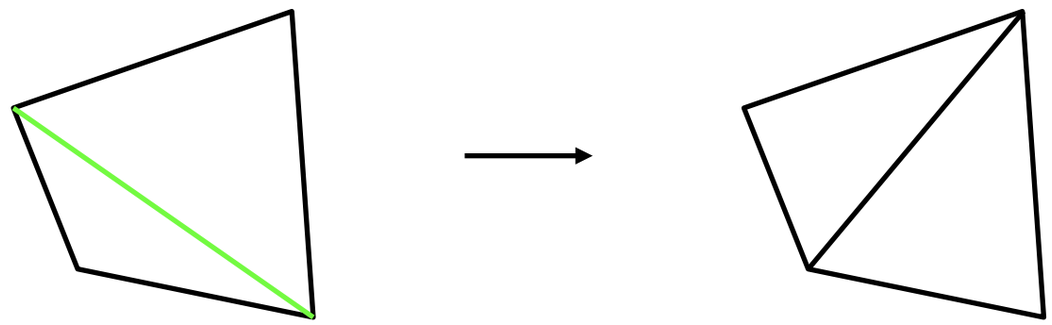
good



bad

Improving a Triangulation

In any convex quadrangle, an *edge flip* is possible. If this flip *improves* the triangulation locally, it also improves the global triangulation.

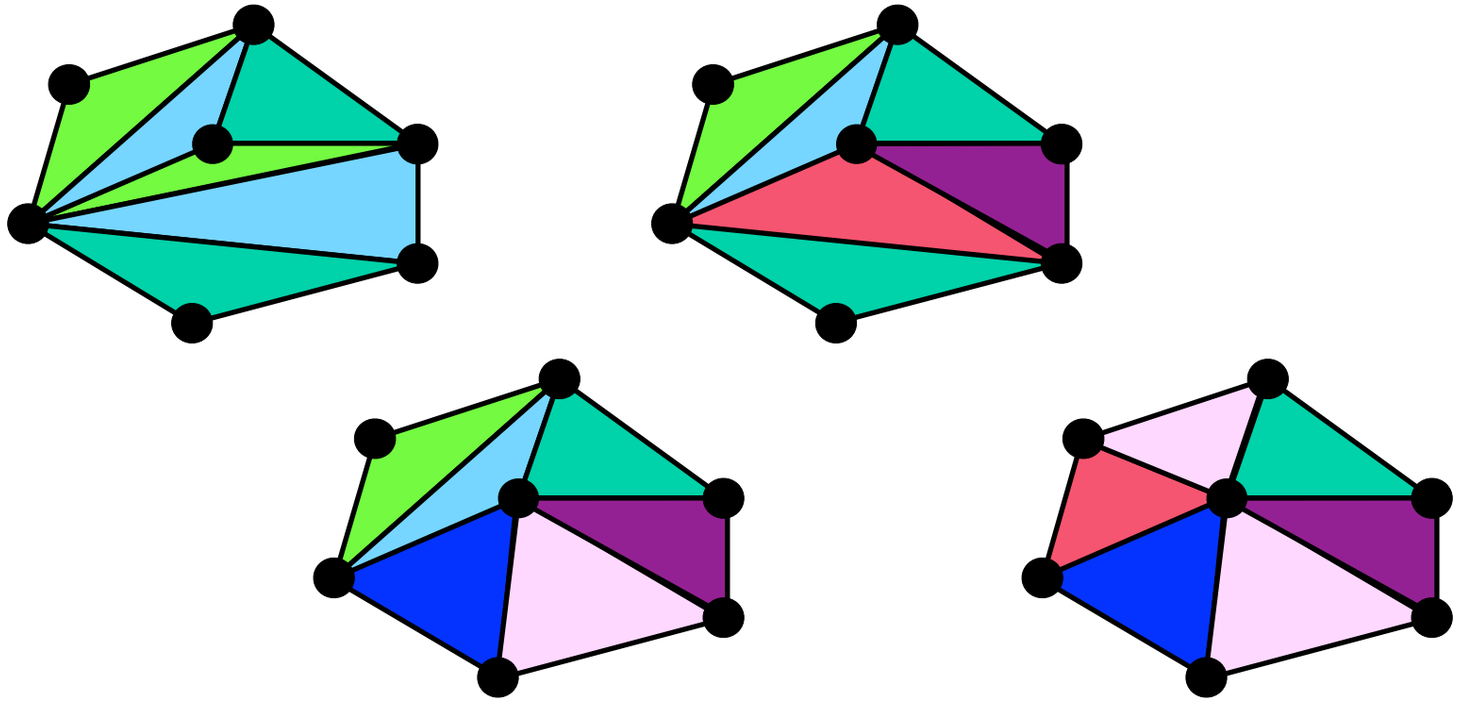


If an edge flip improves the triangulation, the first edge is called "*illegal*".

Naïve Delaunay Algorithm

Start with an arbitrary triangulation. Flip any illegal edge until no more exist.

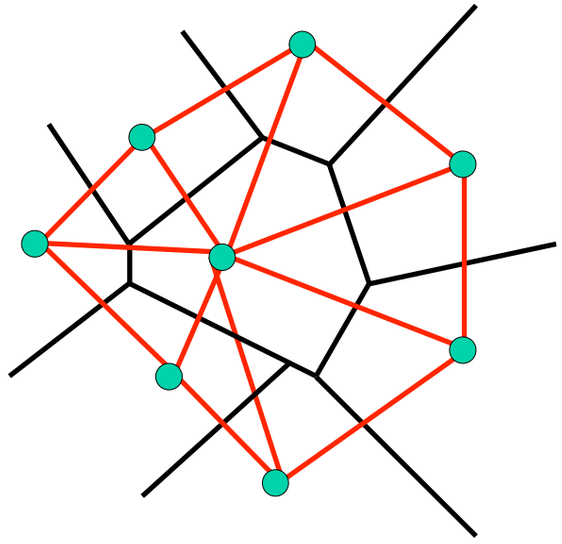
Could take a long time to terminate.



Delaunay Triangulation by Duality

Draw the dual to the Voronoi diagram by connecting each two neighboring sites in the Voronoi diagram.

- The DT may be constructed in O(*n*log*n*) time
- This is what Matlab's `delaunay` function uses



Demo: http://www.cs.cornell.edu/home/chew/Delaunay.html