from DFA to regular expression:

<u>principle</u>: the language recognized by a DFA is the set of all accepted inputs, starting in the <u>start state</u>.

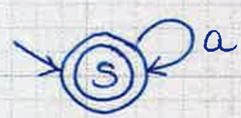
↳ we will express the recognized language as what is accepted from the start state and successively collapse the rest of the states onto the start state.

<u>examples</u>:

(1) 

the start state is final: ε is accepted.

we write $S = \varepsilon$.
this means $L = \{\varepsilon\}$ or ε.

(2) 

here the start state is final, hence ε is accepted.
however more strings are accepted because of the self loop a.
from S, we accept ε or 'a' followed by what is accepted from S:

we write it as:

$$S = \underbrace{aS}_{} + \varepsilon$$

'a' followed by anything that is accepted by S (a self-loop is recursion)

or

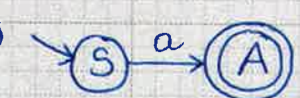
ε because S is final.

↳ we conclude that $S = a^*.\varepsilon = a^*$.

<u>Important</u>: this is generalized by the following only important rule:

$$\boxed{\begin{array}{l} \text{if } X = \alpha X + \beta \\ \text{then } X = \alpha^*.\beta \end{array}}$$

where X is any state
$\alpha \in \Sigma^*$
$\beta \in (\Sigma \cup Q)^*$

(3) 

leads to: $\begin{cases} S = aA \\ A = \varepsilon \end{cases}$

hence: $S = a.\varepsilon = a$.

(4) 

leads to: $\begin{cases} S = aA \\ A = bA + \varepsilon = b^* \end{cases}$

hence: $S = a.b^*$.

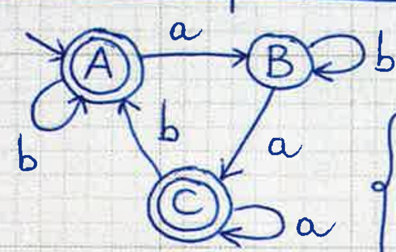
steps to follow:

(1) from DFA, derive equations (one equation per state).

(2) eliminate states from RHS and replace states in equation

(3) determine S = reg. expression = language of DFA.

more complex example:



$$\begin{cases} A = bA + \varepsilon + aB \ (*) \\ B = bB + aC \\ C = aC + \varepsilon + bA \end{cases}$$

(*) we keep this equation for last.

- $C = aC + (\varepsilon + bA) \rightarrow C = a^*(\varepsilon + bA)$

now we can replace C in B's equat°:

- $B = bB + aC = bB + a.a^*(\varepsilon + bA)$
  $= bB + (a^+(\varepsilon + bA))$
  $\rightarrow B = b^* a^+ (\varepsilon + bA)$

we replace B in A's equation:

- $A = bA + \varepsilon + ab^*a^+(\varepsilon + bA)$

we need to isolate A on the RHS:

$A = bA + \varepsilon + ab^*a^+ + ab^*a^+bA$
$= (b + ab^*a^+b)A + (\varepsilon + ab^*a^+)$
$= (b + ab^*a^+b)^*(\varepsilon + ab^*a^+)$.
$=$ language of above DFA.

also: $(b \cup ab^*a^+b)^* . (\varepsilon \cup ab^*a^+)$.