CS3350 – Automata

October 6, 2014 – 1 hour and 20 minutes

NAME: SOLUTION

Exam 1

GRADE: Total = / 100

1. / 30
2. / 50
3. / 20

Rules: This exam is to be done individually. Class notes and books of any kind are not allowed.
You must write legibly and in a structured manner: unreadable answers will be graded 0; messy / hard-to-follow answers will be penalized by up to 75%.

Cell phones should be turned off, no headsets allowed. Turn in your exam by 10:20am sharp: no exam will be accepted past this time.

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1 General Knowledge [30 points]

1.1 Consider the language L described by its regular expression: $(ab^+)^*ab$, where $\Sigma = \{a, b\}$. Give two strings that are members of L and two strings that are not members of L.

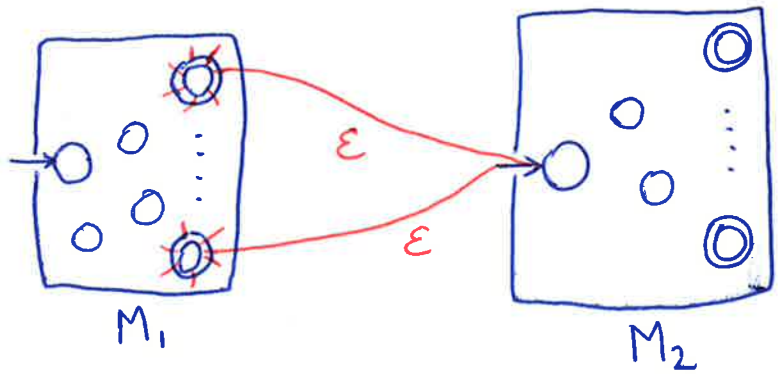
- Member: ab
- Member: abbab
- Not a member: ba
- Not a member: a

1.2 You are given a language L, that is regular. Please answer the following questions (circle your answers when appropriate):

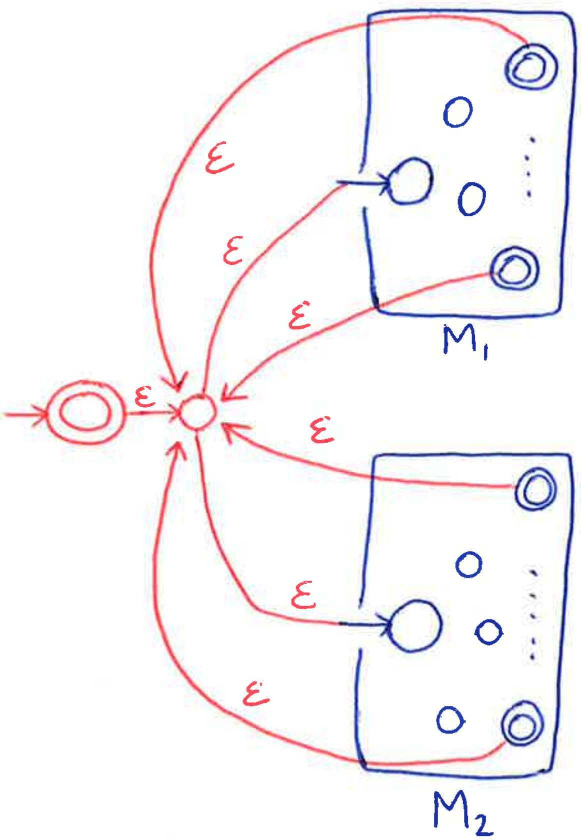
1. Is there an NFA that recognizes L? YES / NO
2. There is only one DFA that recognizes L. TRUE / FALSE

1.3 Apply the requested operators on the abstract finite automata below.

1.3.1 Build the automaton that recognizes $L = L(M_1) \cdot L(M_2)$

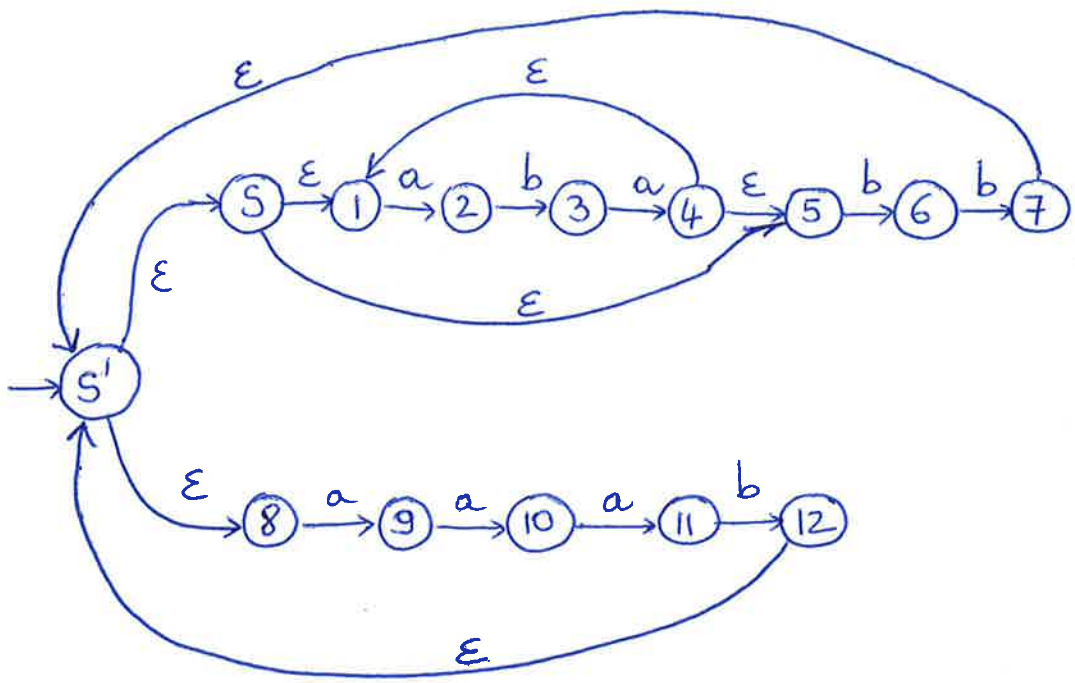


1.3.2 Build the automaton that recognizes $L = (L(M_1) \cup L(M_2))^*$

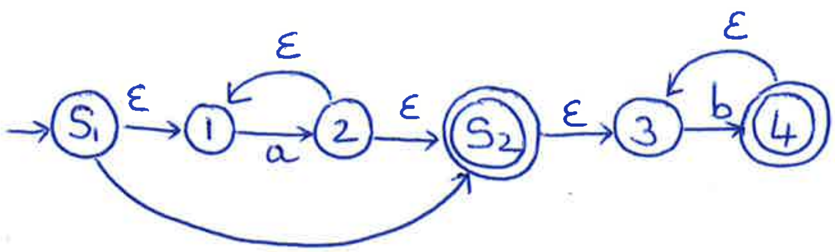


2 From Language to FA, and NFA to DFA [50 points]

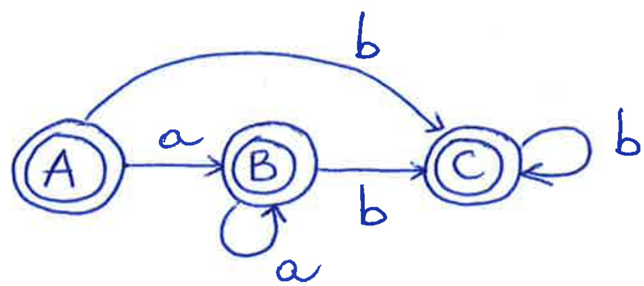
2.1 **Consider the following language $L = (((aba)^* \cdot bb) \cup aaab)^+$ over the alphabet $\Sigma = \{a, b\}$. Build an NFA for L from smaller automata, using the techniques presented in class.**



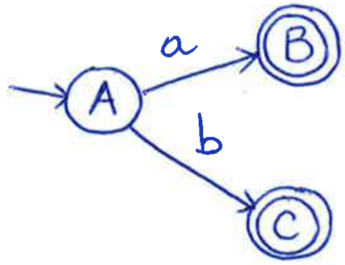
2.2 Consider the following NFA. Convert it into a DFA by following the procedure detailed in class.



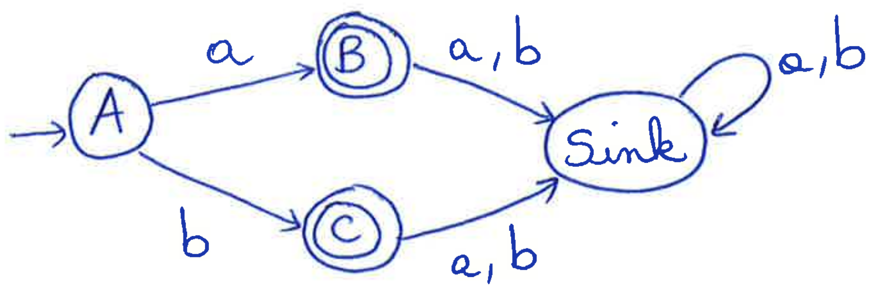
δ	a	b
$\{S_1, 1, S_2, 3\}$ A	$\{2, 1, S_2, 3\}$ B	$\{4, 3\}$ C
$\{2, 1, S_2, 3\}$ B	$\{2, 1, S_2, 3\}$ B	$\{4, 3\}$ C
$\{4, 3\}$ C		$\{4, 3\}$ C
	incomplete	



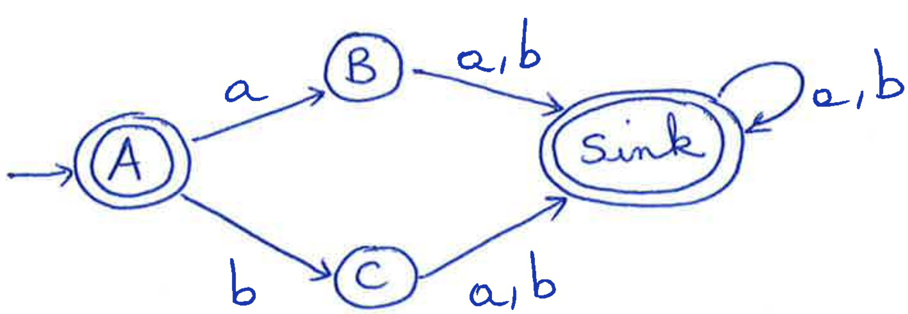
2.2.1 **Consider the following DFA that recognizes some language L. Produce a DFA for $\bar{L}$.**



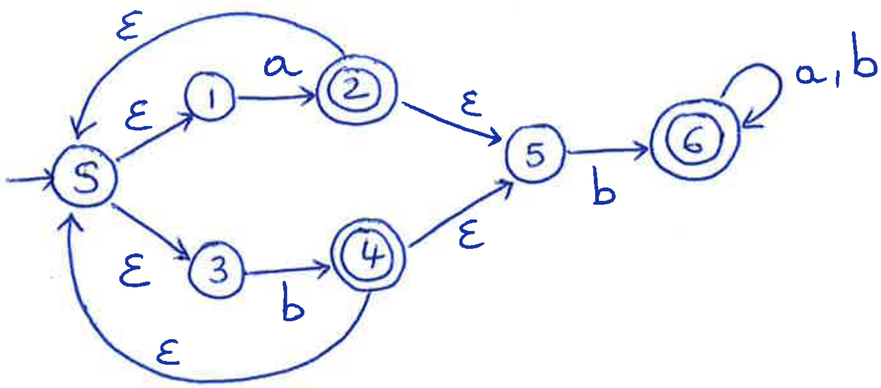
first, we need to complete this DFA:



now we can create its complement:

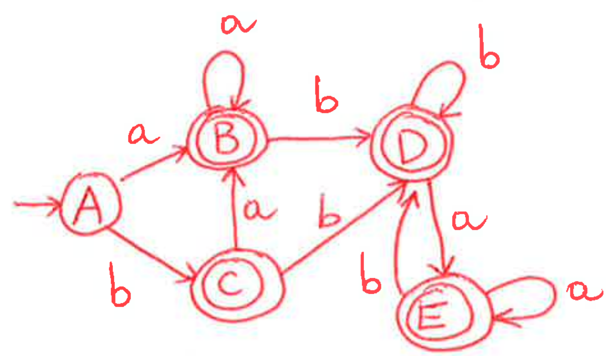


2.2.2 **Consider the following NFA that recognizes some language L. Produce a DFA for $\bar{L}$.**

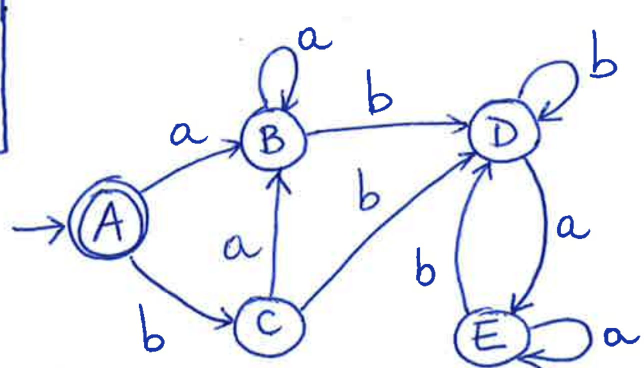


first : NFA → DFA

δ	a	b
{S,1,3} A	{2,S,1,3,5} B	{4,S,1,3,5} C
{2,S,1,3,5} B	{2,S,1,3,5} B	{4,S,1,3,5,6} D
{4,S,1,3,5} C	{2,S,1,3,5} B	{4,S,1,3,5,6} D
{4,S,1,3,5,6} D	{2,S,1,3,5,6} E	{4,S,1,3,5,6} D
{2,S,1,3,5,6} E	{2,S,1,3,5,6} E	{4,5,1,3,S,6} D



this DFA is complete, we can take its complement:



(note: it recognizes ε)

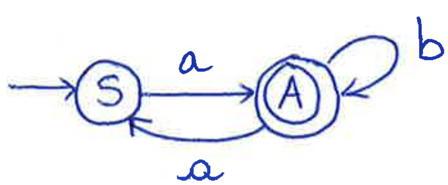
3 From DFA to Language [20 points]

For each of the following DFAs, identify the language they recognize, following the two step approach presented in class: (1) lay out the set of equations, (2) solve the equations using the one important simplification rule.

3.1 DFA 1.

(1). $S = \varepsilon + aS \overset{(2).}{=} a^* . \varepsilon = a^*$

3.2 DFA 2.



(1). $\begin{cases} S = aA \\ A = \varepsilon + bA + aS \end{cases} \xrightarrow{(2)} A = b^*(\varepsilon + aS)$

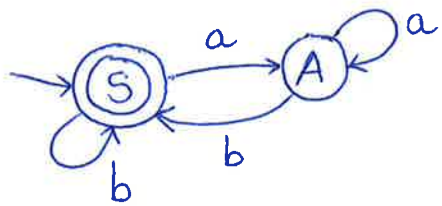
$= b^* + b^* a S.$

therefore: $S = a(b^* + b^* a S)$

$= ab^* + ab^* a S$

$= (ab^* a)^* ab^*.$

3.3 DFA 3.

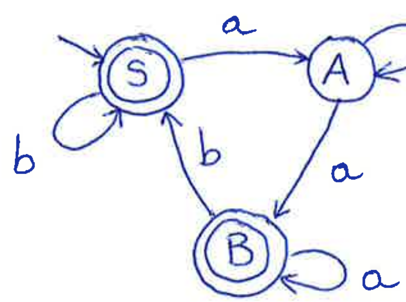


$$(1)\begin{cases} S = \varepsilon + aA + bS \\ A = aA + bS \end{cases} \xrightarrow{(2)} A = a^*bS$$

hence: $S = bS + aa^*bS + \varepsilon = (b + a^+b)S + \varepsilon$

$= (b + a^+b)^* . \varepsilon = (b + a^+b)^*.$

3.4 DFA 4.



$$(1)\begin{cases} S = \varepsilon + aA + bS \\ A = bA + aB \\ B = \varepsilon + aB + bS \end{cases} \xrightarrow{(2)} B = a^*(\varepsilon + bS) = a^* + a^*bS.$$

hence: $A = bA + a^+ + a^+bS$

$= b^*a^+ + b^*a^+bS$

and: $S = \varepsilon + ab^*a^+ + ab^*a^+bS + bS$

$= (ab^*a^+b + b)S + (\varepsilon + ab^*a^+)$

$= (ab^*a^+b + b)^* . (\varepsilon + ab^*a^+).$