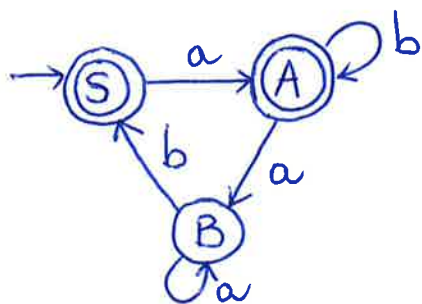


Question 1. [5 points] Use the technique presented in class to identify the language recognized by the following DFA.



$$\begin{cases} S = \epsilon + aA \\ A = \epsilon + aB + bA \\ B = aB + bS \end{cases}$$

$$B = aB + bS \Rightarrow B = a^*bS$$

$$A = \epsilon + aB + bA$$

$$= \epsilon + aa^*bS + bA$$

$$= bA + a^+bS + \epsilon \Rightarrow A = b^*(a^+bS + \epsilon)$$

$$S = \epsilon + aA$$

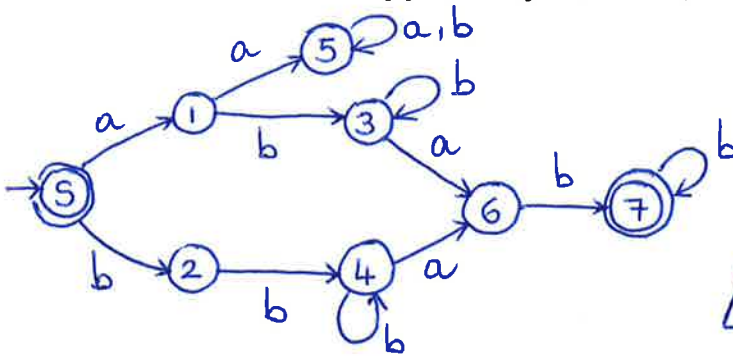
$$= \epsilon + ab^*(a^+bS + \epsilon)$$

$$= \epsilon + ab^*a^+bS + ab^*$$

$$\Rightarrow S = (ab^*a^+b)^* \cdot (\epsilon + ab^*)$$

= the language recognized by
 the automaton above.

Question 2. [5 points] Run the DFA minimization algorithm seen in class on the following DFA. * Important: Make sure that your answer is structured in such a way that it is clear which step you took first, second, etc.



initialization:

$$F = \{5, 7\}$$

$$NF = \{1, 2, 3, 4, 5, 6\}$$

⚠ this automaton is not complete: we add the sink explicitly to NF.

(1) NF

	a	b
1	NF	NF
2	NF	NF
3	NF	NF
4	NF	NF
5	NF	NF
6	NF	F
Sink	NF	NF

split into:
 $NF_1 = \{1, 2, 3, 4, 5, \text{Sink}\}$
 $NF_2 = \{6\}$ ✓

(2) NF₁

	a	b
1	NF ₁	NF ₁
2	NF ₁	NF ₁
3	NF ₂	NF ₁
4	NF ₂	NF ₁
5	NF ₁	NF ₁
Sink	NF ₁	NF ₁

split into:
 $NF_{11} = \{1, 2, 5, \text{Sink}\}$
 $NF_{12} = \{3, 4\}$

(6) NF₁₂

	a	b
3	NF ₂	NF ₁₂
4	NF ₂	NF ₁₂

✓

(7) F

	a	b
5	NF ₁₁₁	NF ₁₁₁
7	NF ₁₁₂	F

split into:
 $F_1 = \{5\}$
 $F_2 = \{7\}$.

(3) NF₁₁

	a	b
1	NF ₁₁	NF ₁₂
2	NF ₁₁	NF ₁₂
5	NF ₁₁	NF ₁₁
Sink	NF ₁₁	NF ₁₁

split into:
 $NF_{111} = \{1, 2\}$
 $NF_{112} = \{5, \text{Sink}\}$

(4) NF₁₁₁

	a	b
1	NF ₁₁₂	NF ₁₂
2	NF ₁₁₂	NF ₁₂

✓

(5) NF₁₁₂

	a	b
5	NF ₁₁₂	NF ₁₁₂
Sink	NF ₁₁₂	NF ₁₁₂

✓

