13 Material balance for oil reservoirs

This course is almost completed. Hopefully you will now have a reasonably understanding of the properties and behaviour of reservoir rocks, hydrocarbons fluids and fluid flow in the reservoir. At the very start of the course we also discussed how reservoirs are formed and throughout the notes we discussed several aspects of reservoir production, but not in great detail. In this chapter we will have a closer look at reservoir production.

The main questions we will try to answer in this chapter are:
- Under primary recovery conditions, what are the main drive mechanisms in the reservoir?
- How can we interpret and predict reservoir performance?

We do this using the material balance equation developed by Schilthuis in 1941 (well, well if that isn't yet another famous old Dutchie!). This material balance equation is still used widely for interpreting and prediction the performance of reservoirs.

13.1 General form of the material balance equation

We will study a layered reservoir: gas cap on top, oil + dissolved gas in the middle, water at the bottom (see figure 13-1a). When we start producing a reservoir, the pressure in the reservoir drops. In this primary recovery stage, hydrocarbons will "automatically" flow from the reservoir. What makes them do this? In other words, what are the prime drive mechanisms?

Drive mechanisms

As a result of the change in pressure, the oil, gas and water in the reservoir will expand. This expansion will push the hydrocarbons (and of course some of the water) out. Also, the rock may be compacted because of the overburden pressure. A decrease in pore volume will squeeze hydrocarbons out of the reservoir. Furthermore, if the reservoir is over an aquifer, aquifer water may encroach on the reservoir. So, the drive mechanisms are:

- Expansion of oil+dissolved gas
- Expansion of gas in the gascap, if we have any
- Expansion of the connate (irreducible) water in the reservoir
- Reduction of volume because of rock compaction
- Encroachment of aquifer water

All of these volume changes will "push" hydrocarbons (and perhaps water) out of the reservoir. We can write down a volume balance equation:

volume of the fluid withdrawn = expansion of oil + solution gas +

expansion of gascap gas +

expansion of connate water +

reduction in hydrocarbon pore volume due to decrease in porosity +

water influx into the reservoir

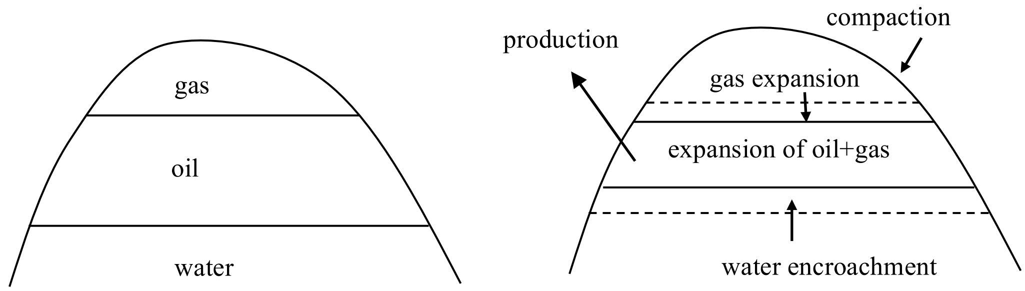

Figure 13-1 a: original reservoir; b: volume changes during production

We will write this material balance in a more formal manner. First we define the following variables

N — initial volume of oil in place at standard conditions in stock tank barrels
N is given by $N = \frac{V\phi(1 - S_{wi})}{B_{oi}}$, where B_{oi} is the initial value of B_o.

m — ratio of the initial hydrocarbon volume in the gas cap over the initial hydrocarbon volume of the oil in the reservoir (so m is constant)

N_p — cumulative oil production at standard conditions

R_p — cumulative gas oil ratio. This is the ratio of the cumulative gas production *over the cumulative oil production (both measured at standard conditions)*

From these definitions it follows that

mNB_{oi} — is the initial gas cap volume, and

NB_{oi} — is the oil+solution gas volume at reservoir conditions

Let's investigate the expansions that occur when the pressure in the reservoir is lowered from pi (the initial pressure) to p (the current pressure)

Expansion of oil + initial dissolved gas above bubble point

The expansion of the oil is $N(B_o\text{-}B_{oi})$, where Bo is the current formation volume factor of oil.

Expansion of the solution gas

Below bubble point pressure, the solution gas (the dissolved gas) will evolve from the oil.
The initial amount of solution gas is $N\ R_{si}$ in standard cubic feet, or $N\ R_{si}\ B_g$ in reservoir barrels. The amount still in solution at the lower pressure $N\ R_s\ B_g$. Therefore the total extra volume due to the free gas is $N\ B_g\ (R_{si} - R_s)$ res bbl.

Expansion of the gas cap

The initial volume of gascap gas is mNB_{oi} res bbl. At the current pressure, the volume is $mNB_{oi}\ B_g/B_{gi}$. Therefore the gas cap expansion is mNBoi(Bg/Bgi – 1).

Connate water expansion and rock compressibility (pore volume reduction)

The compressibility of the water is given by (see these course notes)

$$c_w = -\frac{1}{V_w}\frac{dV_w}{dp},$$

where c_w is the compressibility of the connate water, and V_w is the connate water volume. The total volume change due to connate water expansion can therefore be approximated by

$$c_w V_w \Delta p, \quad \Delta p = p_i - p,$$

The connate water volume is equal to the total pore volume V_f times the connate water saturation, or

$$V_w = S_{wi}\ V_f.$$

In a similar fashion we can express the change in volume because of the rock compressibility as

$$c_f V_f \Delta p,$$

where c_f is the pore compressibility.
V_f is the total pore volume of the reservoir. That means that V_f is equal to the total initial volume of the hydrocarbons (oil + gas cap) divided by the initial saturation of the hydrocarbons which is $1\text{-}S_{wi}$. Therefore

$$V_f = \frac{(1+m)NB_{oi}}{1-S_{wi}}.$$

Water influx

We model this as

$$(W_e - W_p)B_w,$$

where

W_e	is cumulative water influx from the aquifer into the reservoir in STB
W_p	is cumulative amount of aquifer water produced in STB
B_w	is the water formation volume factor in res bbl/.STB

The withdrawal of hydrocarbons

Suppose that we are producing N_p STB of oil and N_pR_p scf of gas at the surface. In the reservoir, the N_p STB of surface oil will occupy a volume of N_p B_o res bbl. We also know that in the reservoir we would have N_p R_s scf of dissolved gas. The remaining produced gas $N_p(R_p\text{-}R_s)$ will therefore be gas in the reservoir (evolved gas because of the pressure drop or gas in the gas cap). In reservoir barrels the gas volume is therefore $N_p(R_p\text{-}R_s)B_g$. Thus, the total withdrawal volume is

$$N_p\left(B_o + \left(R_p - R_s\right)B_g\right)$$

in reservoir barrels.

Now we can put it all together:

$$N_p\left(B_o + \left(R_p - R_s\right)B_g\right) = NB_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \left(R_{si} - R_s\right)B_g}{B_{oi}} + m\left(\frac{B_g}{B_{gi}} - 1\right) + \left(1 + m\right)\left(\frac{c_w S_{wi} + c_f}{1 - S_{wi}}\right)\Delta p\right] + \left(W_e - W_p\right)B_w \, .$$

Note that of course many of the parameters in the above equation depend on pressure (the formation volume factors and the gas-oil ratio).

Can you tell where each of the terms in this total material balance equation come from?

As noted, the terms depend on pressure. Determining a representative average reservoir pressure is not always easy.

Reservoir drive mechanisms

In the balance equation, we included several mechanisms that can drive hydrocarbons out of the reservoir. These are also called reservoir drive mechanisms*. Often, not all the drive mechanisms are equally important in a reservoir. If one or more of the mechanisms can be neglected, the balance equation simplifies of course. We will look at specific cases where the solution gas drive only, the gascap drive only, and the compaction drive are most important.*

13.2 Solution gas drive

In a solution gas drive reservoir the mechanism that drives the oil out is the expansion of the oil and its initial dissolved gas as the pressure in the reservoir drops (as when you are releasing an inflated balloon).
Of course the behaviour of the reservoir will be different above and below the bubble point pressure. Let's look at undersaturated oil first (above the bubble point pressure).

Above the bubble point

We assume there is no initial gas cap and that we can ignore the aquifer water influx.

The simplified material balance equation gives

$$N_p\left(B_o + \cancel{\left(R_p - R_s\right)B_g}\right) =$$
$$NB_{oi}\left[\frac{\left(B_o - B_{oi}\right) + \cancel{\left(R_{si} - R_s\right)}B_g}{B_{oi}} + \cancel{m\left(\frac{B_g}{B_{gi}} - 1\right)} + \left(1 + \cancel{m}\right)\left(\frac{c_w S_{wi} + c_f}{1 - S_{wi}}\right)\Delta p\right] + \cancel{\left(W_e - W_p\right)}B_w$$

or

$$N_p B_o = NB_{oi}\left[\frac{\left(B_o - B_{oi}\right)}{B_{oi}} + \left(\frac{c_w S_{wi} + c_f}{1 - S_{wi}}\right)\Delta p\right].$$

Do you think we could also ignore the term corresponding to connate water expansion and pore compression?

In terms of the oil compressibility we may also write this as

$$N_p B_o = NB_{oi}\left[c_o + \left(\frac{c_w S_{wi} + c_f}{1 - S_{wi}}\right)\Delta p\right].$$

Now there are only two fluids (oil and water) present in the reservoir so we can state that

$$S_o + S_{wi} = 0,$$

so that

$$N_p B_o = NB_{oi}\frac{c_o S_o + c_w S_{wi} + c_f}{S_o}\Delta p = NB_{oi} c_e \Delta p.$$

Here c_e is referred to as the effective compressibility. We write the equation in this form with the effective compressibility because it states now that

produced oil = original volume x compressibility x pressure drop

which is just like a normal compressibility equation. The "compressibility" here is ce, which is a combination of the oil, water and pore compressibility.

In this situation (above bubble point) do you expect the oil recovery to be high or low?

Below the bubble point

Below the bubble point, gas will evolve from the oil as the pressure drops. We know that as a first approximation the compressibility of the gas is 1/p. This compressibility is generally much higher than the connate water compressibility or the pore compressibility and so now these terms in the balance equation can be neglected. We get

$$N_p\left(B_o + (R_p - R_s)B_g\right) =$$
$$NB_{oi}\left[\frac{(B_o - B_{oi}) + (R_{si} - R_s)B_g}{B_{oi}} + \cancel{m\left(\frac{B_g}{B_{gi}} - 1\right)} + \cancel{(1+m)\left(\frac{c_w S_{wi} + c_f}{1 - S_{wi}}\right)\Delta p}\right] + \cancel{(W_e - W_p)B_w}$$

or

$$N_p\left(B_o + (R_p - R_s)B_g\right) = N\left((B_o - B_{oi}) + (R_{si} - R_s)B_g\right).$$

So: underground withdrawal = expansion of oil + expansion of originally dissolved gas.

Example for solution gas drive, unsaturated reservoir

Determine the oil recovery (N_p/N), during depletion down to bubble point pressure, for the reservoir whose PVT parameters are listed in the table and for which $c_w = 3.0 \times 10^{-6}$ / psi, cf = 8.6×10^{-6} / psi, $S_{wc} = 0.20$. Is this oil recovery high or low?

Example for solution gas drive, below bubble point pressure

We are now reducing the same reservoir to 900 psia. Using the data given, find an expression for the recovery at this pressure as a function of the cumulative gas oil ratio R_p. What do you conclude from this expression?
Find an expression for the free gas saturation in the reservoir at this pressure of 900 psia. There are two ways to do this. Method 1 is to consider the overall gas balance. Method 2 is to consider the change of the oil volume in the reservoir. Do both.

In what ways could you try to prevent large quantities of gas being produced?

The graph below gives an example production history of a solution gas drive reservoir.

Can you explain the difference in slope between the pressures curve above and below the bubble point?

13.3 Gas cap drive

Again we assume that the aquifer water influx is negligible, and that the effects of pore and connate water compressibility are minute compared to the gas expansion. Here we will also assume of course that initially the pressure is at bubble point (otherwise the gas and oil can not co-exist in equilibrium). We get

$$N_p\left(B_o + (R_p - R_s)B_g\right) =$$
$$NB_{oi}\left[\frac{(B_o - B_{oi}) + (R_{si} - R_s)B_g}{B_{oi}} + m\left(\frac{B_g}{B_{gi}} - 1\right) + \cancel{(1+m)\left(\frac{c_w S_{wi} + c_f}{1 - S_{wi}}\right)\Delta p}\right] + \cancel{(W_e - W_p)B_w}$$

or

$$N_p\left(B_o + (R_p - R_s)B_g\right) = NB_{oi}\left[\frac{(B_o - B_{oi}) + (R_{si} - R_s)B_g}{B_{oi}} + m\left(\frac{B_g}{B_{gi}} - 1\right)\right].$$

So, this equation has the general form P = N (E_o + m E_g).

For a gascap reservoir the least certain quantity is usually m. It is however often possible to get a reasonably good estimate of N.

How could you find an estimate for m in that case?

An example production history is given in the figure below.

Why is the pressure decline less dramatic?

What could cause the dips?

The oil recovery is generally greater than for a solution gas drive reservoir, typically in the order of 25-30%, depending on the size of the gascap.