Problem Bank 5: Second-Order and Higher Order Linear ODEs

Kreyszig Section No.s	Topics
2.1	**Homogeneous Linear ODEs of Second Order**
2.2	**Homogeneous Linear ODEs with Constant Coefficients**
2.5	**Euler-Cauchy Equations**
2.6	**Existence and Uniqueness of Solutions. Wronskian**
2.7	**Nonhomogeneous ODEs**

1 Constant coefficient ODEs

1. Find the general solutions of the following equations. Show the details of your calculation:

   (a) $y'' - 2y' - 3y = 0$

   (b) $y'' - 2y' + 5y = 0$

   (c) $y'' + 6y' + 13y = 0$

   (d) $4\frac{d^2x}{dt^2} - 20\frac{dx}{dt} + 25x = 0$

2. Solve the ODE with the given initial values:

   (a) $y'' - 4y' + 3y = 0,\ y(0) = 6, y'(0) = 10$

   (b) $4y'' + 4y' + y = 0,\ y(0) = 2, y'(0) = 0$

   (c) $y'' - 3y' - 4y = 0,\ y(0) = 0, y'(0) = -5$

   (d) $y'' + 4y' + 29y = 0,\ y(0) = 0, y'(0) = 15$

   (e) $y'' - 4y' + 13y = 0,\ y(0) = 0, y'(0) = 3$

3. Find a 2-order ODE, the solutions of which contain $1, e^x, 2e^x$.

2 Equations with non-constant coefficients

Find the general solutions of the following equations. Show the details of your calculation.

1. $y'' - \frac{y'}{x} + \frac{y}{x^2} = \frac{2}{x}$

2. $x^2y'' - xy' + 4y = x \sin lnx$

3. Prove that $y_1 = e^{x^2}$ and $y_2 = xe^{x^2}$ are solutions of the ODE $y'' - 4xy' + (4x^2 - 2)y = 0$, and write the general solution. Solve it with the initial conditions $y(1) = 1$ and $y'(1) = 0$.

4. Solve the ODE $(2x-1)y''-(2x+1)y'+2y=0$ in the general case (no initial value), knowing that $y_1(x)=e^x$ is a solution.

5. Solve the ODE $x^2y''-2xy'+2y=0$ in the general case (no initial value), knowing that $y_1(x)=x$ is a solution.

3 Nonhomogeneous ODEs

1. Solve the following ODEs:
   (a) $y''+3y'+2y=10e^{-3x}$
   (b) $y''+5y'+4y=x^2$
   (c) $y''+4y'+4y=\cos x$
   (d) $y''-5y'+6y=xe^{2x}$
   (e) $y''+y=x\cos 2x$
   (f) $y''-y=e^x\cos 2x$

2. Solve the ODE with the given initial values:
   (a) $y''+4y=x^2, y(0)=1, y'(0)=0$
   (b) $y''-2y'=6e^{2x}-4e^{-2x}, y(0)=-1, y'(0)=6$
   (c) $y''+2y'+0.75y=2\cos x-0.25\sin x+0.09x, y(0)=2.78, y'(0)=-0.43$

4 Higher order ODE

1. $y^{(4)}-2y^{(3)}+5y''=0$

2. $y^{(4)}+y=0$

3. $y'''=e^{2x}-\cos x$

4. $y'''+3y''+3y'+y=0$