Objectives

- Understand what SSA form is and how to convert a program into SSA
- Define dominance frontiers and an efficient algorithm for computing DFs
- Use DFs to efficiently covert a program into SSA form
- Understand the notion of control-dependence and how to compute it

Static Single Assignment (SSA) Form

References

- Ron Cytron, Jeanne Ferrante, Barry K. Rosen, Mark N. Wegman, and F. Kenneth Zadeck, "Efficiently Computing Static Single Assignment Form and the Control Dependence Graph," *ACM Trans. on Programming Languages and Systems*, 13(4), Oct. 1991, pp. 451–490.
- Muchnick, Section 8.11 (*partially covered*).

What is SSA?

- Informally, a program can be converted into *SSA form* as follows:
  - Each assignment to a variable is given a unique name
  - All of the uses reached by that assignment are renamed.
- Easy for straight-line code:

$$V \leftarrow 4$$

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$V \leftarrow 4$	$V_0 \leftarrow 4$
$\leftarrow V + 5$	$\leftarrow V_0 + 5$
$V \leftarrow 6$	$V_1 \leftarrow 6$
$\leftarrow V + 7$	$\leftarrow V_1 + 7$

Static Single Assignment with Control Flow

- *2-way branch*:

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if (...)
        X = 5;
else
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- *While loop*:

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j=1;
while (j < X)
    ++j;
N = j;
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```
      j = 1;
      if (j ≥ X)
          goto E;
S:
      j = j+1;
      if (j < X)
          goto S;
E:
      N = j;
```

Static Single Assignment with Control Flow

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      j = j+1;
      if (j < X)
          goto S;
E:
      N = j;
```

```
      j0 = 1;
      if (j0 < X0)
          goto E;
S:
      j1 = φ(j0, j2);
      j2 = j1+1;
      if (j2 < X0)
          goto S;
E:
      j3 = φ(j0, j2);
      N0 = j3;
```

Review of Some Basic Terms

- **Value**:
- **Storage location** (register or memory)
- **Variable**:
- **Pointer**:
- **Alias**:
- **Reference to a variable**:

- **Use of a variable**: A use of variable X is a reference that *may read* the value stored in the location named X.
- **Definition of a variable**: A definition (def) of a variable X is a reference that *may store* a value into the location named X.

  Examples: Assignment; FOR; input I/O

- **Ambiguity**:

Unambiguous def : *guaranteed* to store to X — *must*

Ambiguous def : *may* store to X — *may*

Similarly, ambiguous/unambiguous use.

Q. Where does ambiguity come from?

Definition of SSA Form

- **Definition (ϕ Functions)**:
  In a basic block B with N predecessors, $P_1,\ P_2,\ \ldots,\ P_N$,

  $$X = \phi(V_1, V_2, \ldots, V_N)$$

  assigns $X = V_j$ if control enters block B from P_j, $1 \leq j \leq N$.

- *Properties of ϕ-functions*:
  - ϕ is not an executable operation.
  - ϕ has exactly as many arguments as the number of incoming BB edges
  - Think about ϕ argument V_i as being evaluated on CFG edge from predecessor P_i to B

- **Definition (SSA form)**:
  A program is in SSA form if:
  1. each variable is assigned a value in exactly one statement
  2. each use of a variable is *dominated* by the definition

Which Variables Can We Convert?

Which of these can we convert? And when?

Scalars?

Arrays?

Structures?

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General Criteria

Convert all variables to SSA form, *except* . . .

- *Arrays*: Array elements do not have an explicit name
- *Variables that may have aliases*: do not have a unique name
- *Volatile variables*: can be modified "unexpectedly"

E.g., In LLVM, only variables in virtual registers are in SSA form.

Advantages of SSA

- Explicit def-use and use-def chains:

  Def-use chain: The set of uses reached by a particular definition.

  Use-def chain: The set of defs reaching a particular use.

  These are foundation of many dataflow optimizations.

  $\Longrightarrow$ Specifically **enables sparse optimizations.**
- Compact, *flow-sensitive* def-use information
  $\Longleftarrow$ fewer def-use edges per variable: one per CFG edge
- No anti- and output dependences on SSA variables
- Explicit merging of values (ϕ): key additional information
- Can serve as IR for code transformations

Disadvantages of SSA

- Size of SSA program is O(N^2) for an ordinary program with N variables.
- Usually not used for structures and arrays
- May not be used for scalar variables with aliases
- *If used as IR*: Must be converted back to code (*Not too bad*)
- *Otherwise*: Must be recomputed frequently (*Often bad*)

Constructing *SSA* Form

Simple method

1. insert ϕ-functions for every variable at every join
2. solve REACHING DEFINITIONS
3. rename each use to the def that reaches it (*unique*)

What's wrong with this approach

1. too many ϕ-functions (*precision*)
2. too many ϕ-functions (*space*)
3. too many ϕ-functions (*time*)

Might be good enough for some transformations

Everything else is an optimization

The *SSA*-Construction Algorithm: Intuition

Key question: Where do we place ϕ-functions?

- **Example:**

```
1:  if (...) then {
2:      V = ...;
3:      if (...)  {
4:          U = V + 1;    // No phi for V needed here
5:      } else {
6:          U = V + 2;    // No phi for V needed here
7:      }
8:      W = U + 1;        // No phi for V needed here
9:  }
10: ...                   // phi for V _is_ needed here
```

The SSA-Construction Algorithm: Intuition(cont)

Informal Conditions:

If node X contains an assignment to a variable V, then a ϕ must be inserted in each node Z such that:

1. there is a non-null path X $\rightarrow^+$ Z, and
2. there is a path from ENTRY to Z that does not go through X,
3. Z is the first node on the path X $\rightarrow^+$ Z that satisfies (2).

The Dominance Frontier *of the node X is exactly the set of nodes Z that satisfy the above condition!*

Intuition for Placement Conditions:

(1) $\Longrightarrow$ the value of V computed in X reaches Z

(2) $\Longrightarrow$ there is a path that does not go through X, so some other value of V reaches Z along that path (ignore bugs due to uses of uninitialized variables). So, two values must be merged at X with a ϕ.

(3) $\Longrightarrow$ The ϕ for the value coming from X is placed in Z and not in some earlier node on the path X $\rightarrow^+$ Z.

The *SSA*-Construction Algorithm: Intuition (cont)

Iterating the Placement Conditions:

- After a ϕ is inserted at Z, the above process must be repeated for Z because the ϕ is effectively a new definition of V.
- For each node X and variable V, there must be at most one ϕ for V in X. This means that the above iterative process can be done with a single worklist of nodes for each variable V, initialized to handle all original assignment nodes X simultaneously.

Flavors of SSA:

Minimal SSA : As few as possible, subject to above condition

Pruned SSA : As few as possible, subject to above condition *and* no dead ϕ-functions

Dominance Frontiers

Dominance: *Recall definitions of:*

1. X *dominates* Y, or X *dom* Y or X $\underline{\gg}$ Y
2. X *strictly dominates* Y, or X $\gg$ Y
3. X is the *immediate dominator* of Y, or X = *idom*(Y)
4. *Dominator tree*

Dominance frontiers:

The *dominance frontier* of node X is the set of nodes Y such that X dominates a predecessor of Y, but X does not strictly dominate Y.

DF(X) = {Y | $\exists$ p $\in$ Pred(Y) : X $\underline{\gg}$ p and X $\not\gg$ Y}

The *dominance frontier* can be subdivided into two components:

DF_{local}(X) $\equiv$ {Y $\in$ Succ(X) | X $\not\gg$ Y}

DF_{up}(Z) $\equiv$ {Y $\in$ DF(Z) | *idom*(Z) $\not\gg$ Y}

Then,

$$DF(X) = DF_{local}(X) \bigcup \bigcup_{Z \in Children(X)} DF_{up}(Z)$$

Computing Dominance Frontiers

Algorithm for computing dominance frontiers

for each X in a bottom-up traversal of the dominator tree
    DF(X) $\leftarrow \emptyset$
    for each Y $\in$ succ(X) /* *local* */
        if *idom*(Y) $\neq$ X then
            DF(X) $\leftarrow$ DF(X) $\bigcup$ {Y}
    for each Z $\in$ children(X) /* *up* */
        for each Y $\in$ DF(Z)
            if *idom*(Y) $\neq$ X then
                DF(X) $\leftarrow$ DF(X) $\bigcup$ {Y}

Theorem 1

The dominance frontier algorithm is correct.

Refer to paper for proof.

Iterated Dominance Frontiers

Dominance frontiers for a set of nodes

Extend the dominance frontier mapping from nodes to sets of nodes:

$DF(\mathcal{L}) = \bigcup_{X \in \mathcal{L}} DF(X)$

The *iterated* dominance frontier $DF^+(\mathcal{L})$ is the limit of the sequence:

$DF_1 = DF(\mathcal{L})$

$DF_{i+1} = DF(\mathcal{L} \bigcup DF_i)$

Theorem 2

The set of nodes that need ϕ-functions for any variable V is the iterated dominance frontier $DF^+(\mathcal{L})$, where $\mathcal{L}$ is the set of nodes that may modify V.

Refer to paper for proof.

Computing Static Single Assignment Form

Complete algorithm

Compute the dominance frontiers

Insert ϕ-functions

Rename the variables

Theorem 3

Any program can be put into minimal SSA form using this algorithm.

Refer to paper for proof.

Computing Static Single Assignment Form

Inserting ϕ-functions

```
for each variable V
    HasAlready ← ∅
    EverOnWorkList ← ∅
    WorkList ← ∅
    for each node X that may modify V
        EverOnWorkList ← EverOnWorkList ⋃ {X}
        WorkList ← WorkList ⋃ {X}

    while WorkList ≠ ∅
        remove X from W
        for each Y ∈ DF(X)
            if Y ∉ HasAlready then
                insert a φ-node for V at Y
                HasAlready ← HasAlready ⋃ {Y}
                if Y ∉ EverOnWorkList then
                    EverOnWorkList ← EverOnWorkList ⋃ {Y}
                    WorkList ← WorkList ⋃ {Y}
```

Renaming the Variables

Intuition:

Renaming defs is easy. To rename each use of V:

(a) *Use in a non-ϕ statement*: Use immediately dominating definition of V (+ ϕ nodes inserted for V).

$\Longrightarrow$ *preorder on Dominator Tree!*

(b) *Use in a ϕ operand*: Use definition that immediately dominates incoming CFG edge (not ϕ)

$\Longrightarrow$ rename the ϕ operand when processing the predecessor basic block!

Data Structures:

Stacks - an array of stacks, one for each original variable V

Stacks[V] = subscript of most recent definition of V. Initially, Stacks[V] = EmptyStack, $\forall$ V

Counters - an array of counters, one for each original variable

Counters[V] = number of assignments to V processed, Initially. Counters[V] = 0, $\forall$ V

procedure GenName(Variable V)

1. i $\leftarrow$ Counters[V]
2. replace V by V_i
3. push i onto Stacks[V]
4. Counters[V] $\leftarrow$ i + 1

Computing Static Single Assignment Form

The Renaming Algorithm Initially, call Rename(**Entry**)

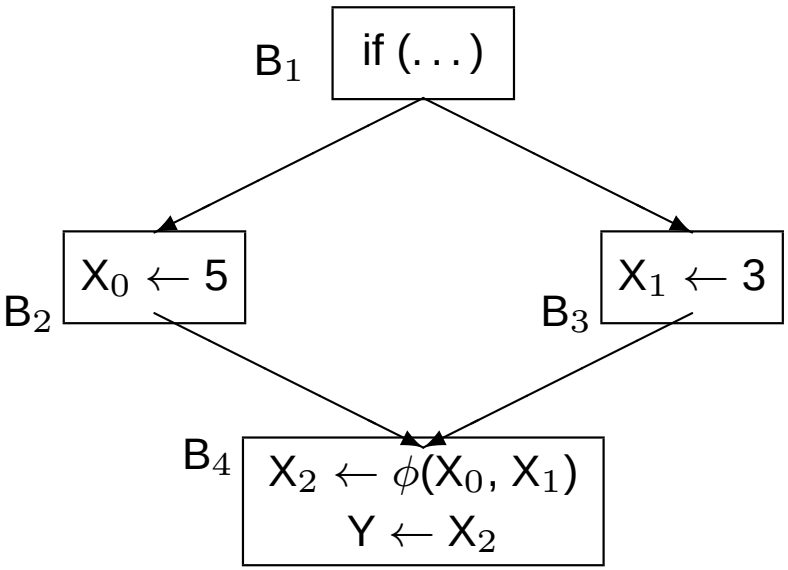
```
procedure Rename(Block X)
    for each φ-node P in X
        GenName(LHS(P))
    for each statement A in X
        for each variable V ∈ RHS(A)
            replace V by V_i, where i = Top(Stacks[V])
        for each variable V ∈ LHS(A)
            GenName(V)
    for each Y ∈ Succ(X)
        j ← position in Y's φ-functions corresponding to X
        for each φ-node P in Y
            replace the j^th operand of RHS(P) by V_i
                where i = Top(Stacks[V])
    for each Y ∈ Children(X)
        Rename(Y)
    for each φ-node or statement A in X
        for each V_i ∈ LHS(A)
            pop Stacks[V]
```

Translating out of Static Single Assignment form

Overview:

1. Dead-code elimination (prune dead ϕs)
2. Replace ϕ-functions with copies in predecessors
3. Register allocation with copy coalescing

Before (2)



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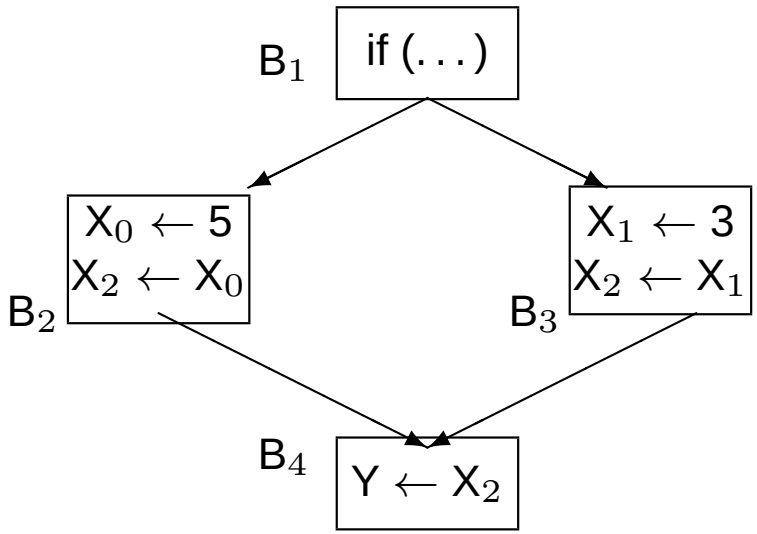

Control Dependence

Definition: In a CFG, node Y is *control-dependent* on node B if

1. There is a non-empty path $N_0 = B, N_1, N_2, ..., N_k = Y$ such that Y postdominates $N_1 \ldots N_k$, *and*
2. Y does not strictly postdominate B

Definition: The *Reverse Control Flow Graph* (RCFG) of a CFG has the same nodes as CFG and has edge $Y \rightarrow X$ if $X \rightarrow Y$ is an edge in CFG.

Control Dependence (continued)

Computing Control Dependence

Key observation: Node Y is control-dependent on B *iff* $B \in DF(Y)$ in RCFG.

Algorithm:

1. Build RCFG
2. Build dominator tree for RCFG
3. Compute dominance frontiers for RCFG
4. Compute $CD(B) \equiv \{Y : B \in DF(Y)\}$.
   $CD(B)$ gives the nodes that are control-dependent on B.