Reliability: Bounds and Life time

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Outline

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Reliability of System

Reliability of system

- $P\{X_i = 1\} = p_i = 1 - P\{X_i = 0\}$
  p_i is reliability of i^{th} component
- Reliability of system
  $r = P\{\phi(\mathbf{X}) = 1\}$ where $\mathbf{X} = (X_1, X, ..., X)$
- If components, RVs X_i are independent then,
  $r = r(\mathbf{p})$, where $\mathbf{p} = (p_1, p_2, ..., p_n)$
- Example- Series, parallel, k-out-of-n system

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Bounds on Reliability Funtion

Bounds

- Consider a bridge system
  - $\phi(\mathbf{x}) = max(x_1, x_2)\, max(x_2, x_3, x_4)\, max(x_1, x_3, x_5)\, max(x_4, x_5)$ expand in terms of x_is (Cut-Sets)
  - $\phi(\mathbf{x}) = min(x_1, x_4) + min(x_2, x_3, x_4)\, min(x_1, x_3, x_5)\, min(x_2, x_5)$ expand in terms of x_is (Min. Paths)
- $r = P\{\phi(\mathbf{X}) = 1\}$ and as X_i is Bernoulli RV with $\{0, 1\}$

$$
\begin{aligned}
r &= P\{\phi(\mathbf{X}) = 1\} \times 1 + P\{\phi(\mathbf{X}) = 0\} \times 0 \\
&= E\{\phi(\mathbf{X}) = 1\}
\end{aligned}
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Bounds

- $E\{AB\} \neq E\{A\}.E\{B\}$, if A and B are not independent
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  - $\phi(\mathbf{x}) = 1 - (1 - x_1x_4)(1 - x_1x_3x_5)(1 - x_2x_5)(1 - x_2x_3x_4)$ expand in terms of x_is (Min. Paths)
- $r(\mathbf{p}) = 1 - E\left[(1 - X_1X_4)(1 - X_1X_3X_5)(1 - X_2X_5)(1 - X_2X_3X_4)\right]$
- Minimal paths overlap, hence probabilities are NOT computed as variables are independent, rather expression is expanded & then Expectation computed

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Bounds on Reliability Funtion

Bounds

- Method of inclusion & Exclusion
- Second Method

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Outline

1. Reliability of System of Components
   - System of Components
2. Computing bounds on reliability
   - Method of inclusion exclusion
   - Second Method
3. System life time computation

Method of inclusion and exclusion

Bounds method-1

- Consider a probability of union of events- $E_1, E_2, E_3, \ldots E_n$

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum P(E_i) - \sum_{i<j}\sum P(E_iE_j) + \sum \sum_{i<j<k} \sum P(E_iE_jE_k) - \ldots + (-1)^{n+1} P(E_i$$

-
$$P\left(\bigcup_{i=1}^{n} E_i\right) \leq \sum P(E_i)$$

-
$$\left(\bigcup_{i=1}^{n} E_i\right) \geq \sum P(E_i) - \sum_{i<j}\sum P(E_iE_j)$$

$$P\left(\bigcup_{i=1}^{n} E_i\right) \leq \sum P(E_i) - \sum\sum P(E_iE_j) + \sum \sum_{i<j<k} \sum P(E_iE_jE_k)$$

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Inclusion-Exclusion method ...

Bounds, method-1

- Lets define indicator variables, $I_j \; j = 1, 2, \ldots n$

  $I_j = \begin{cases} 1 & \text{if } E_j \text{ occurs} \\ 0 & \text{otherwise} \end{cases}$ & let $N = \sum_{j=1}^{n} I_j$ and

  $I = \begin{cases} 1 & \text{if } N > 0 \\ 0 & \text{if } N = 0 \end{cases}$

  then,

$$1 - I = (1-1)^N = \sum_{i=0}^{N} C_i^N (-1)^i$$

- hence, $I = N - {}^N C_2 + {}^N C_3 - {}^N C_4 \ldots \pm - {}^N C_N$; also ${}^N C_i - {}^N C_{i+1} + {}^N C_N = {}^{N-1} C_{i-1} \geq 0$

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Inclusion & exclusion ...

Bounds, method-1

- $I \leq N$
  $I \geq N - {}^N C_2$
  $I \leq N - {}^N C_2 + {}^N C_3$
- Taking expectation on both sides, LHS

$$E[I] = P\{N > 0\} = P\{at\,least\,one\,of\,E_j\,occurs\}$$

$$= P\left(\bigcup_1^n E_j\right)$$

Inclusion & exclusion ...

Bounds, method-1

- Expectation, RHS

$$E[N] = E\left[\sum I_j\right] = \sum P(E_j)$$

$$E\left[{}^N C_2\right] = E[number\ of\ pairs\ that\ occur] = E\left[\sum\sum I_i I_j\right] = \sum\sum P(E_i E_j) \ldots$$

$$E\left[{}^N C_3\right] = E[number\ of\ triplet\ that\ occur] = E\left[\sum\sum I_i I_j I_k\right] = \sum\sum P(E_i E_j E_k) \ldots$$

Inclusion & exclusion ...

Bounds, method-1

- For minimal path formulation, let $A_1, A_2, \ldots A_s$ denote minimal path sets of ϕ, and $E_i = \{all\, components\, in\, A_i\, function\}$
  - Since, system functions iff at least one of E_i occurs,
- $r(\mathbf{p}) = (\bigcup_1^s E_i)$ to be computed using inequalities (bounds) following these terms pre-computed
- Pre-computed terms
  $P(E_i) = \prod_{\ell \in A_i} p_\ell$
  $P(E_i E_j) = \prod_{\ell \in A_i \cup A_j} p_\ell$
  $P(E_i E_j E_k) = \prod_{\ell \in A_i \cup A_j \cup A_k} p_\ell$

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- For minimal cut-set formulation, let $C_1, C_2, \ldots C_r$ denote cut sets of ϕ, and $F_i = \{all\, components\, in\, C\, fail\}$
- Since, system fails iff all components in at least one of C_i are failed,
- $1 - r(\mathbf{p}) = (\bigcup_1^r F)$ to be computed using inequalities (bounds) following these terms $P()$ pre-computed
  $1 - r(\mathbf{p}) \leq \sum P(F_i)$
  $1 - r(\mathbf{p}) \geq \sum P(F_i) - \sum\sum P(F_i F_j)$
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Inclusion-Exclusion method ...

Bounds, method-1

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  $P(F_i) = \prod_{l \in C_i}(1 - p_l)$

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Outline

Second Method of Obtaining bounds

Second Method

- Let's Consider minimal path sets, A_i
  - Let's define D_i as $D_i = \{\textit{at least one component in } A_i \textit{ has failed}\}$

$$1 - r(\mathbf{p}) = P(D_1 D_2 \ldots D_s)$$
$$\geq \prod_i P(D_i)$$

- Let's consider minimal cut-sets, C_i
  - Let's define U_i as $U_i = \{\textit{at least one component in } C \textit{ is functioning}\}$
  - $r(\mathbf{p}) = P(U_1, U_2, \ldots U_r)$
  - $r(\mathbf{p}) \geq \prod_i P(U_i)$

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Second method

Bounds, method-2

- Express desired probability as intersection of events
- For minimal path $A_1, A_2, \ldots A_s$ of ϕ, define events $D_i = \{at\ least\ one\ components\ in\ A_i\ fails\}$
- Since, system fails iff at least one component in each minimal path has failed, $1 - r(\mathbf{p}) = P(D_1, D_2, \ldots D_s) = P(D_1)P(D_2|D_1)\ldots P(D_s|D_1 D_2 .. D_{s-1})$

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Second method

Bounds, method-2

- Intuitively, $P(D_2|D_1) \geq P(D_2)$
- Let,

$$P(D_2) = P(D_2|D_1)P(D_1) + P(D_2|D_1^c)(1 - P(D_1))$$

$$P(D_2|D_1^c) = \{\textit{at least one failed in } A_2 | \textit{all functions in } A_1\}$$

$$= 1 - \prod_{\substack{j \in A_2 \\ j \notin A_1}} p_j$$

$$\leq 1 - \prod_{j \in A_2} p_j = P(D_2)$$

Second method

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Second method

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- Hence,

$$
\begin{aligned}
P(D_2) &\leq P(D_2|D_1)P(D_1) + P(D_2)(1 - P(D_1)) \\
\Rightarrow P\left(D_2|D_1\right) &\geq P(D_2) \\
also\ \Rightarrow P\left(D_i|D_1 D_2, \ldots D_{i-1}\right) &\geq P(D_i)
\end{aligned}
$$

Second method...

Bounds, method-2

- Hence,

$$1 - r(\mathbf{p}) = P(D_1, D_2, ...D_s) = P(D_1)P(D_2|D_1)...P(D_s|D_1 D_2..D_{s-1})$$
$$\geq \prod P(D_i)$$

- equivalently,

$$r(\mathbf{p}) \leq 1 - \prod_i \left(1 - \prod_{j \in A_i} p_j\right)$$

Second method...

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- Express desired probability as intersection of events
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$$\geq \prod P(U_i)$$
$$\geq \prod_i \left[1 - \prod_{j \in C_i} (1 - p_j)\right]$$

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$$\geq \prod P(U_i)$$

$$\geq \prod_i \left[1 - \prod_{j \in C_i} (1 - p_j)\right]$$

Second method

Bounds, method-2

- We have following bounds for reliability

$$\prod_i \left[1 - \prod_{j \in C_i} (1 - p_j)\right] \leq r(\mathbf{p}) \leq 1 - \prod_i \left(1 - \prod_{j \in A_i} p_j\right)$$

System Life in terms of component life

System life time

- For a random variable having distribution function G, lets define $\bar{G}(a) = 1 - G(a)$ to be probability that random variable is greater than a
- Let i^{th}component be having F_i as function for length of time
  - It functions for a random length of time & then fails, and remains so forever
- A system will be functioning for time t or greater, if it is functioning at t

System Life in terms of component life ...

System life time

- Let F denote distribution of life time of system
  - $$\begin{aligned} \bar{F}(t) &= P\{system > t\} \\ &= P\{system\, is\, functioning\, at\, time\, t\} \end{aligned}$$
  - $P\{system\, is\, functioning\, at\, time\, t\} = r(P_1(t), \ldots, P_n(t))$
  - $$\begin{aligned} P_i(t) &= P\{component\, i\;\, is\, functioning\, at\, time\, t\} \\ &= P\{lifetime\, of\, i > t\} = \bar{F}_i(t) \end{aligned}$$

System Life in terms of component life ...

System life time

- Hence,
  - $\bar{F}(t) = r\left(\bar{F}_1(t), \ldots, \bar{F}_n(t)\right)$

Example- System Life

System life time

- Life time of Series system
  - $\bar{F}(t) = \prod_{i=1}^{n} \bar{F}_i(t)$
- Life time of parallel system
  - $\bar{F}(t) = 1 - \prod_{i=1}^{n} \bar{F}_i(t)$

Example- System Life

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  - $\bar{F}(t) = \prod_{i=1}^{n} \bar{F}_i(t)$
- Life time of parallel system
  - $\bar{F}(t) = 1 - \prod_{i=1}^{n} \bar{F}_i(t)$

System Life Time

System life time

- For a continuous distribution G
  - lets define $\lambda(t)$, failure rate function of G
  - $\lambda(t) = \frac{g(t)}{\bar{G}(t)}$; where, $g(t) = \frac{d}{dt}G(t)$
  - $\lambda(t)$ indicates the probability intensity that a t-year old item will fail

Expected System Life Time

Expected system life time

- System life time will be t or larger, iff system is still functioning at time t
  - $P\{system\,life > t\} = r(\mathbf{F}(t))$
  - knowing that $E[X] = \int_0^\infty P\{X \geq x\}\, dx$

$$E\{system\ life\} = \int_0^\infty r(\mathbf{F}(t))\, dt$$

System Life time Computation

Parallel system, exponential

- Life distribution pf parallel system of two independent components
  - i^{th} component having an exponential distribution with mean $1/i$, $i = 1,2$

$$\bar{F}(t) = 1-(1-e^{-t})(1-e^{-2t})$$
$$= e^{-2t} + e^{-t} - e^{-3t}$$

Find $\lambda(t)$

Expected System Life Time

System Life time

- $P\{system\, life \;>\; t\} = r\left(\mathbf{F}(\mathbf{t})\right)$
- For non-negative variable, $E[X] = \int_0^\infty P\{X \geq x\}\, dx$
- Hence,
  $E[system\ life] = \int_0^\infty P\{system\, life \;>\; t\}\, dt = \int_0^\infty r\left(\mathbf{F}(\mathbf{t})\right) dt$

Expected System Life Time ... example

Example- expected life time

- A series system of three independent components uniformly distributed over $(0, 10)$
- $F_i(t) = \begin{cases} t/10 & 0 \le t \le 10 \\ 1 & t > 10 \end{cases}$
- $r(\mathbf{F}(\mathbf{t})) = \begin{cases} \left(\frac{10-t}{10}\right)^3 & 0 \le t \le 10 \\ 0 & t > 10 \end{cases}$
- $E[\textit{system life}] == \int_0^\infty r(\mathbf{F}(\mathbf{t}))\, dt = \int_0^{10} \left(\frac{10-t}{10}\right)^3 dt = 10 \int_0^1 y^3 dy = \frac{10}{4}$

Completed!

Completed.

Best Wishes!