Maths for ECE – I

Special ~~MTE~~

1. $f(t) = \frac{1}{\sqrt{2\pi}\,\sigma} \cdot e^{-\left(\frac{t-\mu}{\sqrt{2}\,\sigma}\right)^2}$ $E[T] = \mu = 90$, $\sigma_T = 5$ $\Rightarrow f(t) = \frac{1}{5\sqrt{2\pi}} e^{-\left(\frac{t-90}{\sqrt{2}\times 5}\right)^2}$

$R(t) = \int_0^T f(t)\,dt$

$erf(T)$

$R(T)$	T
90%	
99%	
99.9%	

from Table

2. $\alpha = 0.02$ $R(t) = e^{-\alpha t} = e^{-\frac{t}{50}} = e^{-\alpha t}$

Using $(1-x)^n = 1 - xn + x^2\frac{n(n-1)}{2}$

$R_{All}(t) = 1 - [1 - R(t)]^3 = 1 - (1 - e^{-\alpha t})^3 = 3e^{-\alpha t} - 3 \cdot e^{-2\alpha t}$

explore for

	$R(t)$	$R_{All}(t)$
$T = 1$		
$T = 10$		
$T = 100$		

$\frac{R_{All}(t)}{R_1(t)} = \frac{3\left(e^{-\alpha t} - e^{-2\alpha t}\right)}{e^{-\alpha t}} = 3 \cdot \left(1 - e^{-\alpha t}\right)$

3. $P_1 = \frac{5}{16}$ $P_2 = \frac{1}{16}$ $P_3 = \frac{3}{16}$ $P_4 = \frac{7}{16}$ $\Rightarrow n = 4.$

$P_2 < \frac{1}{n-1}$ $P_2 + P_1 > \frac{1}{n-1}$ Lemma satisfied,

Defining $P = \frac{1}{(n-1)} \sum_{k=1}^{n-1} Q^{(k)}$ & $\bar{P} = \frac{1}{n-1} Q^{(1)} + \frac{n-2}{n-1} \cdot P^{(n-1)}$

$= \frac{1}{3} \sum_1^3 Q^{(k)}$

	1	2	3	4
$Q^{(1)}$	$\frac{13}{16}$	$\frac{3}{16}$	X	X
$Q^{(3)}$	X	X	$\frac{9}{16}$	$\frac{7}{16}$
$Q^{(2)}$	$\frac{2}{16}$	X	X	$\frac{14}{16}$

(I) Let P_2 carry only from $Q_2^{(1)}$

$P_2 = \frac{1}{3} Q_2^{(1)} \Rightarrow Q_2^{(1)} = \frac{3}{16}$

$\Rightarrow Q_1^{(1)} = \frac{13}{16}.$

(I) $i=2,\ j=1$.

$p_2^{(n-1)} = 0$

$p_1^{(n-1)} = \left(\frac{n-1}{n-2}\right)\left(P_1 - \frac{1}{3} Q_1^{(1)}\right)$

$= \frac{3}{2}\cdot\left(\frac{5}{16} - \frac{13}{16 \cdot 3}\right) = \frac{1}{2}\cdot\frac{2}{16} = \frac{1}{16}$.

$p_3^{(3)} = \frac{3}{2} P_3 = \frac{9}{32}$

$p_4^{(3)} = \frac{3}{2} P_4 = \frac{21}{32}$

(II) Let $Q_1^{(2)}$ be sole contributor to $P_1^{(3)}$.

$\bar{p}^{(n-1)} = \frac{1}{n-2} Q^{(3)} + \frac{n-3}{n-2}\cdot \bar{p}^{(n-2)}$

$p^{(3)} = \frac{1}{2} Q^{(3)} + \frac{1}{2}\cdot p^{(2)}$

~~i.e.~~ $\cancel{P_1^{(3)} = \frac{1}{2} Q_1^{(3)}}$

$\cancel{P_4^{(3)}}$

	$P_1^{(3)}$	$P_2^{(3)}$	$P_3^{(3)}$	$P_3^{(4)}$
	$\frac{1}{16}$	[illegible]	$\frac{9}{32}$	$\frac{21}{32}$
$Q_1^{(2)}$	$\frac{2}{16}$	x	x	$\frac{14}{16}$

$i=1,\ j=4$

$P_1^{(2)} = 0$

$P_4^{(2)} = \frac{n-2}{n-3}\left(P_4^{(3)} - \frac{1}{n-2} Q_4^{(3)}\right) = 2\cdot\left(\frac{21}{32} - \frac{14}{32\,(3)}\right) = \frac{7}{16}$

$P_3^{(2)} = \frac{n-2}{n-3} P_3 = 2\cdot P_3^{(3)} = 2\cdot\frac{9}{32} = \frac{18}{32}$

$P_1^{(2)}$	$P_2^{(2)}$	$P_3^{(2)}$	$P_4^{(2)}$
x	x	$\frac{18}{32}$	$\frac{7}{16}$

III

$p^{(n-2)} = \frac{1}{n-3}\cdot Q^{(2)} \Rightarrow p^{(2)} = Q^{(2)}$

$Q_3^{(2)} = P_3^{(2)} = \frac{18}{32} = \frac{9}{16}$

$Q_4^{(2)} = P_4^{(2)} = \frac{14}{32} = \frac{7}{16}$.

found & QED ——