Math 141

Lecture 1

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Spring 2015

Outline

1. Review of trigonometry
   - The Trigonometric Functions
   - Trigonometric Identities
   - Trigonometric Identities and Complex Numbers
   - Graphs of the Trigonometric Functions

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   - The Trigonometric Functions
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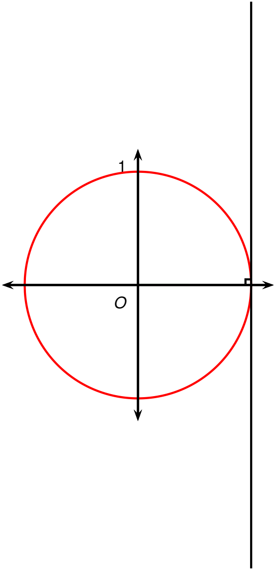
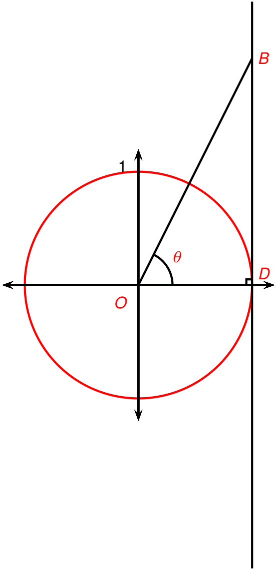
   - Graphs of the Trigonometric Functions
2. Inverse Trigonometric Functions

Geometric interpretation of all trigonometric functions



On picture: circle of radius 1 centered at point O with coordinates $(0, 0)$.

Geometric interpretation of all trigonometric functions



On picture: circle of radius 1 centered at point O with coordinates $(0, 0)$. Let $\angle DOB = \theta$.

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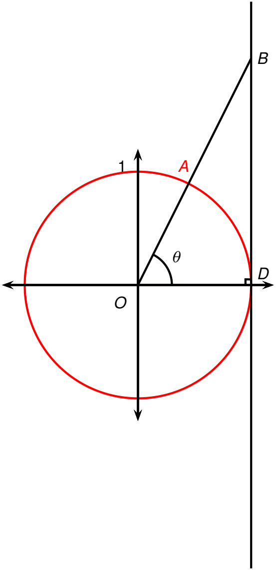

On picture: circle of radius 1 centered at point *O* with coordinates $(0,0)$. Let $\angle DOB = \theta$. Let *OB* intersect the circle at point *A*.

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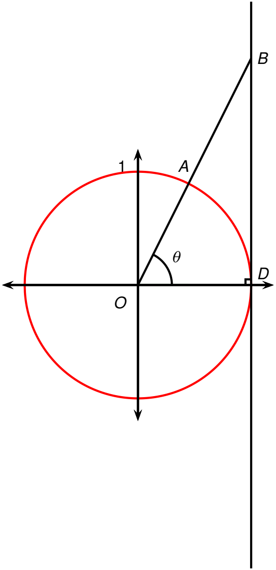

On picture: circle of radius 1 centered at point O with coordinates $(0, 0)$. Let $\angle DOB = \theta$. Let OB intersect the circle at point A. Then the coordinates of A are $(\cos\theta, \sin\theta)$.

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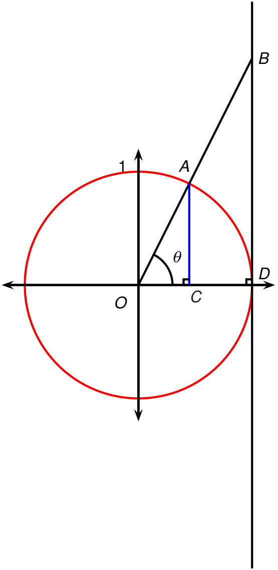

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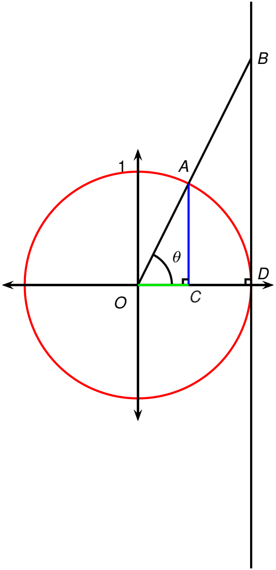

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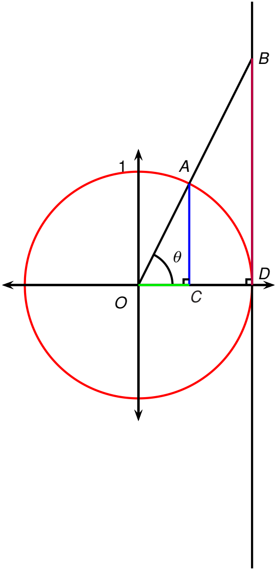

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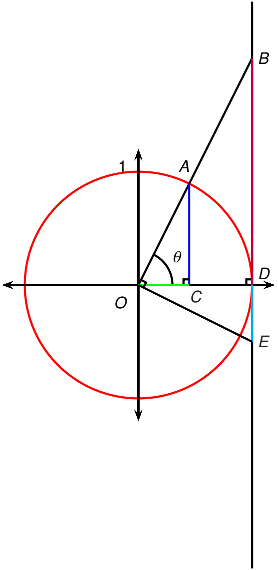

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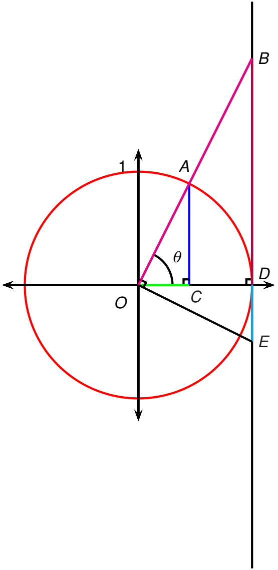

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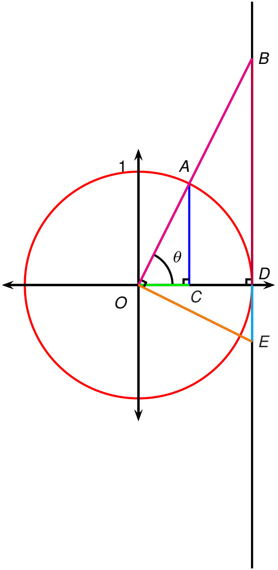

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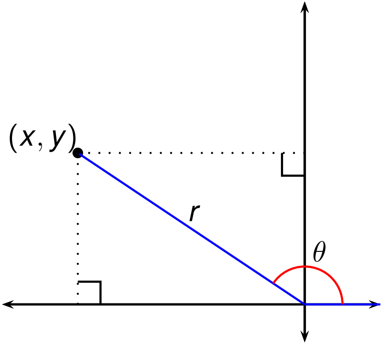
$$\sec\theta = \frac{|OB|}{|OD|} = \frac{|OB|}{1} = |OB|.$$

$$\csc\theta = \frac{|OE|}{|DO|} = \frac{|OE|}{1} = |OE|.$$

Trigonometric Identities

Definition (Trigonometric Identity)

A trigonometric identity is a relationship among the trigonometric functions that is true for any value of the independent variable.



$$\sin\theta = \frac{y}{r} \qquad \csc\theta = \frac{r}{y}$$
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- $\csc\theta = \dfrac{1}{\sin\theta}$
- $\sec\theta = \dfrac{1}{\cos\theta}$
- $\cot\theta = \dfrac{1}{\tan\theta}$
- $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$
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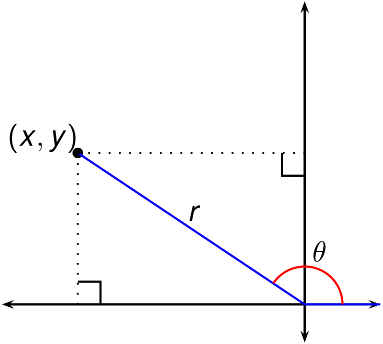

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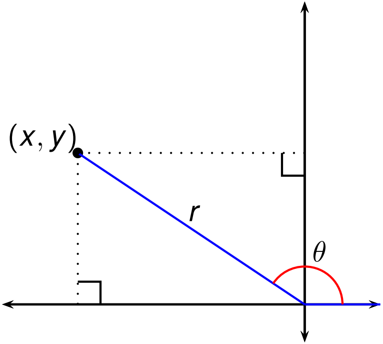

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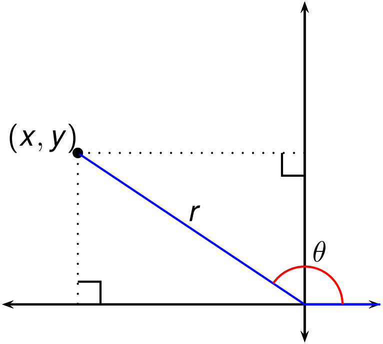
$$\sin^2\theta + \cos^2\theta$$



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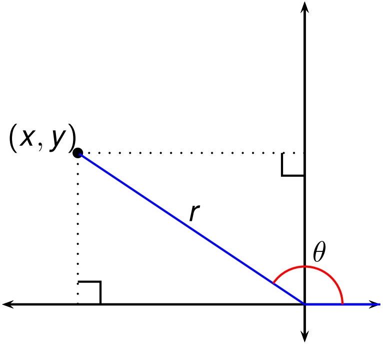
$$\begin{aligned} & \sin^2\theta + \cos^2\theta \\ = \quad & \frac{y^2}{r^2} + \frac{x^2}{r^2} \end{aligned}$$



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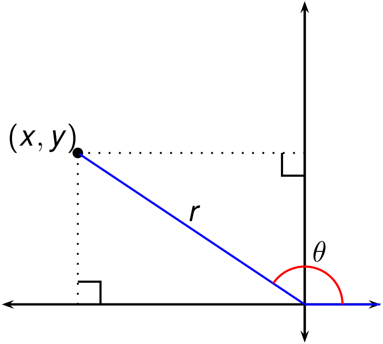
$$\begin{aligned} & \sin^2\theta + \cos^2\theta \\ = \; & \frac{y^2}{r^2} + \frac{x^2}{r^2} \\ = \; & \frac{y^2 + x^2}{r^2} \end{aligned}$$



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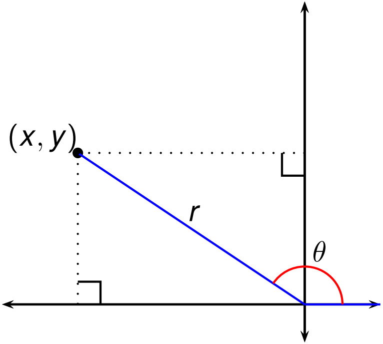

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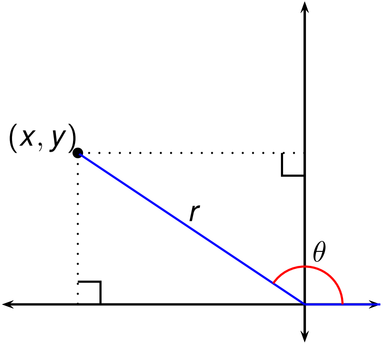

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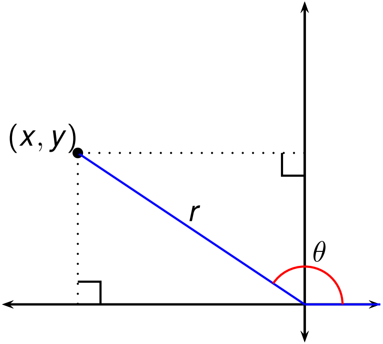
Therefore $\sin^2\theta + \cos^2\theta = 1$.



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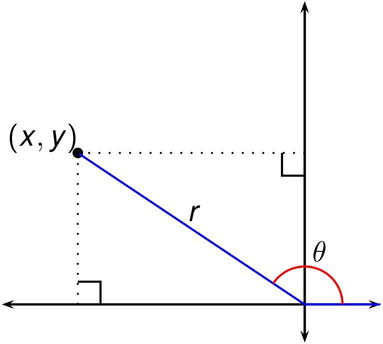
Example ($\tan^2\theta + 1 = \sec^2\theta$)

Prove the identity
$\tan^2\theta + 1 = \sec^2\theta$.



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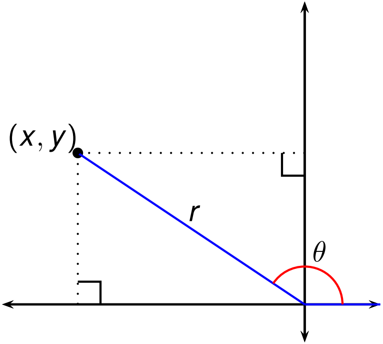
$$\sin^2\theta + \cos^2\theta = 1$$



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Example ($\tan^2\theta + 1 = \sec^2\theta$)

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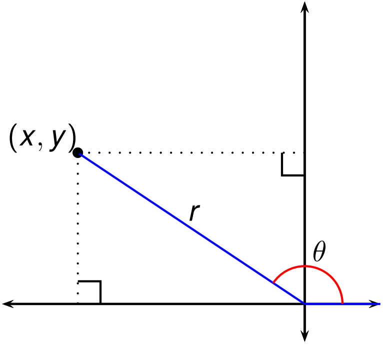
$\tan^2\theta + 1 = \sec^2\theta$.

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= 1 \\ \frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} &= \frac{1}{\cos^2\theta} \end{aligned}$$



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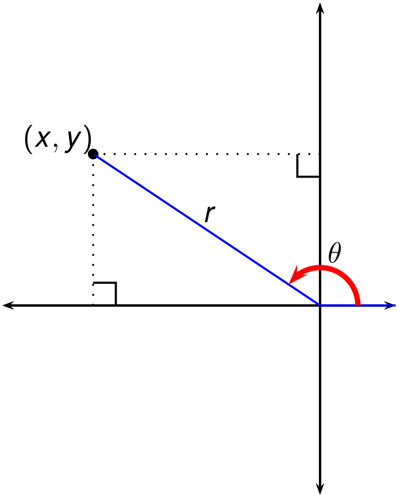

- Positive angles are obtained by rotating counterclockwise.

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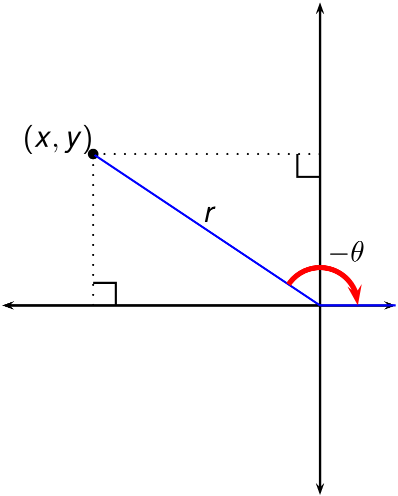

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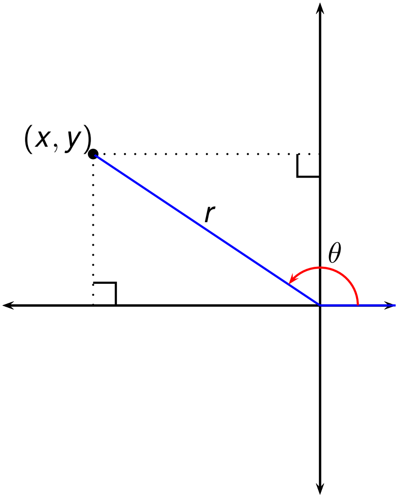

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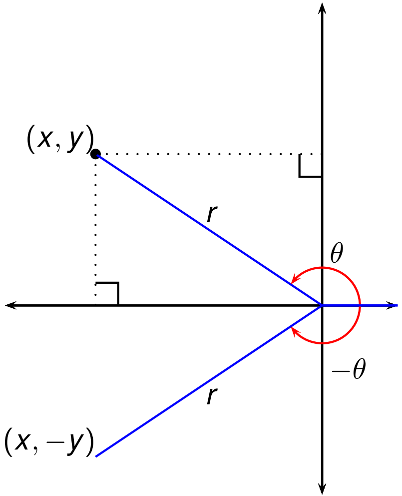
- If (x, y) is on the terminal arm of the angle θ, then $(x, -y)$ is on the terminal arm of $-\theta$.



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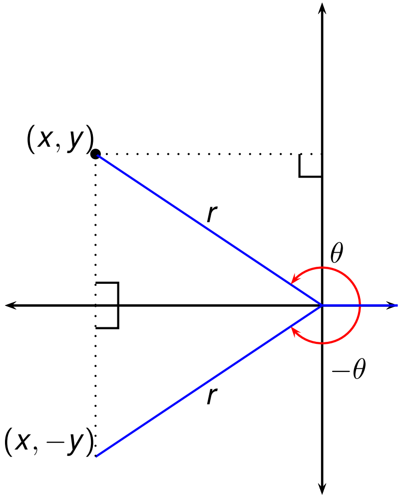

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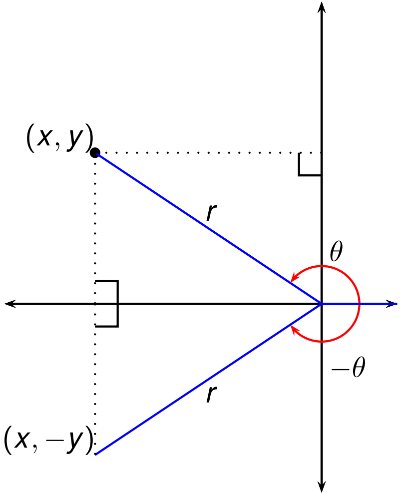

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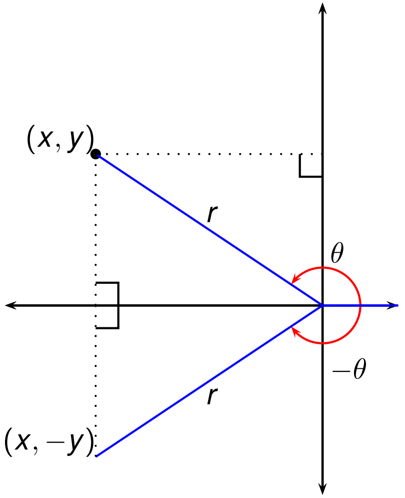

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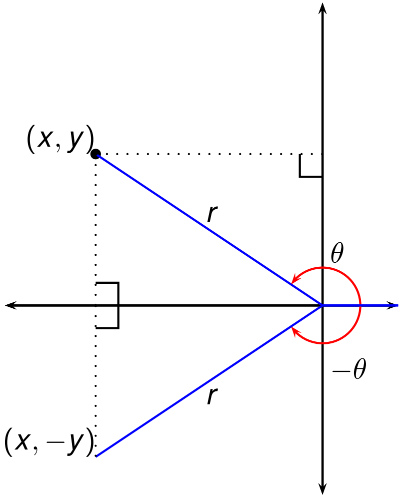
- $\cos(-\theta) = \frac{x}{r} = \cos\theta$.
- sin is an odd function.



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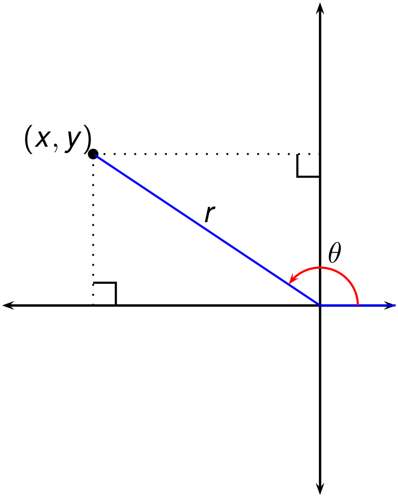
- cos is an even function.



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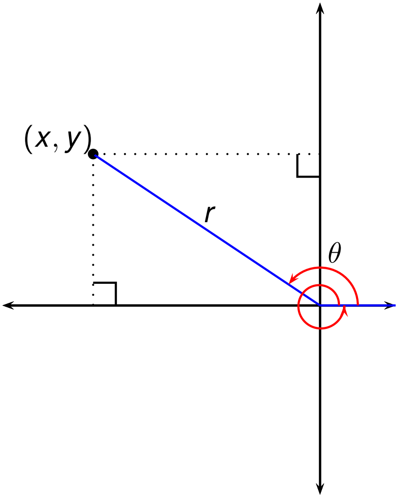

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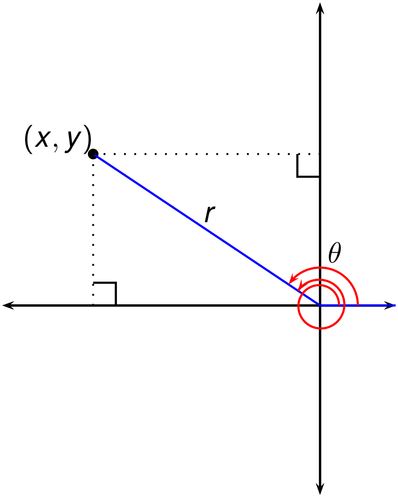
- 2π represents a full rotation.



$$\sin\theta = \frac{y}{r} \quad \csc\theta = \frac{r}{y}$$
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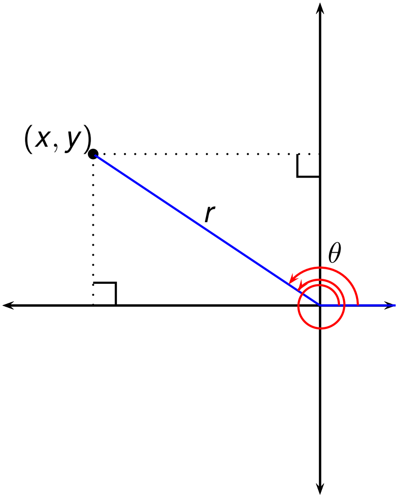

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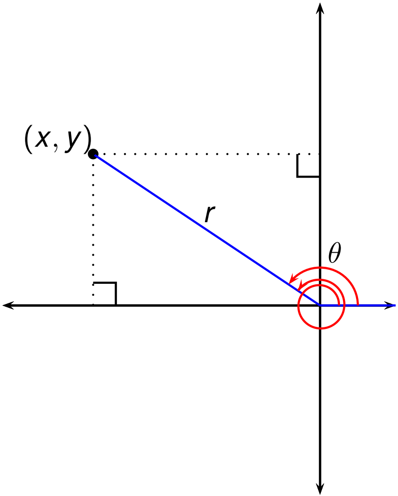

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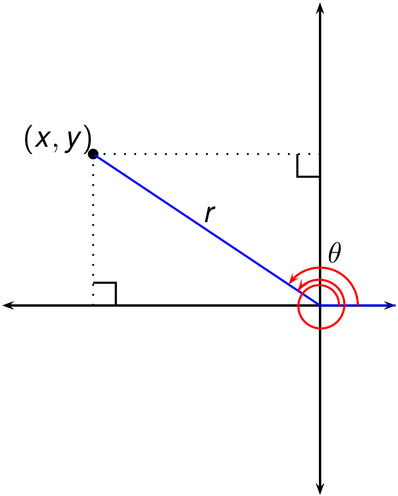

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- We say sin and cos are 2π-periodic.

The remaining identities are consequences of the addition formulas:

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Substitute $-y$ for y, and use the fact that $\sin(-y) = -\sin y$ and $\cos(-y) = \cos y$:

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Rewrite the second double angle formula in two ways, using $\cos^2 x = 1 - \sin^2 x$ and $\sin^2 x = 1 - \cos^2 x$:

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To get the half-angle formulas, solve these equations for $\cos^2 x$ and $\sin^2 x$ respectively.

$$\cos^2 x = \frac{1+\cos 2x}{2}, \qquad \sin^2 x = \frac{1-\cos 2x}{2}$$

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Divide the first equation by the second, and then cancel $\cos x \cos y$ from the top and bottom:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

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Divide the first equation by the second, and then cancel $\cos x \cos y$ from the top and bottom:

$$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Do the same for the subtraction formulas:

$$\tan(x-y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$$

Complex numbers definition

Definition

The set of complex numbers $\mathbb{C}$ is defined as the set

$$\{a + bi | a, b - \text{real numbers}\},$$

where the number i is a number for which

$$i^2 = -1 \quad .$$

The number i is called the imaginary unit.

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You will not be tested on the material in the following slide.

Euler's Formula

Theorem (Euler's Formula)

$$e^{ix} = \cos x + i \sin x,$$

where $e \approx 2.71828$ *is Euler's/Napier's constant .*

Proof.

Recall $n! = 1 \cdot 2 \cdot 3 \cdot \dots \cdot (n-1) * n$. Borrow from Calc II the f-las:

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$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + \frac{(-1)^n x^{2n+1}}{(2n+1)!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots + \frac{(-1)^n x^{2n}}{(2n)!} + \dots$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots$$

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Rearrange.

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e^{ix} & = & 1 & +ix & -\frac{x^2}{2!} & -i\frac{x^3}{3!} & +\frac{x^4}{4!} & +i\frac{x^5}{5!} & -\dots
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Rearrange. Plug-in $z = ix$. Use $i^2 = -1$. $\square$

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Rearrange. Plug-in $z = ix$. Use $i^2 = -1$. Multiply $\sin x$ by i. Add to get $e^{ix} = \cos x + i \sin x$. □

You will not be tested on the material in the following slide.

Trigonometric Identities Revisited

- $e^{ix} = \cos x + i \sin x$ (Euler's Formula).
- $e^{ix} e^{iy} = e^{ix+iy} = e^{i(x+y)}$ (exponentiation rule: valid for $\mathbb{C}$).
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All trigonometric formulas can be easily derived using the above formulas.

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$$\begin{aligned}\sin(x+y) &= \sin x \cos y + \sin y \cos x\\ \cos(x+y) &= \cos x \cos y - \sin x \sin y \quad .\end{aligned}$$

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$$e^{i(x+y)} =$$

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$$e^{i(x+y)} = \cos(x+y) + i\sin(x+y)$$

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Proof.

$$\begin{aligned} e^{i(x+y)} &= \cos(x+y) + i \sin(x+y) \\ e^{ix}e^{iy} &= \cos(x+y) + i \sin(x+y) \end{aligned}$$

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Compare coefficient in front of i and

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Compare coefficient in front of i and remaining terms to get the desired equalities. □

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Example

$$\sin^2 x + \cos^2 x = 1$$

Proof.

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$$1 = e^{0}$$

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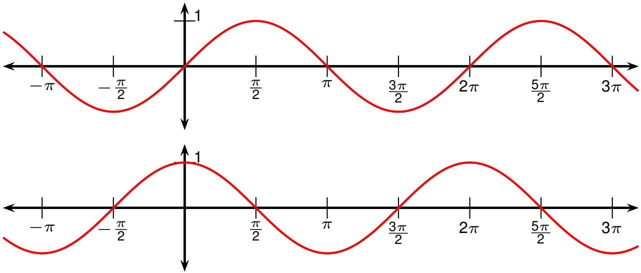
Compare coefficient in front of i and remaining terms to get the desired equalities. □

Graphs of the Trigonometric Functions



$y = \sin x$

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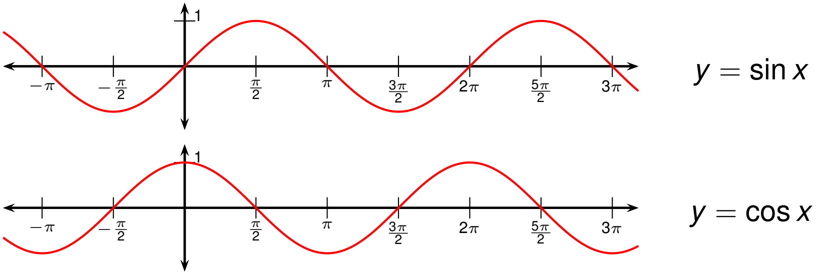

- $\sin x$ has zeroes at $n\pi$ for all integers n.

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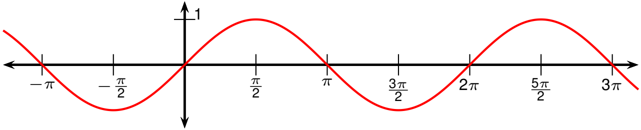

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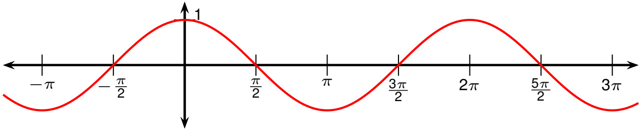

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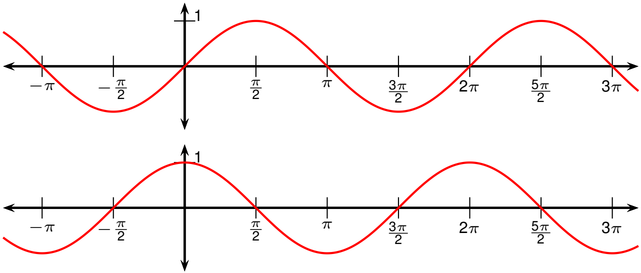

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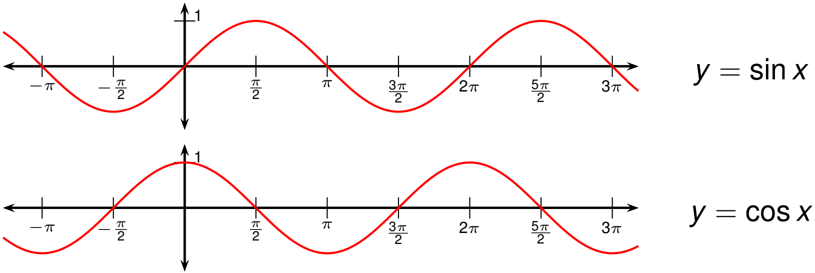

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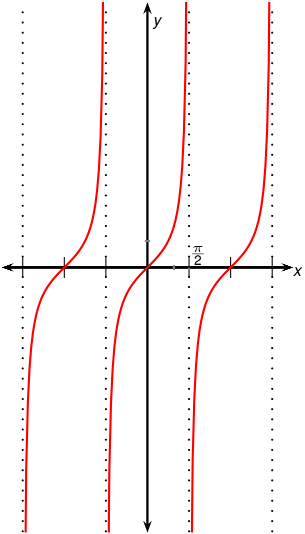

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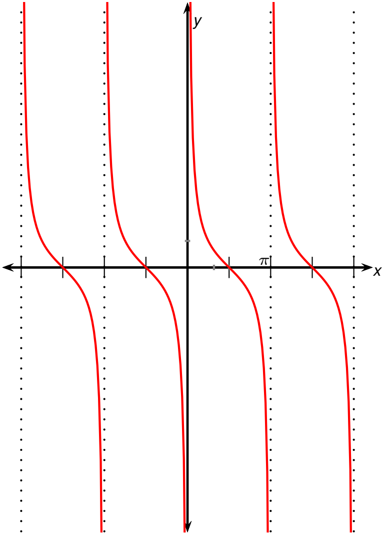

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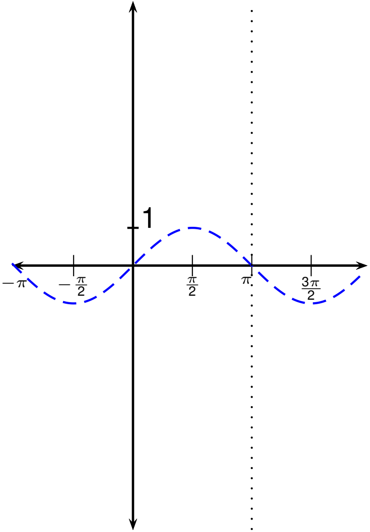
1
$-\pi$
$-\frac{\pi}{2}$
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π
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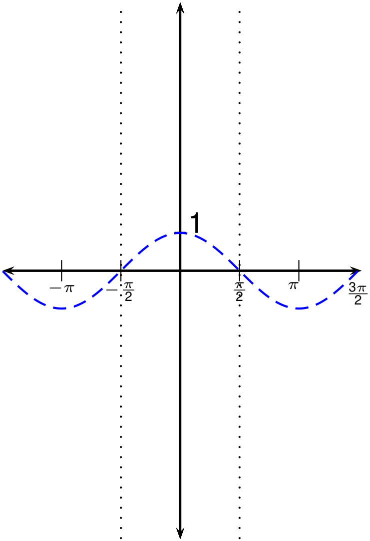
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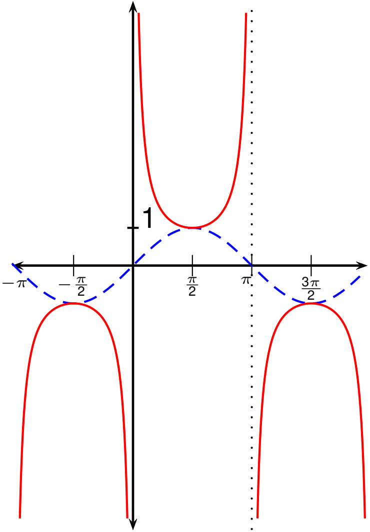
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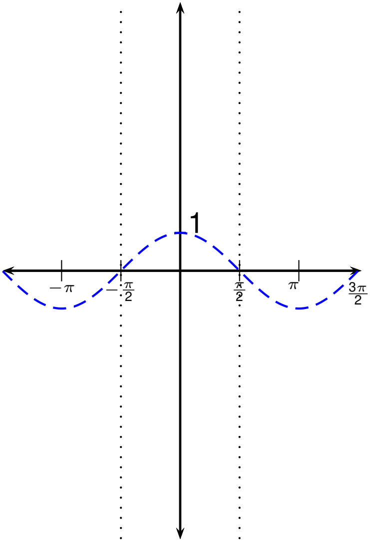
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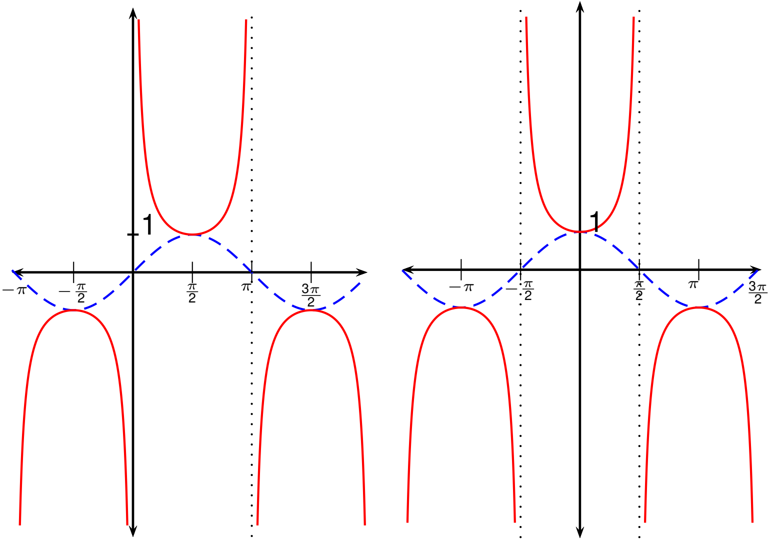
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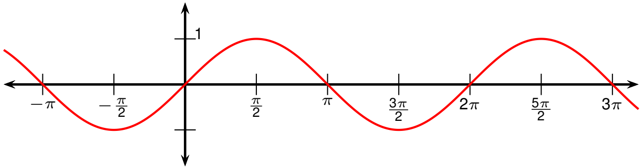
Inverse Trigonometric Functions



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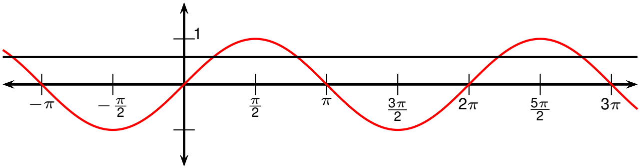

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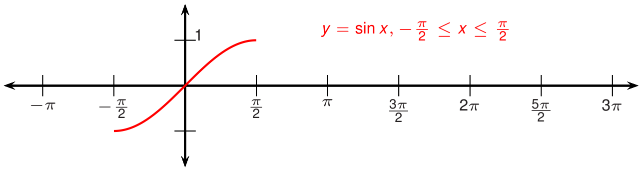

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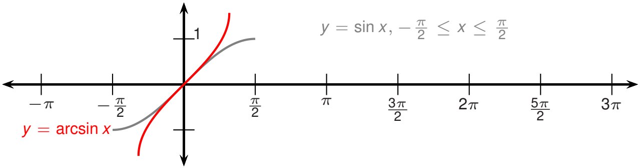

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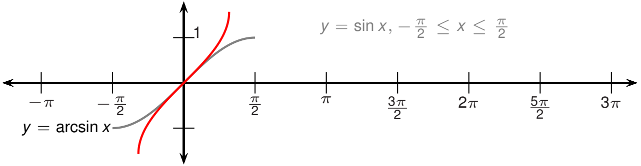

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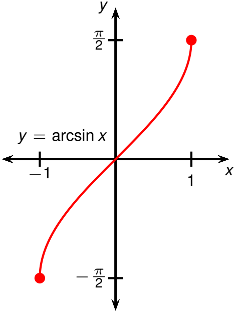

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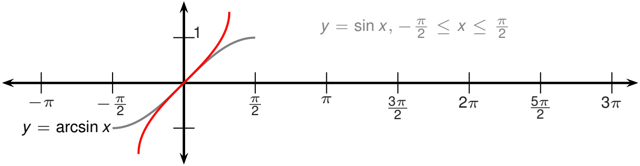

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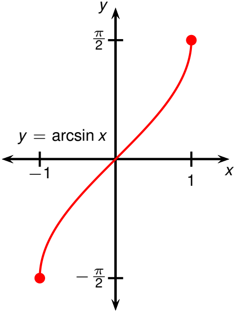
- $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\pi/2 \le y \le \pi/2$.



Example

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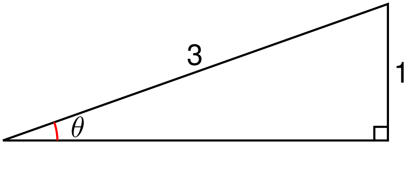

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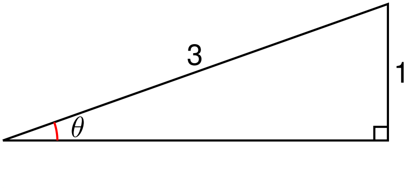

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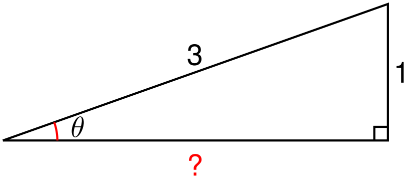

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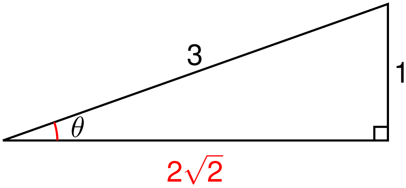

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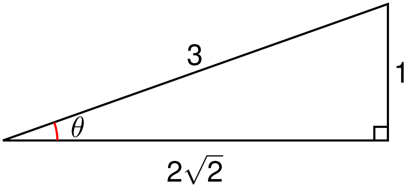

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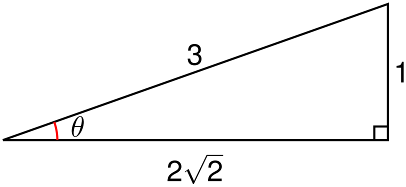

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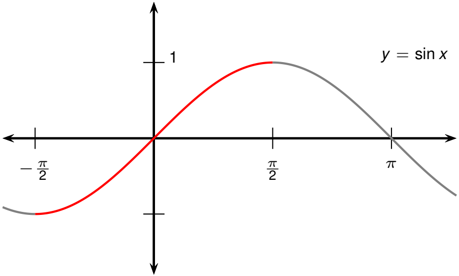

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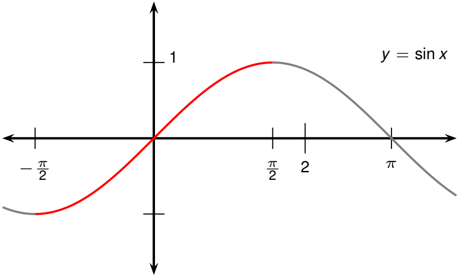

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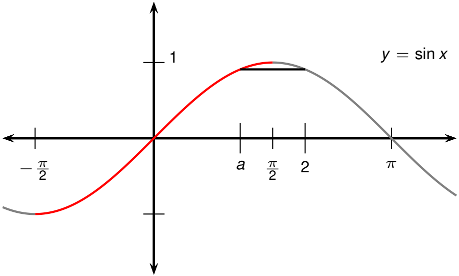

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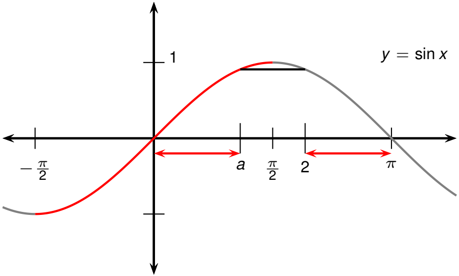
$$a - 0 = \pi - 2.$$



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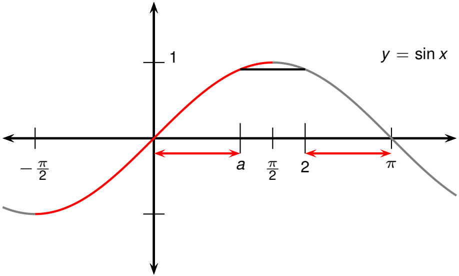
$$a - 0 = \pi - 2.$$

Therefore $\quad \arcsin(\sin 2) = \arcsin(\sin a)$



Example

Find arcsin(sin 2).

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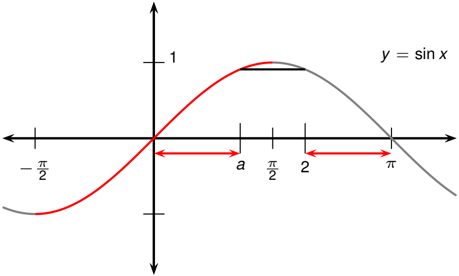
$$a - 0 = \pi - 2.$$

Therefore $$\arcsin(\sin 2) = \arcsin(\sin a) = a$$



Example

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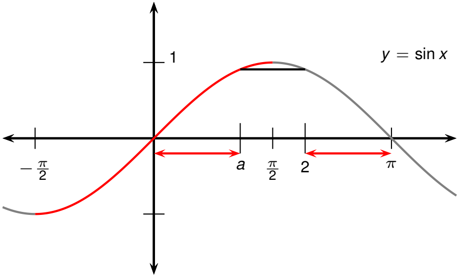

Theorem (The Derivative of $\arcsin x$)

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}, \qquad -1 < x < 1.$$

Proof.

$\square$

Theorem (The Derivative of arcsin x)

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Proof.

$$\text{Let} \quad y = \arcsin x.$$
$$\text{Then} \quad \sin y = x \quad \text{and} \quad -\pi/2 \le y \le \pi/2.$$

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Differentiate implicitly: $=$

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$$\text{But } \cos y > 0: \quad = \frac{1}{\sqrt{1-\sin^2 y}}$$

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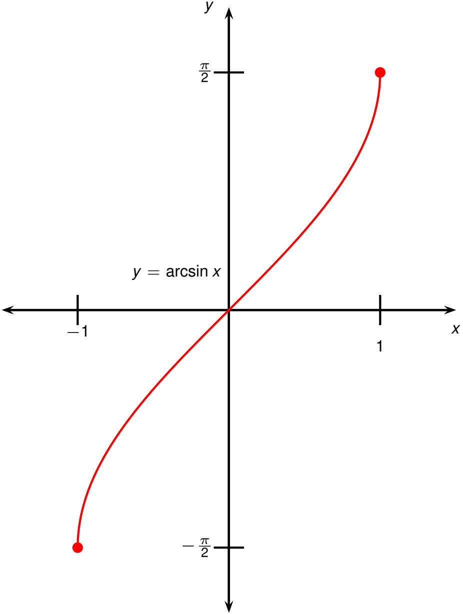
\text{But } \cos y > 0\text{:}\quad & = \frac{1}{\sqrt{1-\sin^2 y}} = \frac{1}{\sqrt{1-x^2}}.
\end{aligned}$$

$\square$

Important facts about arcsin:



1. Domain:
2. Range:
3. $\arcsin x = y \Leftrightarrow \sin y = x$ and $-\pi/2 \leq y \leq \pi/2$.
4. $\arcsin(\sin x) = x$ for $-\pi/2 \leq x \leq \pi/2$.
5. $\sin(\arcsin x) = x$ for $-1 \leq x \leq 1$.
6. $\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$.

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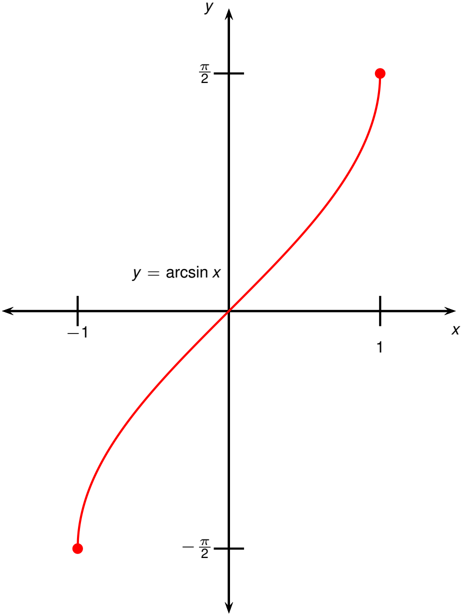

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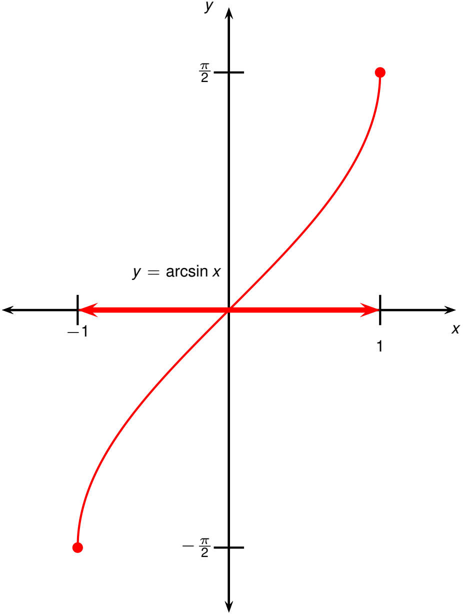

1. Domain: $[-1, 1]$.
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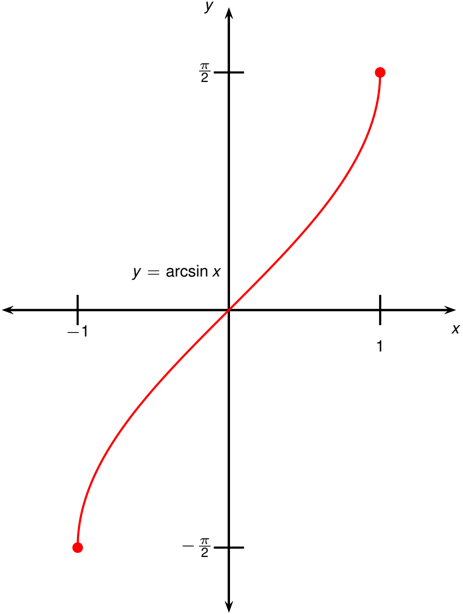

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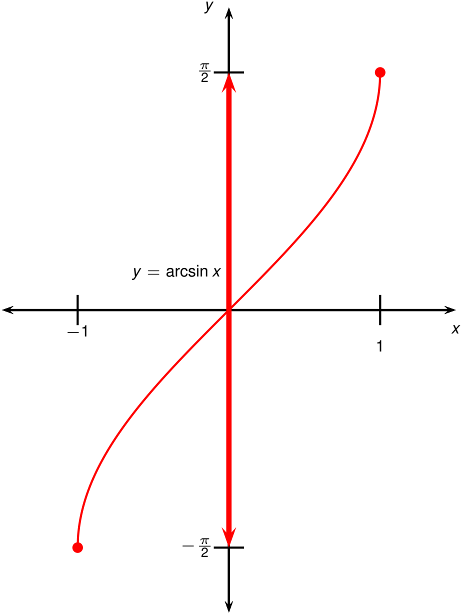

1. Domain: $[-1, 1]$.
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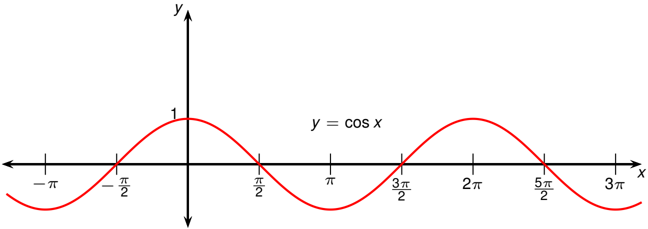
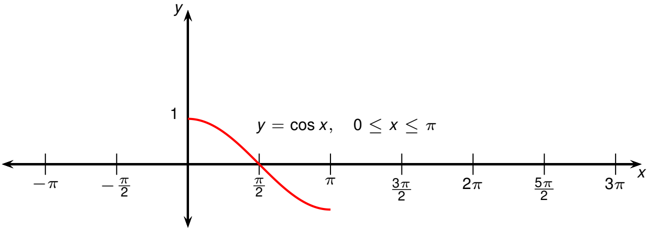


- Same for $\cos x$.



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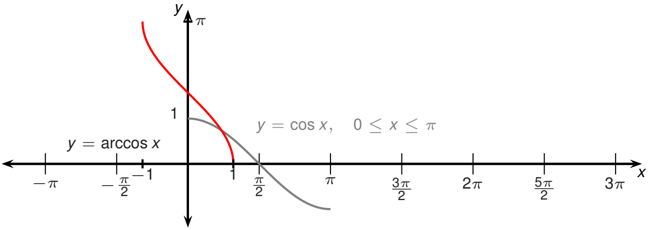
- Restrict the domain to $[0, \pi]$.



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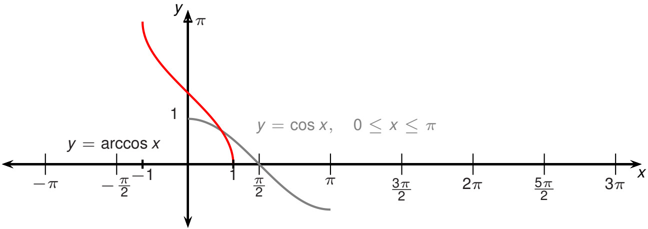
- Restrict the domain to $[0, \pi]$.
- The inverse is called arccos or $\cos^{-1}$.



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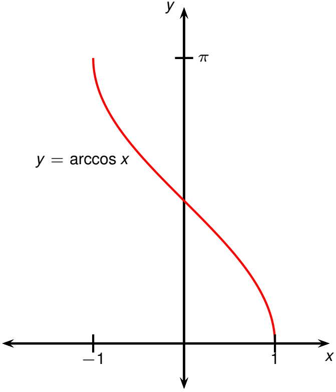

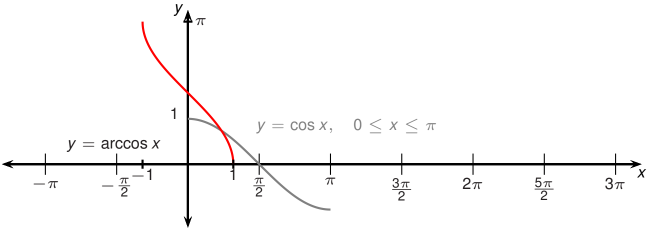

- Same for $\cos x$.
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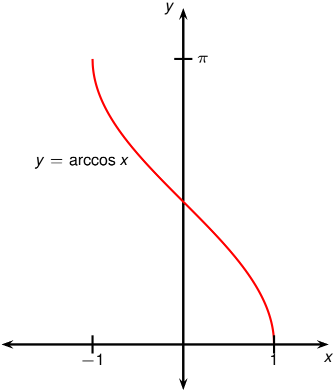
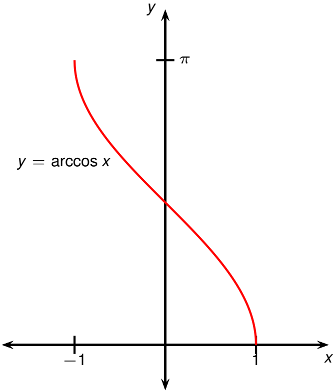
- The inverse is called arccos or $\cos^{-1}$.
- $\arccos(x) = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.



Important facts about arccos:



1. Domain:
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3. $\arccos x = y \Leftrightarrow \cos y = x$ and $0 \leq y \leq \pi$.
4. $\arccos(\cos x) = x$ for $0 \leq x \leq \pi$.
5. $\cos(\arccos x) = x$ for $-1 \leq x \leq 1$.
6. $\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$.

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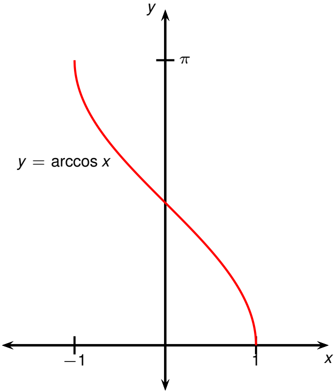

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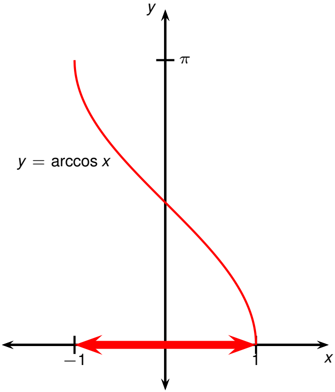

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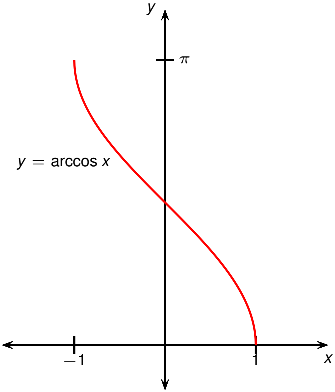

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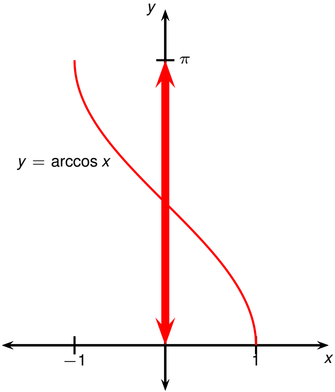

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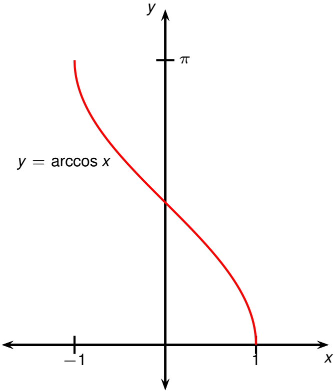

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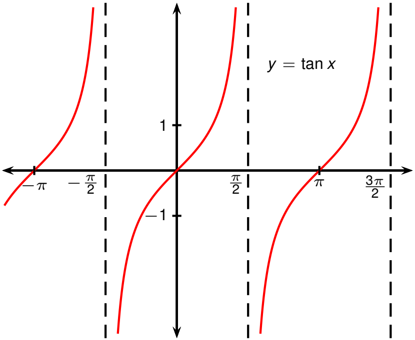
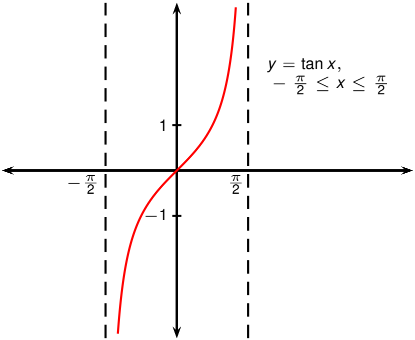
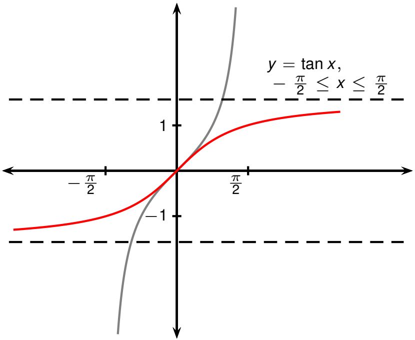
6. $\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}}$. (The proof is similar to the proof of the formula for the derivative of $\arcsin x$.)



- $\tan x$ isn't one-to-one.



- $\tan x$ isn't one-to-one.
- Restrict the domain to $(-\frac{\pi}{2}, \frac{\pi}{2})$.



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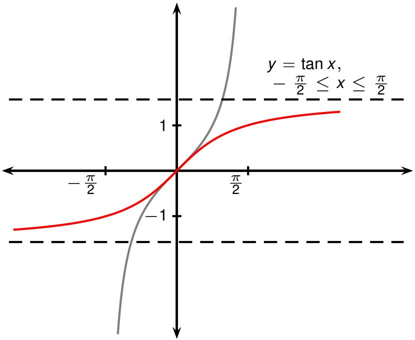

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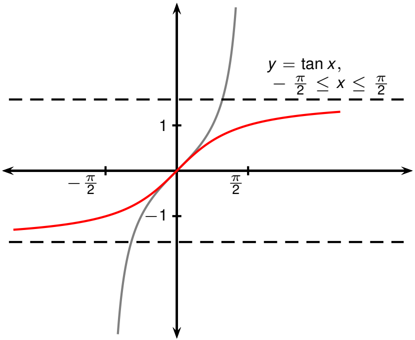

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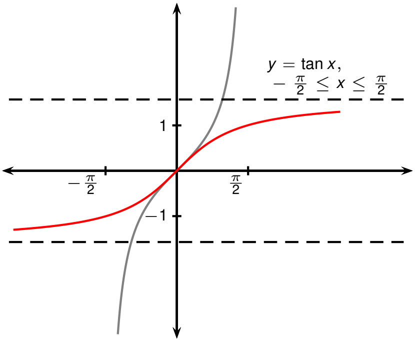
- Domain of arctan:
- Range of arctan:



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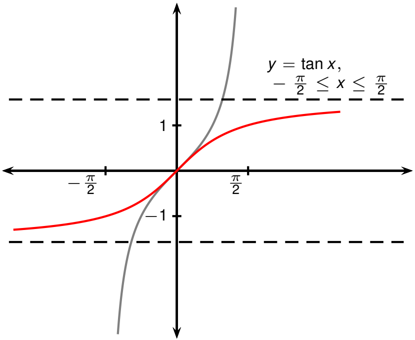
- Domain of arctan: $(-\infty, \infty)$.
- Range of arctan:



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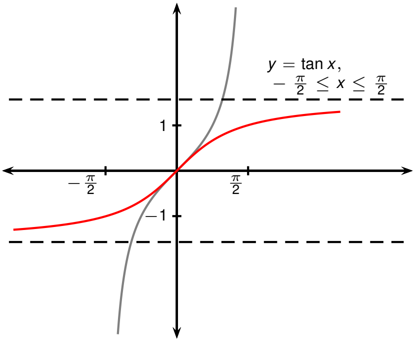

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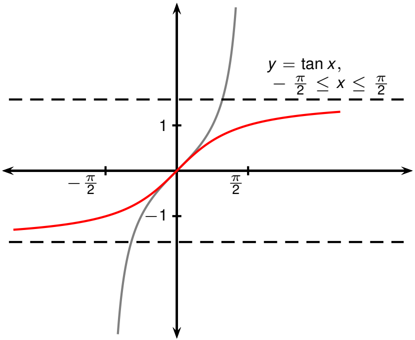

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- $\lim_{x\to\infty} \arctan x =$
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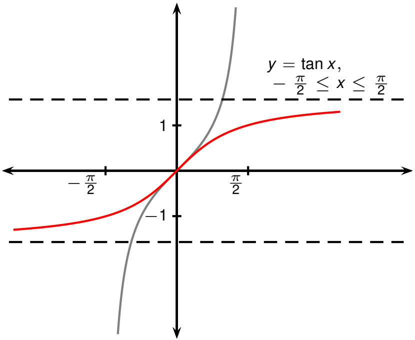

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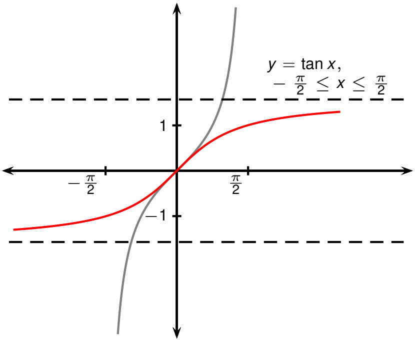

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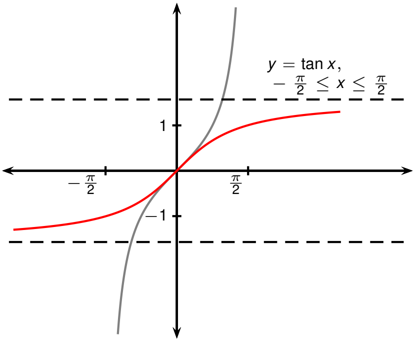

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- Range of arctan: $(-\frac{\pi}{2}, \frac{\pi}{2})$.
- $\lim_{x\to\infty} \arctan x = \dfrac{\pi}{2}$.
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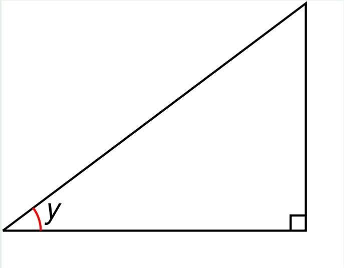
Example

Simplify the expression $\cos(\arctan x)$.



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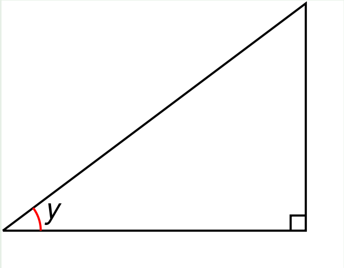

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- Let $y = \arctan x$, so $\tan y = x$.
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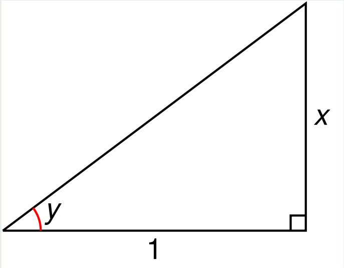

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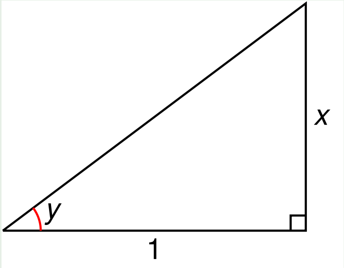

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- Let $y = \arctan x$, so $\tan y = x$.
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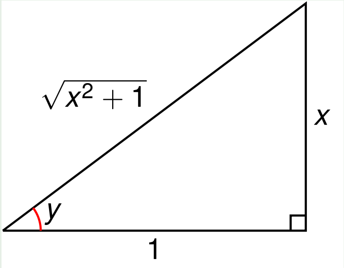
- Length of hypotenuse $= \sqrt{1^2 + x^2}$.



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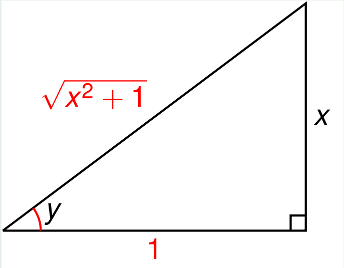

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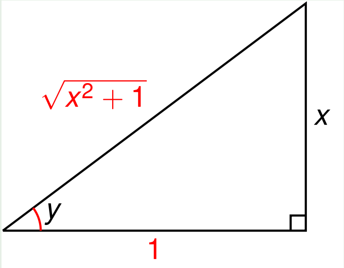

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Theorem (The Derivative of arctan x)

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2}.$$

Proof.

$\square$

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The remaining inverse trigonometric functions aren't used as often:

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y = \operatorname{arccsc} x & (|x| \geq 1) & \Leftrightarrow & \csc y = x & \text{and} \quad y \in \left(0, \frac{\pi}{2}\right] \cup \left(\pi, \frac{3\pi}{2}\right] \\
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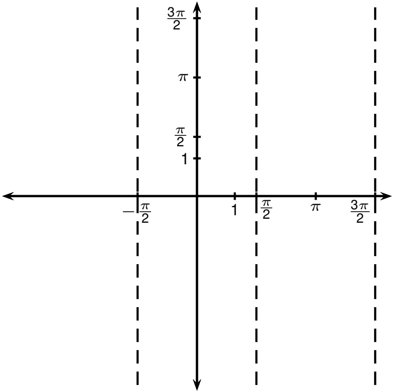

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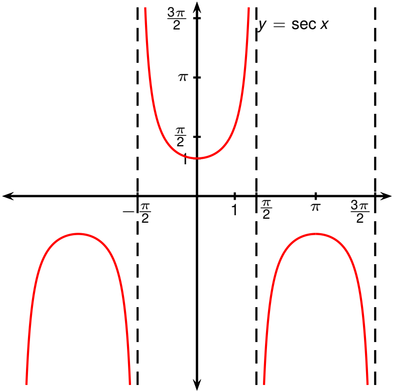

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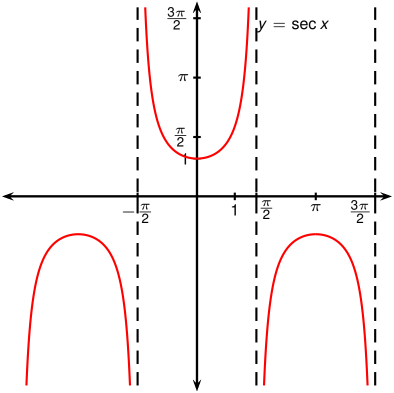

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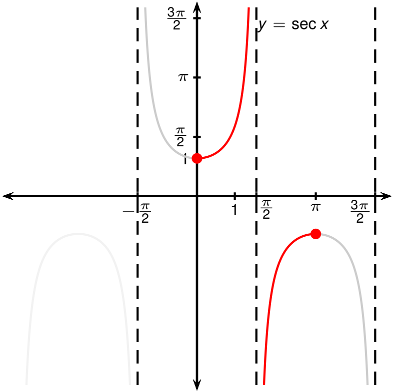

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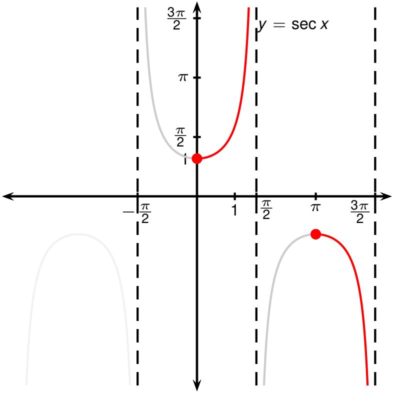

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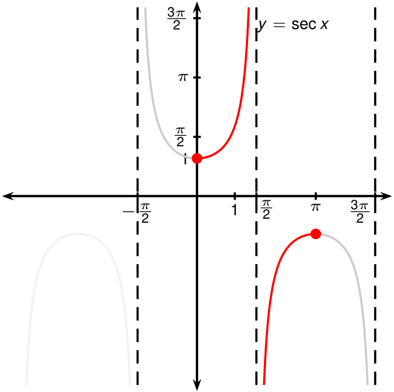

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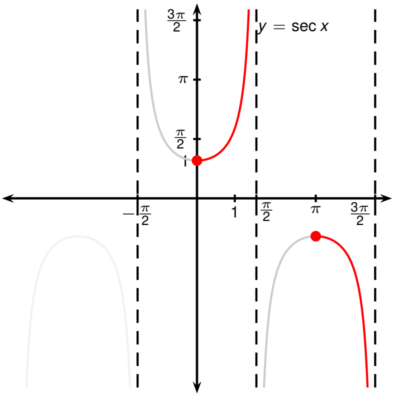

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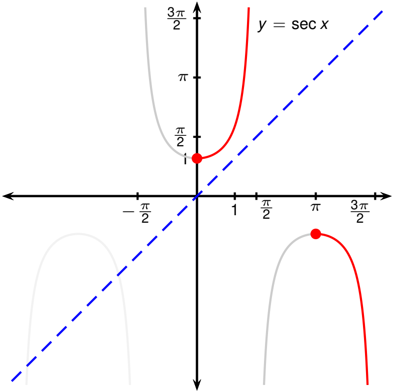

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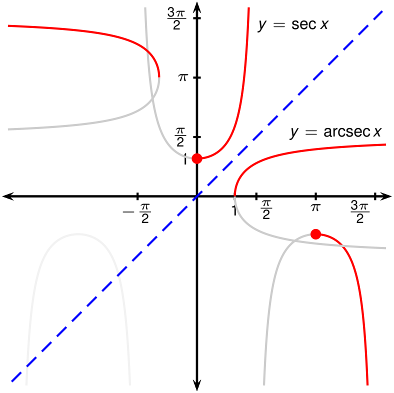

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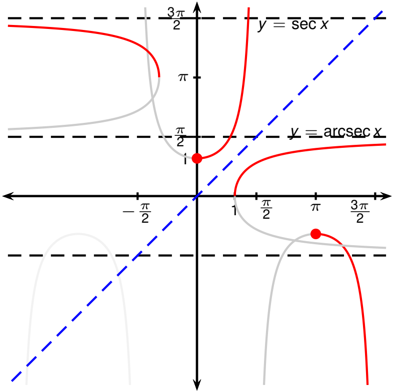

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Table of derivatives of inverse trigonometric functions:

$$\frac{d}{dx}(\arcsin x) = \frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\operatorname{arccsc} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\arccos x) = -\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}(\operatorname{arcsec} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\arctan x) = \frac{1}{1+x^2} \qquad \frac{d}{dx}(\operatorname{arccot} x) = -\frac{1}{1+x^2}$$

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Differentiate $\quad y = \dfrac{1}{\arcsin x}.$

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$$\text{Let} \quad u = \arcsin x.$$

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$$\text{Chain Rule:} \quad \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x}$$

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Differentiate $y = \dfrac{1}{\arcsin x}$.

Let $u = \arcsin x$.

Then $y = u^{-1}$.

Chain Rule: $\dfrac{dy}{dx} = \dfrac{dy}{du}\dfrac{du}{dx}$

$$= \left(\qquad \right)\left(\qquad \qquad \right)$$

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All of the inverse trigonometric derivatives also give rise to integration formulas. These two are the most important:

$$\int \frac{1}{\sqrt{1-x^2}}\mathrm{d}x = \arcsin x + C.$$

$$\int \frac{1}{x^2+1}\mathrm{d}x = \arctan x + C.$$