Math 141

Lecture 2

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Outline

1. Integration, Review
   - The Evaluation Theorem (FTC part 2)

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   - The Evaluation Theorem (FTC part 2)

2. Integration Techniques from Calc I, Review
   - Differential Forms, Review

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   - The Evaluation Theorem (FTC part 2)
2. Integration Techniques from Calc I, Review
   - Differential Forms, Review
3. Integration and Logarithms, Review

Antiderivatives

Definition (Antiderivative)

A function F is called an antiderivative of f on an interval I if $F'(x) = f(x)$ for all x in I.

Theorem (The Evaluation Theorem (FTC part 2))

If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\mathrm{d}x = F(b) - F(a),$$

where F is any antiderivative of f.

$$\int_a^b f(x)dx \text{ exists for any continuous (over } [a, b]) $$
function f.

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Theorem

Let f be a continuous function on $[a, b]$. Then f is integrable over $[a, b]$.

In other words, $\int_a^b f(x)dx$ exists for any continuous (over $[a, b]$) function f.

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Indefinite Integrals

- The Evaluation Theorem establishes a connection between antiderivatives and definite integrals.
- It says that $\int_a^b f(x)\mathrm{d}x$ equals $F(b) - F(a)$, where F is an antiderivative of f.
- We need convenient notation for writing antiderivatives.
- This is what the indefinite integral is.

Definition (Indefinite Integral)

The indefinite integral of f is another way of saying the antiderivative of f, and is written $\int f(x)\mathrm{d}x$. In other words,

$$\int f(x)\mathrm{d}x = F(x) \qquad \text{means} \qquad F'(x) = f(x).$$

Example

$$\int x^4 \mathrm{d}x =$$

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$$\int x^4 \mathrm{d}x = \frac{x^5}{5}$$

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$$\int x^4 \mathrm{d}x = \frac{x^5}{5} + C$$

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$$\int x^4 dx = \frac{x^5}{5} + C$$

because

$$\frac{d}{dx}\left(\frac{x^5}{5} + C\right) = x^4.$$

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- Example: the general antiderivative of $\frac{1}{x}$ is

$$F(x) = \begin{cases} \ln|x| + C_1 & \text{if} \quad x > 0 \\ \ln|x| + C_2 & \text{if} \quad x < 0 \end{cases}$$

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- We adopt the convention that the constant participating in an indefinite integral is only valid on one interval.
- $\int \frac{1}{x}\mathrm{d}x = \ln|x| + C$, and this is valid either on $(-\infty, 0)$ or $(0, \infty)$.

Differentials

- Recall $\Delta y, \Delta x$ stand for change of x, y. Recall: $\Delta y \approx \frac{\mathrm{d}y}{\mathrm{d}x}\Delta x$
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- Define the *differential* d and the *differential forms* $\mathrm{d}x$, $\mathrm{d}(f(x))$ by requesting that d and $\mathrm{d}x$ satisfy the transformation law

$$\mathrm{d}(f(x)) = f'(x)\mathrm{d}x$$

for any differentiable function $f(x)$. In abbreviated notation:

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- Define the *differential* d and the *differential forms* dx, $d(f(x))$ by requesting that d and dx satisfy the transformation law

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$$df = f'dx$$

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- Nonetheless, what we studied is completely sufficient for practical purposes and carrying out computations.
- **Do not confuse differentials with derivatives.** The correct equality is this.

~~$\mathrm{d}f(x) = f'(x)$~~ $\qquad \mathrm{d}f(x) = f'(x)\mathrm{d}x$

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- Formally, the expression $f(x)\mathrm{d}x$ is a differential form (the same differential forms discussed in the preceding slides).
- We did not give a complete formal definition of a differential form, but we showed how to compute with those.
- Computing with differential forms is consistent with computing with integrals: the integrals of equal differential forms are equal. This follows directly from the Net Change Theorem (the substitution rule for integrals), which in turn follows from the Fundamental Theorem of Calculus and the Chain Rule.

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- The rules for computing differential forms are a direct consequence of the corresponding derivative rules and the transformation law $\mathrm{d}(f(x)) = f'(x)\mathrm{d}x$.

Rule name: product rule.

Differential rule

Derivative rule

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Differential rule

$d(fg) = \quad g\,df + \quad f\,dg$

Derivative rule

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Let c be a constant. Rule name: constant derivative rule.

Differential rule	Derivative rule
$d(fg) = \quad gdf + \quad fdg$	$(fg)' = f'g + fg'$
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	$(f(g(x)))' = f'(g(x))g'(x)$

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$\mathrm{d}f(g(x)) = f'(g(x))\mathrm{d}g(x)$	
$= f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) = f'(g)\mathrm{d}g$	

Let c be a constant. Rule name: power rule.

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$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) \quad = \quad f'(g)\mathrm{d}g$	
	$(x^n)' = nx^{n-1}$

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$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$

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$= f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) = f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
	$(e^x)' = e^x$

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$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = \quad f'(g(x))\mathrm{d}g(x)$	
$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) \quad = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = \quad f'(g(x))\mathrm{d}g(x)$	
$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
	$(\sin x)' = \cos x$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
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$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = \quad f'(g(x))\mathrm{d}g(x)$	
$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) \quad = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
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$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) \quad = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
	$(\cos x)' = -\sin x$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$d(fg) = \quad gdf + \quad fdg$	$(fg)' = f'g + fg'$
$dc = 0$	$(c)' = 0$
$d(cf) = c \quad df$	$(cf)' = cf'$
$d(f+g) = \quad df + \quad dg$	$(f+g)' = f' + g'$
$df(g(x)) = \quad f'(g(x))dg(x)$	
$= \quad f'(g(x))g'(x)dx$	$(f(g(x)))' = f'(g(x))g'(x)$
$df(g) \quad = \quad f'(g)dg$	
$dx^n = nx^{n-1}dx$	$(x^n)' = nx^{n-1}$
$de^x = e^x dx$	$(e^x)' = e^x$
$d\sin x = \cos x dx$	$(\sin x)' = \cos x$
$d\cos x = -\sin x dx$	$(\cos x)' = -\sin x$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$\mathrm{d}(fg) = \quad g\mathrm{d}f + \quad f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\mathrm{d}c = 0$	$(c)' = 0$
$\mathrm{d}(cf) = c \quad \mathrm{d}f$	$(cf)' = cf'$
$\mathrm{d}(f+g) = \quad \mathrm{d}f + \quad \mathrm{d}g$	$(f+g)' = f' + g'$
$\mathrm{d}f(g(x)) = \quad f'(g(x))\mathrm{d}g(x)$	
$= \quad f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\mathrm{d}f(g) \quad = \quad f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name:

Differential rule	Derivative rule
$d(fg) = \quad g df + \quad f dg$	$(fg)' = f'g + fg'$
$dc = 0$	$(c)' = 0$
$d(cf) = c \quad df$	$(cf)' = cf'$
$d(f+g) = \quad df + \quad dg$	$(f+g)' = f' + g'$
$df(g(x)) = \quad f'(g(x))dg(x)$	
$= \quad f'(g(x))g'(x)dx$	$(f(g(x)))' = f'(g(x))g'(x)$
$df(g) \quad = \quad f'(g)dg$	
$dx^n = nx^{n-1}dx$	$(x^n)' = nx^{n-1}$
$de^x = e^x dx$	$(e^x)' = e^x$
$d\sin x = \cos x dx$	$(\sin x)' = \cos x$
$d\cos x = -\sin x dx$	$(\cos x)' = -\sin x$
$d\ln x = \frac{1}{x}dx$	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name:
Corresponding integration rules.

Integration rule	Derivative rule
$\int \mathrm{d}(fg) = \int g\mathrm{d}f + \int f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\int \mathrm{d}c = 0$	$(c)' = 0$
$\int \mathrm{d}(cf) = c\int \mathrm{d}f$	$(cf)' = cf'$
$\int \mathrm{d}(f+g) = \int \mathrm{d}f + \int \mathrm{d}g$	$(f+g)' = f' + g'$
$\int \mathrm{d}f(g(x)) = \int f'(g(x))\mathrm{d}g(x)$	
$= \int f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\int \mathrm{d}f(g) = \int f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
$\mathrm{d}\ln x = \frac{1}{x}\mathrm{d}x$	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name: Integration by parts.
Corresponding integration rules.

Integration rule	Derivative rule
$\int \mathrm{d}(fg) = \int g\mathrm{d}f + \int f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\int \mathrm{d}c = 0$	$(c)' = 0$
$\int \mathrm{d}(cf) = c\int \mathrm{d}f$	$(cf)' = cf'$
$\int \mathrm{d}(f+g) = \int \mathrm{d}f + \int \mathrm{d}g$	$(f+g)' = f' + g'$
$\int \mathrm{d}f(g(x)) = \int f'(g(x))\mathrm{d}g(x)$	
$= \int f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\int \mathrm{d}f(g) = \int f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
$\mathrm{d}\ln x = \frac{1}{x}\mathrm{d}x$	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name: Corresponding integration rules.

Integration is linear.

Integration rule	Derivative rule
$\int d(fg) = \int g df + \int f dg$	$(fg)' = f'g + fg'$
$\int dc = 0$	$(c)' = 0$
$\int d(cf) = c \int df$	$(cf)' = cf'$
$\int d(f+g) = \int df + \int dg$	$(f+g)' = f' + g'$
$\int df(g(x)) = \int f'(g(x)) dg(x)$	
$= \int f'(g(x)) g'(x) dx$	$(f(g(x)))' = f'(g(x)) g'(x)$
$\int df(g) = \int f'(g) dg$	
$dx^n = nx^{n-1} dx$	$(x^n)' = nx^{n-1}$
$de^x = e^x dx$	$(e^x)' = e^x$
$d\sin x = \cos x dx$	$(\sin x)' = \cos x$
$d\cos x = -\sin x dx$	$(\cos x)' = -\sin x$
$d\ln x = \frac{1}{x} dx$	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name: Substitution rule.
Corresponding integration rules.

Integration rule	Derivative rule
$\int \mathrm{d}(fg) = \int g\mathrm{d}f + \int f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\int \mathrm{d}c = 0$	$(c)' = 0$
$\int \mathrm{d}(cf) = c\int \mathrm{d}f$	$(cf)' = cf'$
$\int \mathrm{d}(f+g) = \int \mathrm{d}f + \int \mathrm{d}g$	$(f+g)' = f' + g'$
$\int \mathrm{d}f(g(x)) = \int f'(g(x))\mathrm{d}g(x)$	
$= \int f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\int \mathrm{d}f(g) = \int f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
$\mathrm{d}\ln x = \frac{1}{x}\mathrm{d}x$	$(\ln x)' = \frac{1}{x}$

Let c be a constant. Rule name:
Corresponding integration rules. Integration rules justified via the Fundamental Theorem of Calculus

Integration rule	Derivative rule
$\int \mathrm{d}(fg) = \int g\mathrm{d}f + \int f\mathrm{d}g$	$(fg)' = f'g + fg'$
$\int \mathrm{d}c = 0$	$(c)' = 0$
$\int \mathrm{d}(cf) = c\int \mathrm{d}f$	$(cf)' = cf'$
$\int \mathrm{d}(f+g) = \int \mathrm{d}f + \int \mathrm{d}g$	$(f+g)' = f' + g'$
$\int \mathrm{d}f(g(x)) = \int f'(g(x))\mathrm{d}g(x)$	
$= \int f'(g(x))g'(x)\mathrm{d}x$	$(f(g(x)))' = f'(g(x))g'(x)$
$\int \mathrm{d}f(g) = \int f'(g)\mathrm{d}g$	
$\mathrm{d}x^n = nx^{n-1}\mathrm{d}x$	$(x^n)' = nx^{n-1}$
$\mathrm{d}e^x = e^x\mathrm{d}x$	$(e^x)' = e^x$
$\mathrm{d}\sin x = \cos x\mathrm{d}x$	$(\sin x)' = \cos x$
$\mathrm{d}\cos x = -\sin x\mathrm{d}x$	$(\cos x)' = -\sin x$
$\mathrm{d}\ln x = \frac{1}{x}\mathrm{d}x$	$(\ln x)' = \frac{1}{x}$

We recall from previous slides that

$$\frac{\mathrm{d}}{\mathrm{d}x}(\ln|x|) = \frac{1}{x}.$$

This formula has a special application to integration:

Theorem (The Integral of $1/x$)

$$\int \frac{1}{x}\mathrm{d}x = \ln|x| + C.$$

We recall from previous slides that

$$\frac{d}{dx}(\ln|x|) = \frac{1}{x}.$$

This formula has a special application to integration:

Theorem (The Integral of $1/x$)

$$\int \frac{1}{x}dx = \ln|x| + C.$$

This fills in the gap in the rule for integrating power functions:

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \qquad n \neq -1.$$

Now we know the formula for $n = -1$ too.