Math 141

Lecture 3

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Outline

Integration by Parts

Every differentiation rule

Integration by Parts

Every differentiation rule

$$(uv)' = u'v + uv' \quad \Big| \text{ Product Rule}$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule

$(uv)'$	$=$	$u'v + uv'$	Product Rule
$\mathrm{d}(uv)$	$=$	$v\mathrm{d}u + u\mathrm{d}v$	Differential Prod. Rule

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

$$
\begin{array}{rcl|l}
(uv)' & = & u'v + uv' & \text{Product Rule} \\
\mathrm{d}(uv) & = & v\mathrm{d}u + u\mathrm{d}v & \text{Differential Prod. Rule} \\
\displaystyle\int \mathrm{d}(uv) & = & \displaystyle\int v\mathrm{d}u + \int u\mathrm{d}v & \text{integration of the above}
\end{array}
$$

Integration by Parts

Every differentiation rule corresponds to a differential form rule which in turn corresponds to an integration rule.

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uv & = & \displaystyle\int v\mathrm{d}u + \int u\mathrm{d}v &
\end{array}
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uv & = & \displaystyle\int v\mathrm{d}u + \int u\mathrm{d}v & \text{rearrange} \\
\displaystyle\int u\mathrm{d}v & = & uv - \displaystyle\int v\mathrm{d}u &
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uv & = & \int v\mathrm{d}u + \int u\mathrm{d}v & \text{rearrange} \\
\int u\mathrm{d}v & = & uv - \int v\mathrm{d}u &
\end{array}
$$

We just proved the following.

Proposition ((Rule of) Integration by Parts)

$$\int u\mathrm{d}v = uv - \int v\mathrm{d}u$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$\int x \sin x \mathrm{d}x =$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$\int x \sin x \mathrm{d}x =$ | $\sin x \mathrm{d}x = \mathrm{d}(\textbf{?} \qquad)$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$\int x \sin x \mathrm{d}x =$ | $\sin x \mathrm{d}x = \mathrm{d}(\quad - \cos x)$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \quad \Big| \quad \sin x\mathrm{d}x = \mathrm{d}(\quad -\cos x)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \qquad \Big| \quad \sin x\mathrm{d}x = \mathrm{d}(\ -\cos x)$$

$$= x(-\cos x) - \int (\ -\cos x)\mathrm{d}x$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \qquad \Big| \quad \sin x\mathrm{d}x = \mathrm{d}(\ -\cos x)$$

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$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \quad \Big| \quad \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$

$$= x(-\cos x) - \int (-\cos x)\mathrm{d}x$$

$$= -x\cos x + \int \cos x\mathrm{d}x$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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$$= -x\cos x + \textbf{?}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int x\sin x\mathrm{d}x = \int x\mathrm{d}(-\cos x) \quad \Big| \quad \sin x\mathrm{d}x = \mathrm{d}(-\cos x)$$

$$= x(-\cos x) - \int (-\cos x)\mathrm{d}x$$

$$= -x\cos x + \int \cos x\mathrm{d}x$$

$$= -x\cos x + \sin x + C$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int \ln x\mathrm{d}x \quad =$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int \ln x \mathrm{d}x = (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int \ln x \mathrm{d}x = (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int \ln x\mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\ &= x\ln x - \int x(\ln x)'\mathrm{d}x\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u$.

Example

$$\begin{aligned}
\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x\textbf{?}\quad \mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x \ \frac{1}{x}\mathrm{d}x
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\
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&= x\ln x - \int x\ \frac{1}{x}\mathrm{d}x \\
&= x\ln x - \int \mathrm{d}x \\
&= x\ln x - \textbf{?}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u$.

Example

$$\begin{aligned}
\int \ln x \mathrm{d}x &= (\ln x)x - \int x\mathrm{d}(\ln x) \quad \Big| \text{ integrate by parts} \\
&= x\ln x - \int x(\ln x)'\mathrm{d}x \\
&= x\ln x - \int x\ \frac{1}{x}\mathrm{d}x \\
&= x\ln x - \int \mathrm{d}x \\
&= x\ln x - x + C \quad .
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t \quad = \quad \int t^2 \mathrm{d}\left(\textbf{?} \quad\right)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t = \int t^2 \mathrm{d}\left(e^t\right)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t = \int t^2 \mathrm{d}\left(\ e^t\right) \quad \Big| \quad \text{integrate by parts}$$

$$= \textbf{?}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int t^2 e^t \mathrm{d}t = \int t^2 \mathrm{d}\left(e^t\right) \quad \Big| \text{ integrate by parts}$$

$$= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right)$$

Integration by parts: $\int u\,dv = uv - \int v\,du$.

Example

$$\begin{aligned} \int t^2 e^t\,dt &= \int t^2\,d\left(e^t\right) \\ &= t^2 e^t - \int e^t\,d\left(t^2\right) \end{aligned}$$

integrate by parts

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned} \int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) \\ &= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \end{aligned}$$

integrate by parts

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(\ e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t \textbf{?}\ \ \mathrm{d}t
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t \ 2t\mathrm{d}t
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(\ e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t \ \ 2t\mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(\textbf{?}\quad)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(\ e^t\right) && \text{integrate by parts} \\
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&= t^2 e^t - 2\int t\mathrm{d}(\ e^t)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(\ e^t\right) && \text{integrate by parts} \\
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&= t^2 e^t - 2\int t\mathrm{d}(\ \ e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\textbf{?}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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$$\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(\ e^t\right) && \text{integrate by parts} \\
&= t^2 e^t - \int e^t \mathrm{d}\left(t^2\right) \\
&= t^2 e^t - \int e^t \ 2t\mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(\ e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\left(te^t - \int e^t \mathrm{d}t\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u$.

Example

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\begin{aligned}
\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(\ e^t\right) && \text{integrate by parts} \\
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\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
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&= t^2 e^t - \int e^t \ 2t\mathrm{d}t \\
&= t^2 e^t - 2\int t\mathrm{d}(\ e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\left(te^t - \int e^t \mathrm{d}t\right) \\
&= t^2 e^t - 2te^t + 2e^t + C
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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\int t^2 e^t \mathrm{d}t &= \int t^2 \mathrm{d}\left(e^t\right) && \text{integrate by parts} \\
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&= t^2 e^t - 2\int t\mathrm{d}(\ e^t) && \text{integrate by parts} \\
&= t^2 e^t - 2\left(te^t - \int e^t \mathrm{d}t\right) \\
&= t^2 e^t - 2te^t + 2e^t + C
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int e^x \sin x \mathrm{d}x = \int \sin x \, \mathrm{d}(\quad)$$

$$=$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int e^x \sin x \mathrm{d}x = \int \sin x \, \mathrm{d}(\textbf{?}\quad)$$

$$=$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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$$= \textbf{?}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}(\ e^x) \\ &= (\sin x)e^x - \int e^x \mathrm{d}(\sin x)\end{aligned}$$

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Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}(\ e^x) \\ &= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\ &= e^x \sin x - \int e^x \ \cos x \mathrm{d}x\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}(\ \ e^x) \\
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&= e^x \sin x - \int \cos x \mathrm{d}(\textbf{?}\ \ )
\end{aligned}$$

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&= e^x \sin x - \int \cos x \mathrm{d}(\ e^x) \\
&= e^x \sin x - \left(\textbf{?} \right)
\end{aligned}$$

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&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \ \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}(\ e^x) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x) \right)
\end{aligned}$$

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&= e^x \sin x - \int \cos x \mathrm{d}(\ e^x) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x) \right) \\
&= e^x \sin x - \cos x e^x + \int e^x \textbf{?}
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

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\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}(\ e^x) \\
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&= e^x \sin x - \int e^x \ \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}(\ e^x) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x) \right) \\
&= e^x \sin x - \cos x e^x + \int e^x \textbf{?}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$
\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}(\ \ e^x) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
&= e^x \sin x - \int e^x \ \ \cos x \mathrm{d}x \\
&= e^x \sin x - \int \cos x \mathrm{d}(\ \ e^x) \\
&= e^x \sin x - \left(\ \ (\cos x)e^x - \int e^x \mathrm{d}(\cos x) \right) \\
&= e^x \sin x - \cos x e^x + \int e^x \ \ (-\sin x)\mathrm{d}x
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$
\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}(\ \ e^x) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
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&= e^x \sin x - \int \cos x \mathrm{d}(\ \ e^x) \\
&= e^x \sin x - \left(\ \ (\cos x)e^x - \int e^x \mathrm{d}(\cos x) \right) \\
&= e^x \sin x - \cos x e^x + \int e^x \ \ (-\sin x)\mathrm{d}x \\
&= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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$$
\begin{aligned}
\int e^x \sin x \mathrm{d}x &= \int \sin x \, \mathrm{d}(\ e^x) \\
&= (\sin x)e^x - \int e^x \mathrm{d}(\sin x) \\
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&= e^x \sin x - \int \cos x \mathrm{d}(\ e^x) \\
&= e^x \sin x - \left((\cos x)e^x - \int e^x \mathrm{d}(\cos x) \right) \\
&= e^x \sin x - \cos x e^x + \int e^x \ (-\sin x)\mathrm{d}x \\
&= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x
\end{aligned}
$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int e^x \sin x \mathrm{d}x = e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned} \int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x \\ 2\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x \end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x \\
2\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x \\
\int e^x \sin x \mathrm{d}x &= \frac{1}{2}\left(e^x \sin x - \cos x e^x\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x - \int e^x \sin x \mathrm{d}x \\
2\int e^x \sin x \mathrm{d}x &= e^x \sin x - \cos x e^x \\
\int e^x \sin x \mathrm{d}x &= \frac{1}{2}\left(e^x \sin x - \cos x e^x\right) + C
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x\mathrm{d}x \quad =$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x\mathrm{d}x \quad = \quad \textbf{?}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x\mathrm{d}x \quad = \quad [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\,(\arctan x)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\int_0^1 \arctan x\mathrm{d}x = [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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$$\int_0^1 \arctan x\mathrm{d}x \quad = \quad [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}\,(\arctan x)$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\ &= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\textbf{?}\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\ &= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\textbf{?}\end{aligned}$$

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Example

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Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\
&= 1\cdot \arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x \quad \frac{1}{1+x^2}\mathrm{d}x \\
&= \textbf{?} \quad - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\textbf{?} \quad\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\
&= 1\cdot \arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x \ \frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\ \textbf{?}\ \right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

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\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\
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&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\ ?\ \right)
\end{aligned}$$

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\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\
&= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\ \frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\
&= 1\cdot\arctan 1 - 0\cdot\arctan 0 - \int_{x=0}^{x=1} x\ \frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(\qquad)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

$$\begin{aligned}
\int_0^1 \arctan x\mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\
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&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

Set $w = 1 + x^2$.

$$\begin{aligned}
\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2)
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&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

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\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\
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&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1}
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\
&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}\left[\ln|w|\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}(\ln 2 - \ln 1)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

Example

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\int_0^1 \arctan x \mathrm{d}x &= [(\arctan x)x]_{x=0}^{x=1} - \int_0^1 x\mathrm{d}(\arctan x) \\
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&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}[\ln|w|]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}(\ln 2 - \ln 1)
\end{aligned}$$

Integration by parts: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u.$

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&= 1 \cdot \arctan 1 - 0 \cdot \arctan 0 - \int_{x=0}^{x=1} x \frac{1}{1+x^2}\mathrm{d}x \\
&= \frac{\pi}{4} - \int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}\left(\frac{x^2}{2}\right) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{1+x^2}\mathrm{d}(1+x^2) \\
&= \frac{\pi}{4} - \frac{1}{2}\int_{x=0}^{x=1} \frac{1}{w}\mathrm{d}w = \frac{\pi}{4} - \frac{1}{2}[\ln|w|]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}\left[\ln\left(1+x^2\right)\right]_{x=0}^{x=1} \\
&= \frac{\pi}{4} - \frac{1}{2}(\ln 2 - \ln 1) = \frac{\pi}{4} - \frac{1}{2}\ln 2 \quad .
\end{aligned}$$