Math 141

Lecture 8

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Outline

Example

Find $\lim_{x\to 1} \frac{\ln x}{x-1}$.

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- We don't get any cancellation between top and bottom.
- We need new techniques.

Theorem (L'Hospital's Rule)

Suppose that f and g are differentiable and $g'(x) \neq 0$ on an open interval that contains a (except possibly at a). Suppose that

$$\lim_{x\to a} f(x) = 0 \qquad \textit{and} \qquad \lim_{x\to a} g(x) = 0$$

$$\textit{or that} \qquad \lim_{x\to a} f(x) = \pm\infty \qquad \textit{and} \qquad \lim_{x\to a} g(x) = \pm\infty$$

(In other words, we have an indeterminate form of type $0/0$ or ∞/∞.) Then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}.$$

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This converts the given limit into an indeterminate form of type $0/0$ or ∞/∞.

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$$\begin{aligned}\lim_{x\to 0^+} x\ln x &= \lim_{x\to 0^+} \frac{\ln x}{1/x} = \lim_{x\to 0^+} \frac{\frac{d}{dx}(\ln x)}{\frac{d}{dx}(1/x)} \\ &= \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+}(-x) = 0.\end{aligned}$$

Indeterminate Differences

If $\lim_{x\to a} f(x) = \infty$ and $\lim_{x\to a} g(x) = \infty$, then the limit

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is called an indeterminate form of type $\infty - \infty$.
To compute such a limit, try to convert it into a quotient (by using a common denominator, or by rationalizing, or by factoring out a common factor).

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Evaluate $\lim_{x \to (\pi/2)^-} (\sec x - \tan x)$.

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Indeterminate Powers

Several indeterminate forms arise from the limit $\lim_{x\to a} f(x)^{g(x)}$.

$$\lim_{x\to a} f(x) = 0 \quad \text{and} \quad \lim_{x\to a} g(x) = 0 \qquad \text{type } 0^0$$

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These can all be solved either by taking the natural logarithm:

$$\text{let} \;\; y = [f(x)]^{g(x)}, \;\; \text{then} \;\; \ln y = g(x) \ln f(x)$$

or by writing the function as an exponential:

$$[f(x)]^{g(x)} = e^{g(x) \ln f(x)}.$$

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