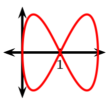
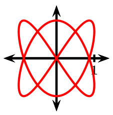
Freecalc
Homework Reduced Lecture 16, will not be quizzed

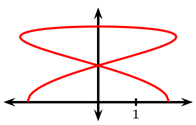
1. Match the graphs of the parametric equations $x = f(t)$, $y = g(t)$ with the graph of the parametric curve $\gamma \left| \begin{array}{lcl} x &=& f(t) \\ y &=& g(t) \end{array} \right.$

(a) 

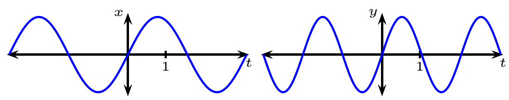
answer: matches to 1c

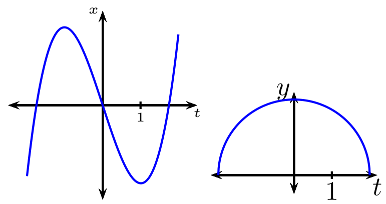
(b) 

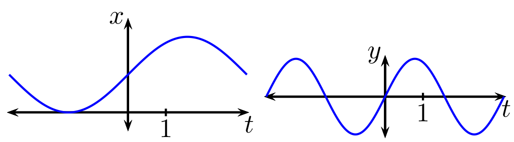
answer: matches to 1a

(c) 

answer: matches to 1b

(a) 

(b) 

(c) 

2. Plot the curve. Set up an integral that expresses its length. Find the length of the curve.

   (a) $y = \sqrt{x}$, $x \in [1, 2]$.

   (b) $y = x^2$, $x \in [1, 2]$.

   (c) $\gamma : \left| \begin{array}{lcl} x(t) &=& \frac{1}{t} + \frac{t^3}{3} \\ y(t) &=& 2t \end{array} \right. , t \in [1, 2]$.

   (d) $x = \sqrt{t} - 2t$ and $y = \dfrac{8}{3} t^{\frac{3}{4}}$ from $t = 1$ to $t = 4$.

Solution. 2.d. The length of the parametric curve is given by

$$L = \int_1^4 \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2} \mathrm{d}t \quad .$$

We have that

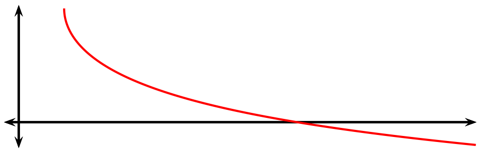
$$\begin{array}{rcl}
\frac{\mathrm{d}x}{\mathrm{d}t} &=& \frac{1}{2\sqrt{t}} - 2 \\
\frac{\mathrm{d}y}{\mathrm{d}t} &=& 2t^{-\frac{1}{4}} \\
\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 &=& \frac{1}{4t} - \frac{2}{\sqrt{t}} + 4 \\
\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 &=& 4t^{-\frac{1}{2}} = \frac{4}{\sqrt{t}} \\
\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 &=& \frac{1}{4t} + 2\frac{1}{\sqrt{t}} + 4 = \left(\frac{1}{2\sqrt{t}} + 2\right)^2 \quad .
\end{array}$$

$\frac{1}{2\sqrt{t}} + 2$ is positive and $\sqrt{\left(\frac{1}{2\sqrt{t}} + 2\right)^2} = \frac{1}{2\sqrt{t}} + 2$. So the integral becomes

$$L = \int_1^4 \left(\frac{1}{2\sqrt{t}} + 2\right) \mathrm{d}t = \left[\sqrt{t} + 2t\right]_{t=1}^{t=4} = (2 + 8) - (1 + 2) = 7 \quad .$$

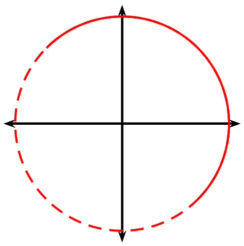
3. Set up an integral that expresses the length of the curve and find the length of the curve.

(a) $\left| \begin{array}{lcl} x(t) & = & e^t + e^{-t} \\ y(t) & = & 5 - 2t \end{array} \right. , t \in [0, 3]$



answer: $e^3 - e^{-3}$

(b) $\left| \begin{array}{lcl} x(t) & = & \sin t + \cos t \\ y(t) & = & \sin t - \cos t \end{array} \right. , t \in [0, \pi]$



answer: $\sqrt{2}\pi$