Math 141

Lecture 18[material skipped, included on final as bonus]

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University of Massachusetts Boston

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Outline

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Polar Coordinates

- The polar coordinate system is an alternative to the Cartesian coordinate system.

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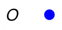
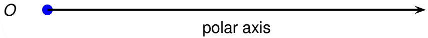
- Choose a point in the plane called O (the origin).



Polar Coordinates

- The polar coordinate system is an alternative to the Cartesian coordinate system.
- Choose a point in the plane called O (the origin).
- Draw a ray starting at O. The ray is called the polar axis. This ray is usually drawn horizontally to the right.



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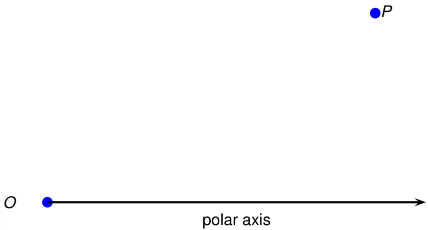

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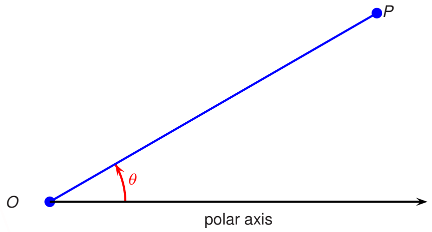

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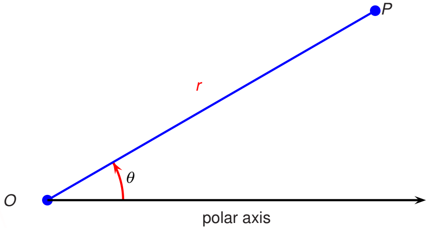

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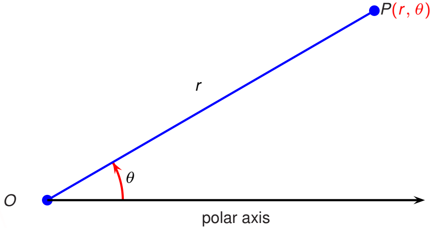

- Let P be a point in the plane.
- Let θ denote the angle between the polar axis and the line OP.
- Let r denote the length of the segment OP.
- Then P is represented by the ordered pair (r, θ).

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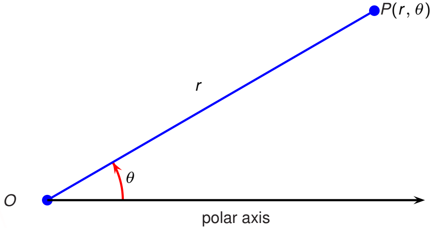

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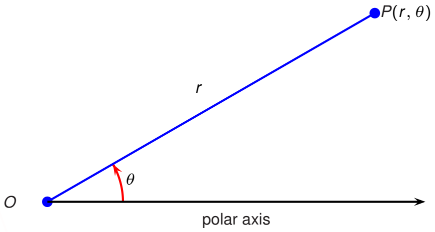

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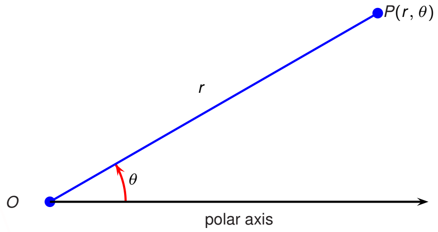

- The letters (x, y) imply Cartesian coordinates and the letters (r, θ)- polar. When we use other letters, it should be clear from context whether we mean Cartesian or polar coordinates. If not, one must request clarification.

1. What if θ is negative?
2. What if r is negative?
3. What if r is 0?

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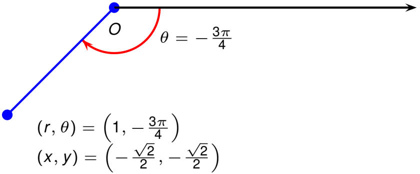

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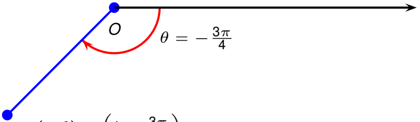

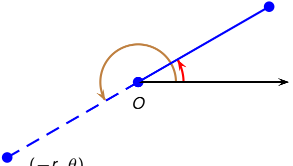

1. Positive angles θ are measured in the counterclockwise direction from O. Negative angles are measured in the clockwise direction.
2. Points with polar coordinates $(-r,\theta)$ and (r,θ) lie on the same line through O and at the same distance from O, but on opposite sides.

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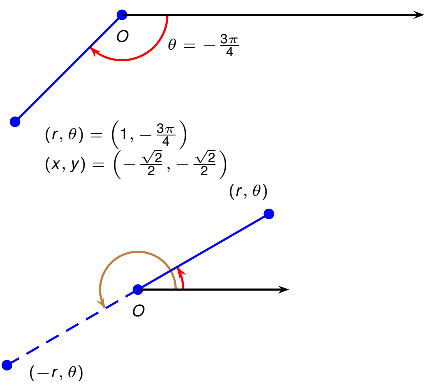
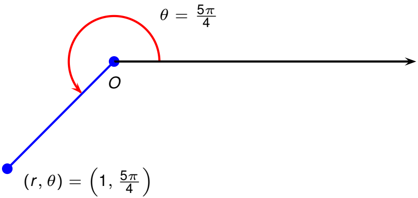
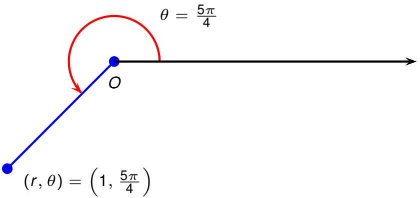
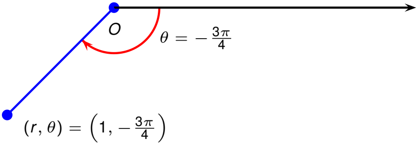


1. Positive angles θ are measured in the counterclockwise direction from O. Negative angles are measured in the clockwise direction.
2. Points with polar coordinates $(-r, \theta)$ and (r, θ) lie on the same line through O and at the same distance from O, but on opposite sides.
3. If $r = 0$, then $(0, \theta)$ represents O for all values of θ.

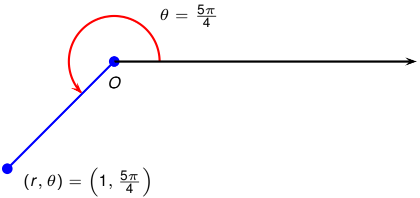


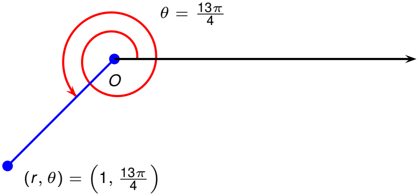
- There are many ways to represent the same point.

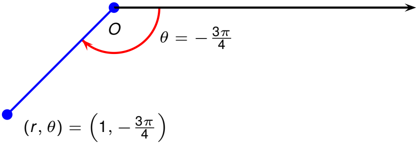




- There are many ways to represent the same point.
- We could use a negative θ.







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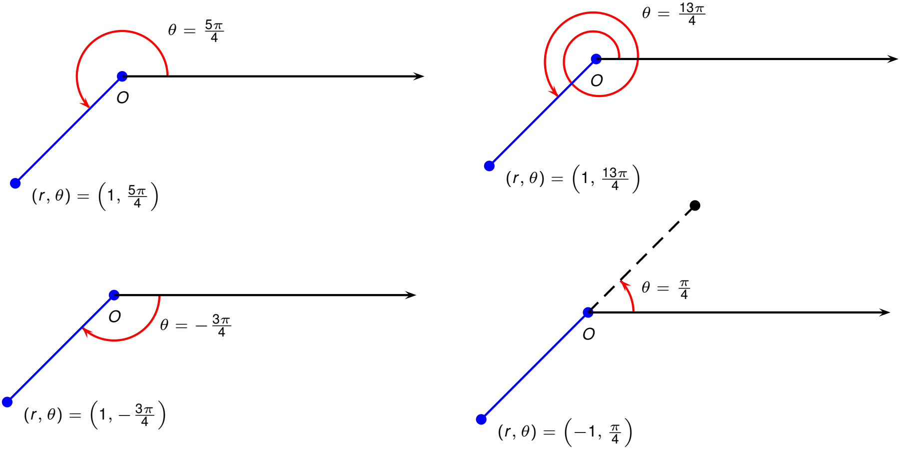
- We could use a negative θ.
- We could go around more than once.



- There are many ways to represent the same point.
- We could use a negative θ.
- We could go around more than once.
- We could use a negative r.

- Let P_1 be point with polar coordinates (r_1, θ_1).
- Let P_2 be point with polar coordinates (r_2, θ_2).

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Observation

P_1 coincides with P_2 if one of the three mutually exclusive possibilities holds:

- $r_1 = r_2 \neq 0$ *and* $\theta_2 = \theta_1 + 2k\pi, k \in \mathbb{Z}$,
- $r_1 = -r_2 \neq 0$ *and* $\theta_2 = \theta_1 + (2k+1)\pi, k \in \mathbb{Z}$,
- $r_1 = r_2 = 0$ *and* θ *is arbitrary.*

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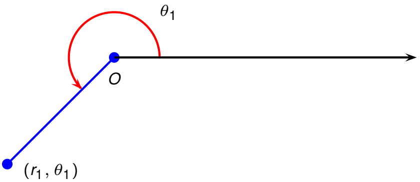

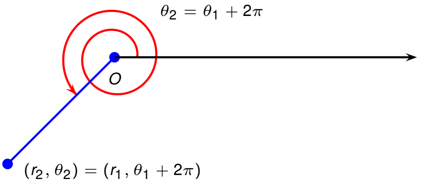

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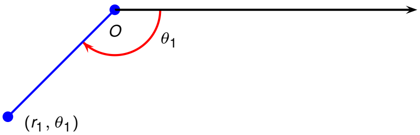
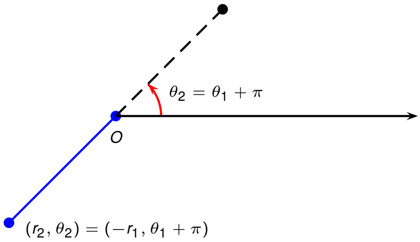
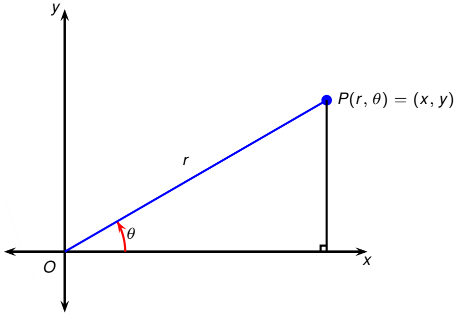




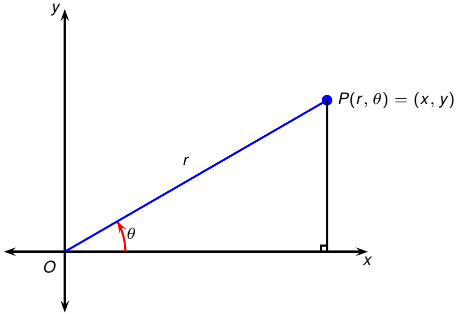
- How do we go from polar coordinates to Cartesian coordinates?



$$x =$$
$$y =$$

$$r =$$
$$\theta =$$

- How do we go from polar coordinates to Cartesian coordinates?
- Suppose a point has polar coordinates (r, θ) and Cartesian coordinates (x, y).



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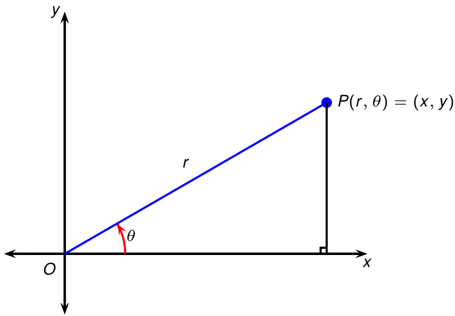

$$x =$$
$$y =$$

$$\cos\theta =$$
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$$r =$$
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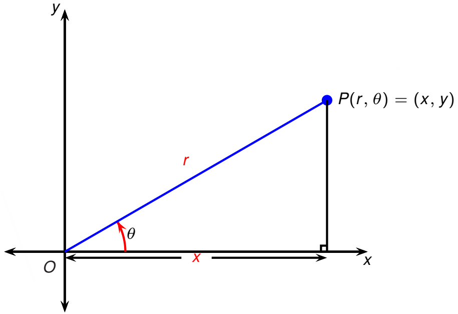

$$x =$$
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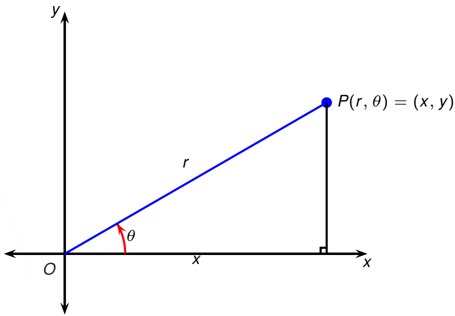

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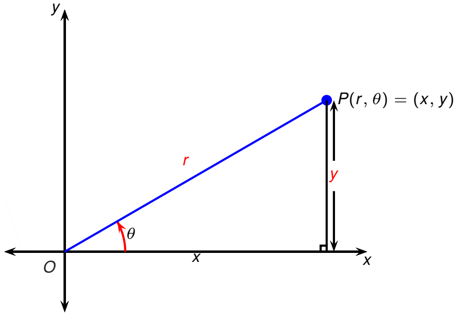

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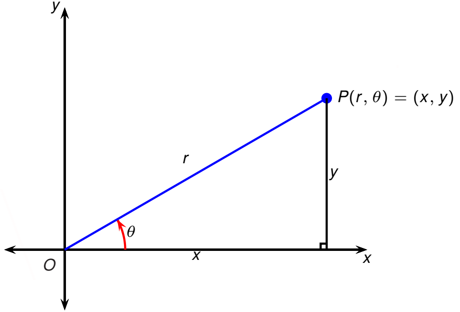

$$x = r\cos\theta$$
$$y = r\sin\theta$$

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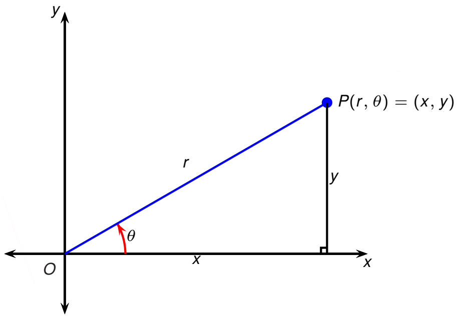

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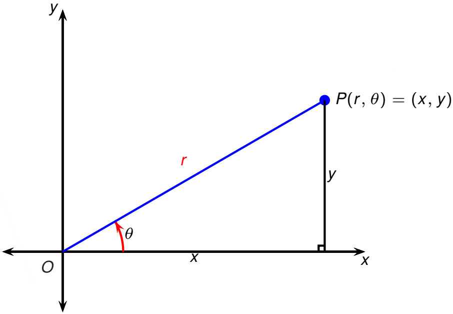

$$x = r\cos\theta$$
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$$\cos\theta = \frac{x}{r}$$
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$$r^2 =$$

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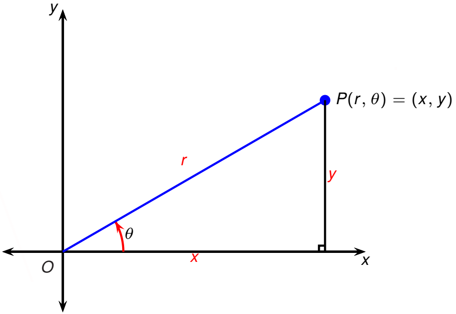

$$
\begin{aligned}
x &= r\cos\theta \\
y &= r\sin\theta \\
\hline
\cos\theta &= \frac{x}{r} \\
\sin\theta &= \frac{y}{r} \\
r^2 &= x^2 + y^2 \\
\hline
r &= \\
\theta &=
\end{aligned}
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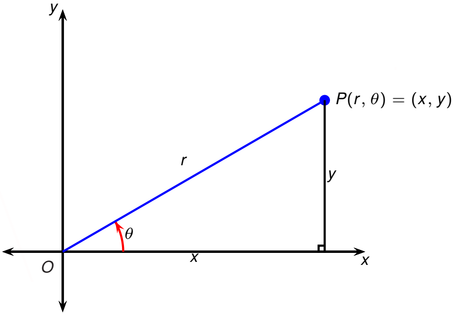

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\hline
r &= \sqrt{x^2 + y^2} \\
\theta &=
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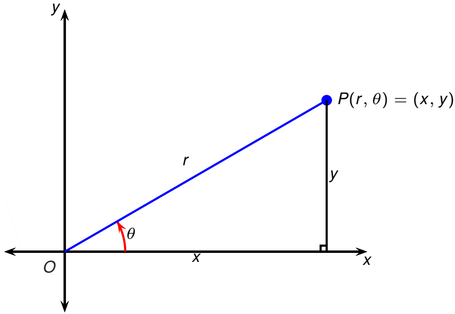

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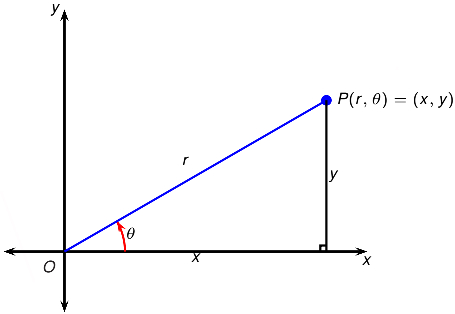

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r^2 &= x^2 + y^2 \\
\hline
r &= \sqrt{x^2 + y^2} \\
\theta &= \arcsin\left(\frac{y}{r}\right) \quad \text{if } x > 0 \\
&= \arccos\left(\frac{x}{r}\right) \quad \text{if } y > 0 \\
&= \arctan\left(\frac{y}{x}\right) \quad \text{if } x > 0
\end{aligned}
$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

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$$x = r\cos\theta =$$

$$y = r\sin\theta =$$

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$$x = r\cos\theta = \quad \cos$$

$$y = r\sin\theta =$$

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$$x = r\cos\theta = 2\cos$$

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Example

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$$x = r\cos\theta = 2\cos\frac{\pi}{3}$$

$$y = r\sin\theta =$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\quad \right)$$

$$y = r\sin\theta =$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right)$$

$$y = r\sin\theta =$$

Example

Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right) = 1$$

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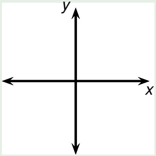
Convert the point $(2, \frac{\pi}{3})$ from polar to Cartesian coordinates.

$$x = r\cos\theta = 2\cos\frac{\pi}{3} = 2\left(\frac{1}{2}\right) = 1$$

$$y = r\sin\theta = 2\sin\frac{\pi}{3} = 2\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}$$

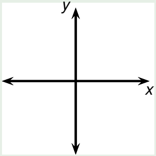
Therefore the point with polar coordinates $(2, \frac{\pi}{3})$ has Cartesian coordinates $(1, \sqrt{3})$.

Example



Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

Example

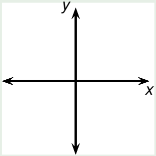


Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

$$r = \pm\sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

Example



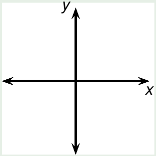
Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.

$$r = \pm\sqrt{x^2 + y^2}$$

$$\tan\theta = \frac{y}{x}$$

Example



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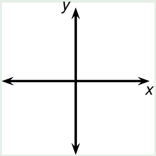


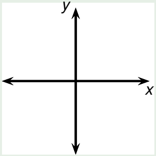
Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\tan\theta = \frac{y}{x}$$

Example

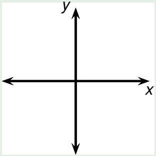


Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.

$$
\begin{aligned}
r &= \sqrt{x^2 + y^2} \\
&= \sqrt{1^2 + (-1)^2} \\
&= \sqrt{2} \\
\tan\theta &= \frac{y}{x} \\
&= -1
\end{aligned}
$$

Example



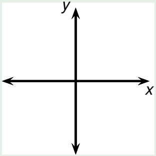
Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ &= -1 \end{aligned}$$

Example



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- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the quadrant.

$$
\begin{aligned}
r &= \sqrt{x^2 + y^2} \\
&= \sqrt{1^2 + (-1)^2} \\
&= \sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
\tan\theta &= \frac{y}{x} \\
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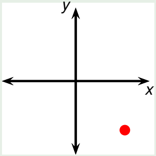
Example



Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the fourth quadrant.

$$
\begin{aligned}
r &= \sqrt{x^2 + y^2} \\
&= \sqrt{1^2 + (-1)^2} \\
&= \sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
\tan\theta &= \frac{y}{x} \\
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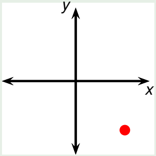
Example



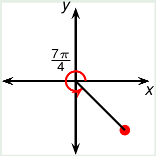
Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the fourth quadrant.
- Of the two values above, only $\theta =$ gives a point in the fourth quadrant.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ &= -1 \end{aligned}$$

Example



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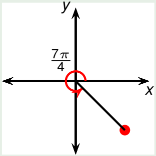
- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the fourth quadrant.
- Of the two values above, only $\theta = \frac{7\pi}{4}$ gives a point in the fourth quadrant.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ &= -1 \end{aligned}$$

Example



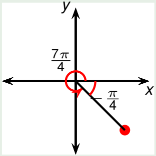
Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the fourth quadrant.
- Of the two values above, only $\theta = \frac{7\pi}{4}$ gives a point in the fourth quadrant.
- Therefore one possible representation of $(1, -1)$ in polar coordinates is $(\sqrt{2}, 7\pi/4)$.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ &= -1 \end{aligned}$$

Example



Represent the point with Cartesian coordinates $(1, -1)$ in terms of polar coordinates.

- Suppose r is positive.
- $\tan\theta = -1$ for $\theta = \frac{3\pi}{4}, \frac{7\pi}{4}$, and many other angles.
- $(1, -1)$ is in the fourth quadrant.
- Of the two values above, only $\theta = \frac{7\pi}{4}$ gives a point in the fourth quadrant.
- Therefore one possible representation of $(1, -1)$ in polar coordinates is $(\sqrt{2}, 7\pi/4)$.
- $(\sqrt{2}, -\pi/4)$ is another.

$$\begin{aligned} r &= \sqrt{x^2 + y^2} \\ &= \sqrt{1^2 + (-1)^2} \\ &= \sqrt{2} \end{aligned}$$

$$\begin{aligned} \tan\theta &= \frac{y}{x} \\ &= -1 \end{aligned}$$

- Recall polar coordinates:

$$\left| \begin{array}{lcl} x & = & r\cos\theta \\ y & = & r\sin\theta \end{array} \right.$$

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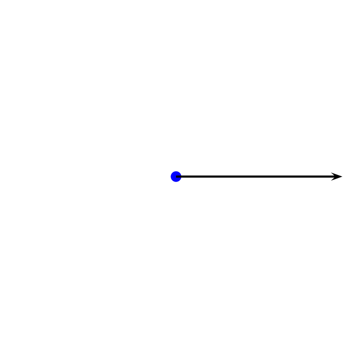
$$\left| \begin{array}{lcl} x & = & r\cos\theta \\ y & = & r\sin\theta \end{array} \right.$$

- A curve in polar coordinates is given by specifying explicit or implicit equations in polar coordinates.

Example

What curve is represented by the polar equation $r = 2$?



Example

What curve is represented by the polar equation $r = 2$?

- The equation describes all points that are 2 units away from O.

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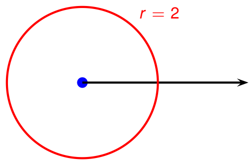

- The equation describes all points that are 2 units away from O.
- This is the circle with center O and radius 2.

Example

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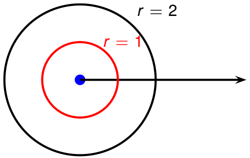

- The equation describes all points that are 2 units away from O.
- This is the circle with center O and radius 2.
- The equation $r = 1$ describes the unit circle.

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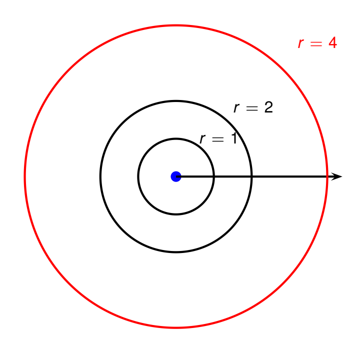
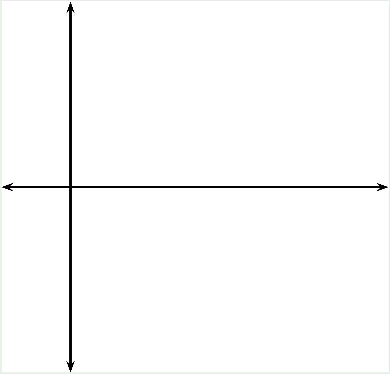


- The equation describes all points that are 2 units away from O.
- This is the circle with center O and radius 2.
- The equation $r = 1$ describes the unit circle.
- The equation $r = 4$ describes the circle with center O and radius 4.

Example

1. Sketch the curve with polar equation $r = 2\cos\theta$.
2. Find a Cartesian equation for this curve.



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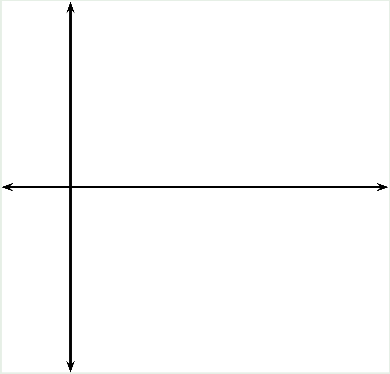

θ	r
0	
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

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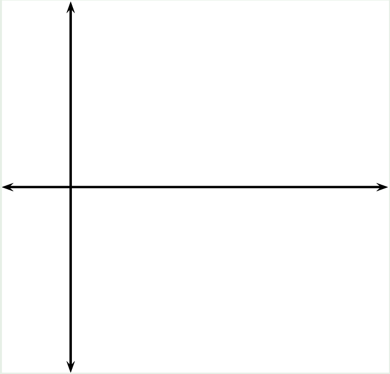

θ	r
0	
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

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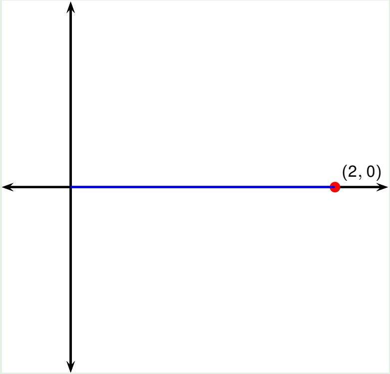

θ	r
0	2
$\pi/6$	
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

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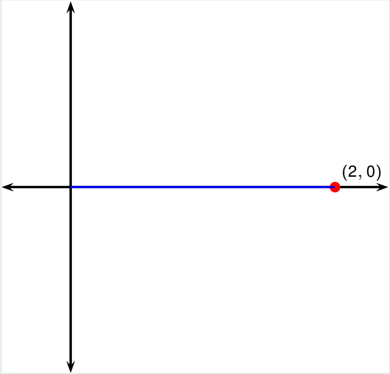

θ	r
0	2
$\pi/6$	
$\pi/4$	
$\pi/3$	
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$2\pi/3$	
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π	

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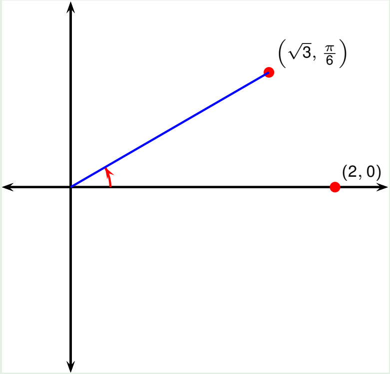

θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

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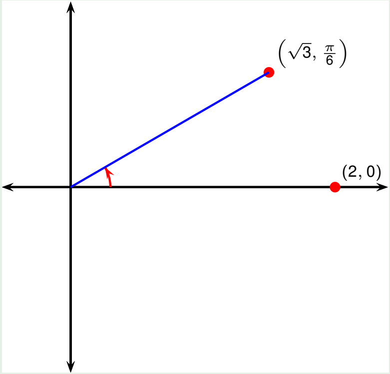

θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	
$\pi/3$	
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$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

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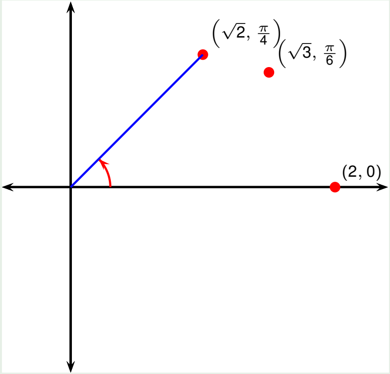

θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

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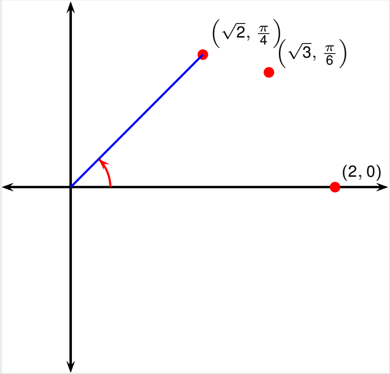

θ	r
0	2
$\pi/6$	$\sqrt{3}$
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$\pi/3$	
$\pi/2$	
$2\pi/3$	
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$5\pi/6$	
π	

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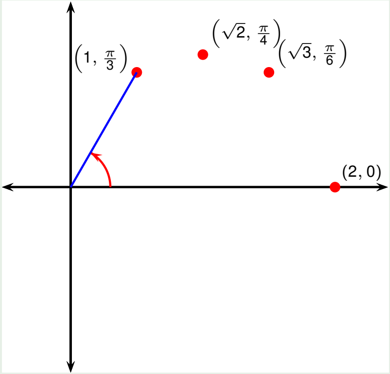

θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	
$2\pi/3$	
$3\pi/4$	
$5\pi/6$	
π	

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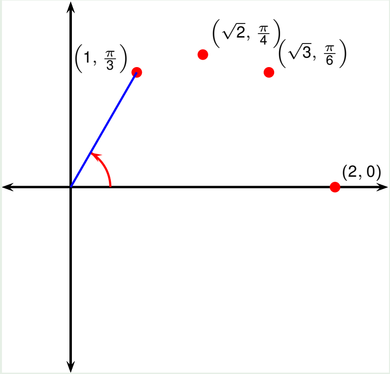

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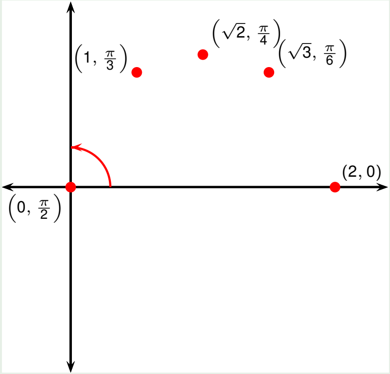

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0	2
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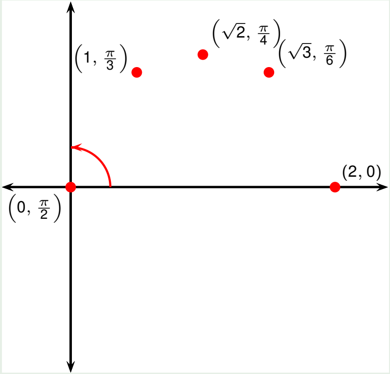

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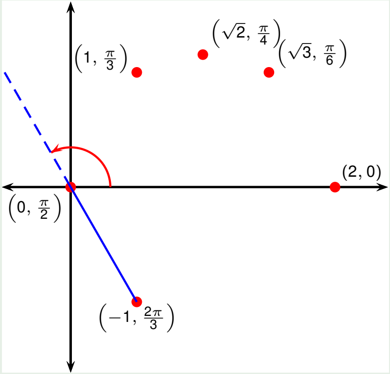

θ	r
0	2
$\pi/6$	$\sqrt{3}$
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$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
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$5\pi/6$	
π	

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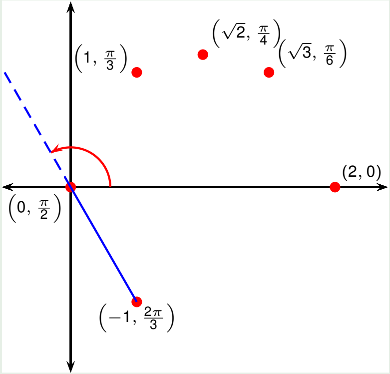

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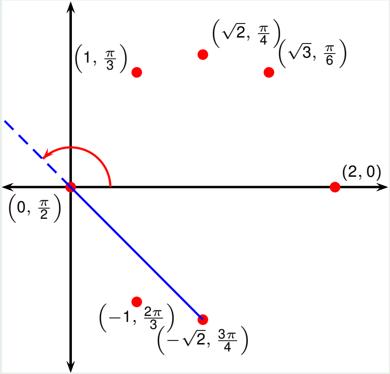

θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
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$2\pi/3$	-1
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$5\pi/6$	
π	

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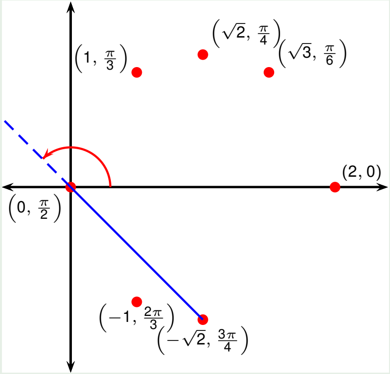

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$5\pi/6$	
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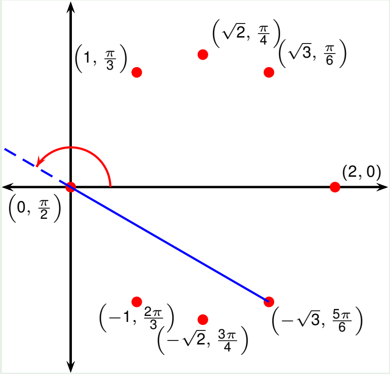

θ	r
0	2
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$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	

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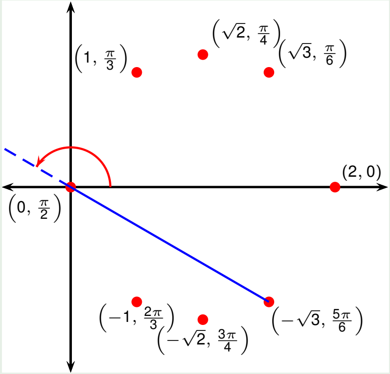

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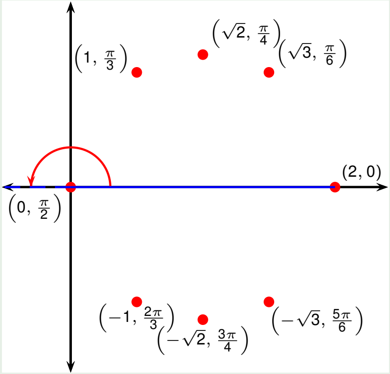
$\pi/2$	0
$2\pi/3$	-1
$3\pi/4$	$-\sqrt{2}$
$5\pi/6$	$-\sqrt{3}$
π	

Example

1. Sketch the curve with polar equation $r = 2\cos\theta$.
2. Find a Cartesian equation for this curve.



θ	r
0	2
$\pi/6$	$\sqrt{3}$
$\pi/4$	$\sqrt{2}$
$\pi/3$	1
$\pi/2$	0
$2\pi/3$	-1
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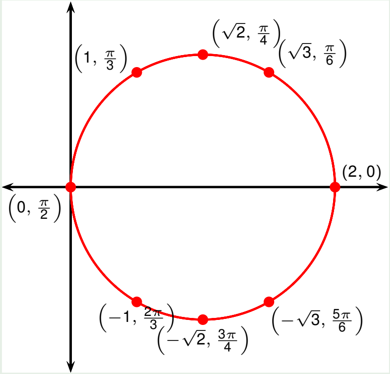

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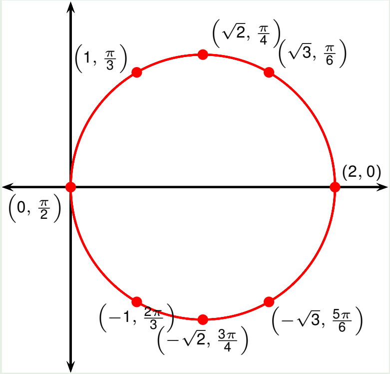

- $x =$

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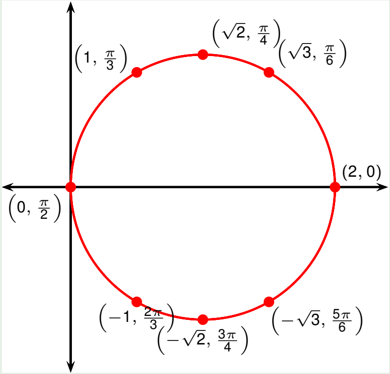

- $x = r\cos\theta$.

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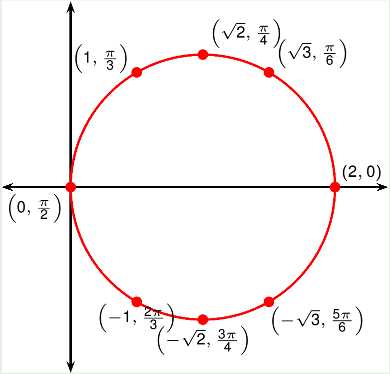

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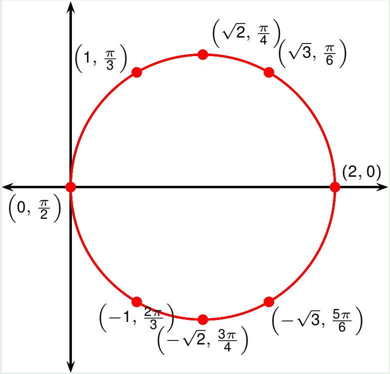

- $x = r\cos\theta$.
- $\cos\theta = x/r$.

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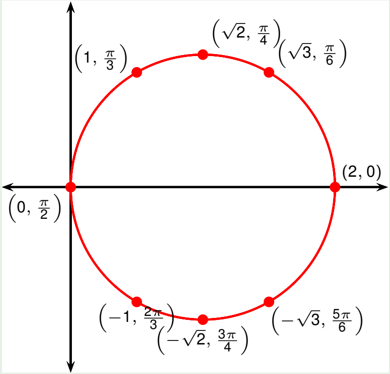

- $x = r\cos\theta$.
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- $r = 2\cos\theta =$

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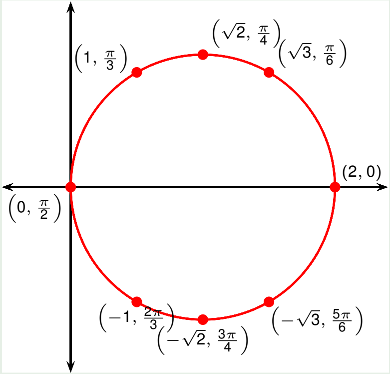

- $x = r\cos\theta$.
- $\cos\theta = x/r$.
- $r = 2\cos\theta = 2x/r$.

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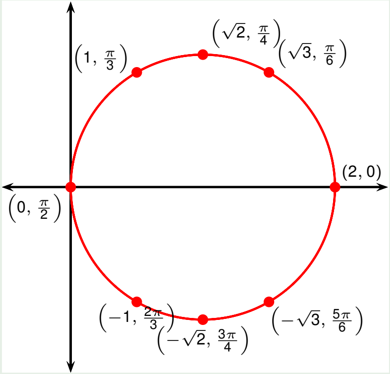

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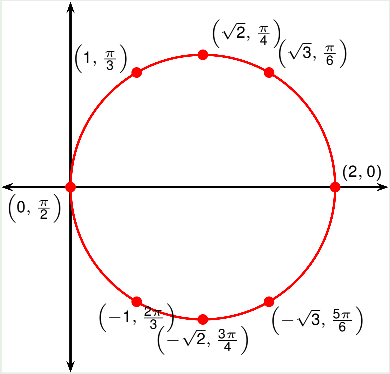

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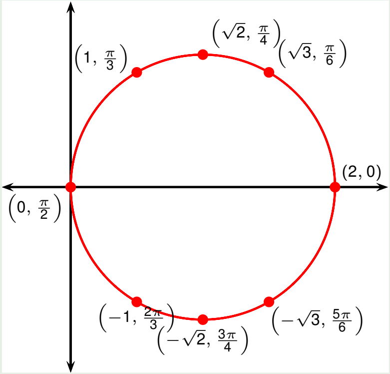

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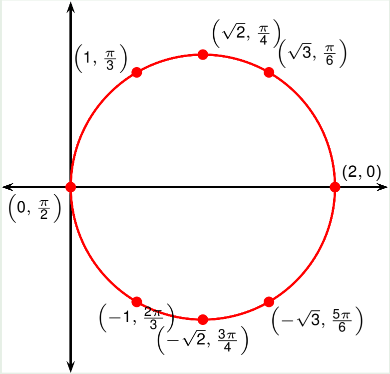

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- $x^2 + y^2 - 2x = 0$.

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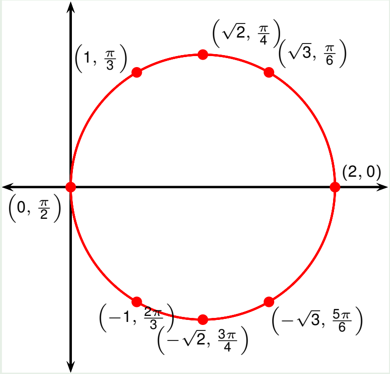

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- Complete the square:

$$(x^2 - 2x \qquad) + y^2 \quad = \quad 0$$

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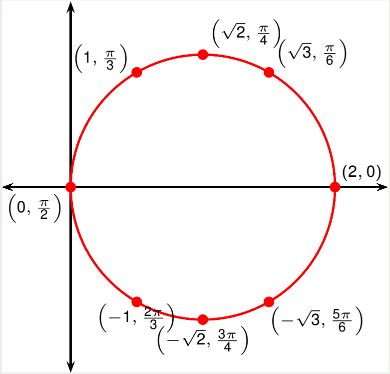

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$$(x^2 - 2x + 1) + y^2 = 0 + 1$$

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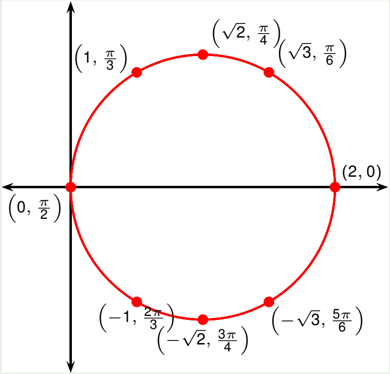

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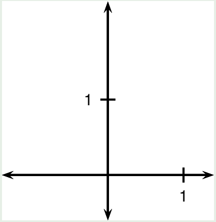
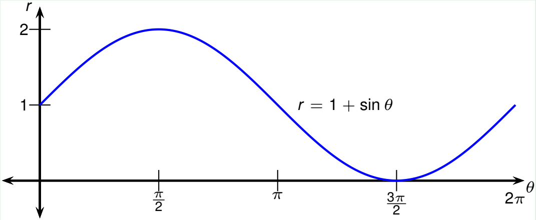
- $2x = r^2 = x^2 + y^2$.
- $x^2 + y^2 - 2x = 0$.
- Complete the square:

$$\begin{aligned}(x^2 - 2x + 1) + y^2 &= 0 + 1\\ (x-1)^2 + y^2 &= 1\end{aligned}$$

Example ( )

Sketch the curve $r = 1 + \sin\theta$.





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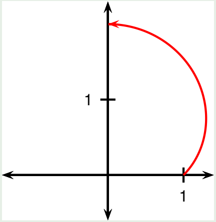

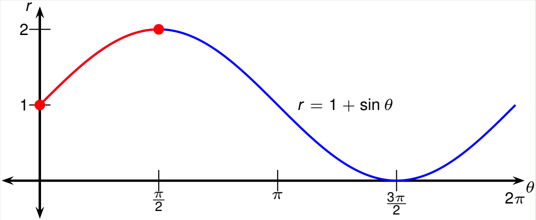

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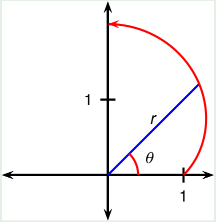

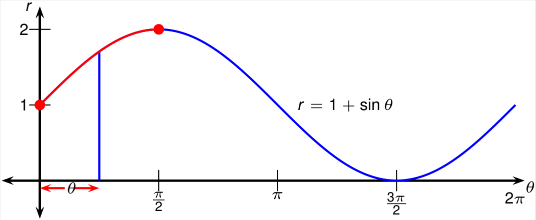

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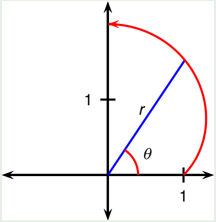

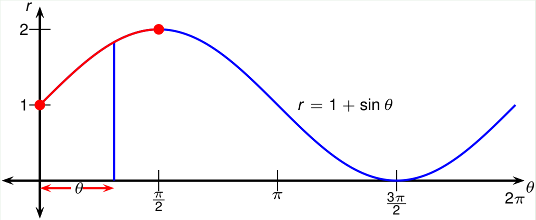

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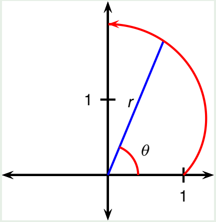

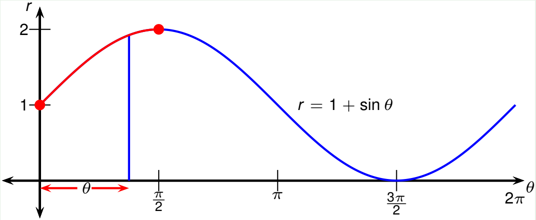

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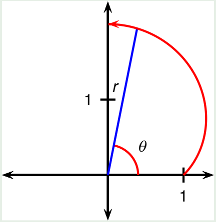

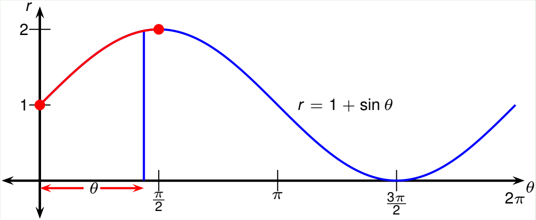

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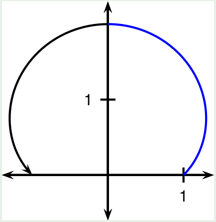

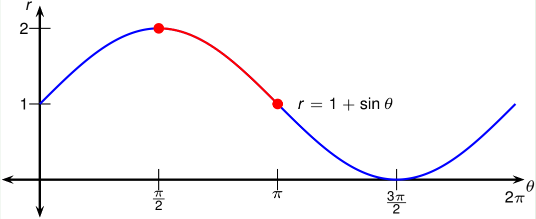

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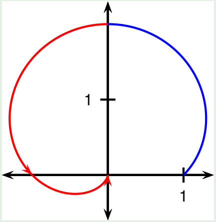

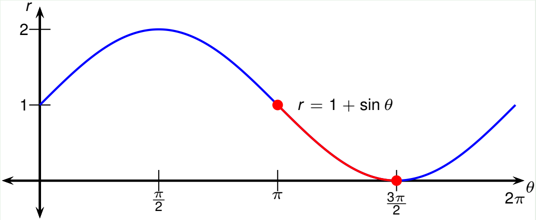

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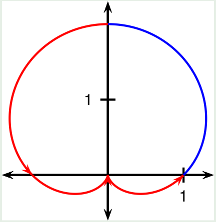
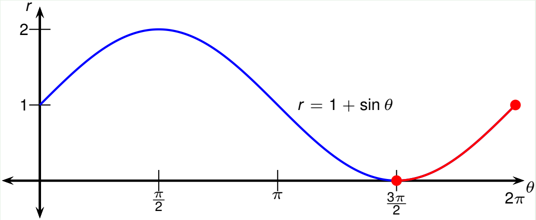
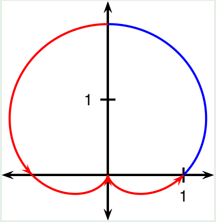
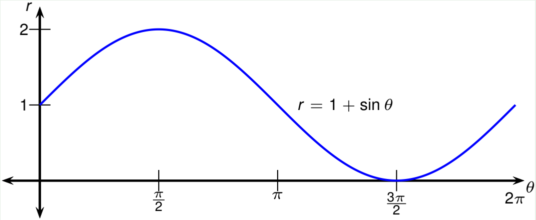




Example (Cardioid)

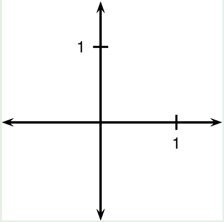
Sketch the curve $r = 1 + \sin\theta$.

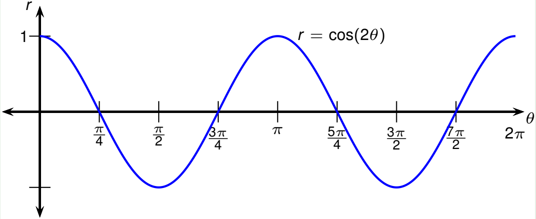




Example

Sketch the curve $r = \cos(2\theta)$.





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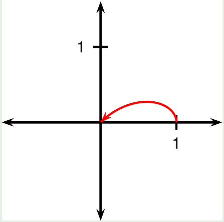

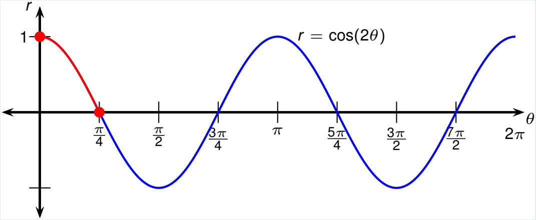

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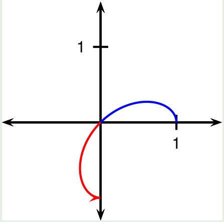

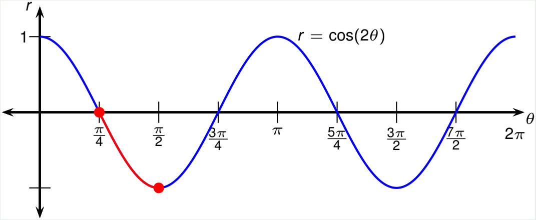

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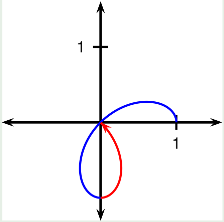

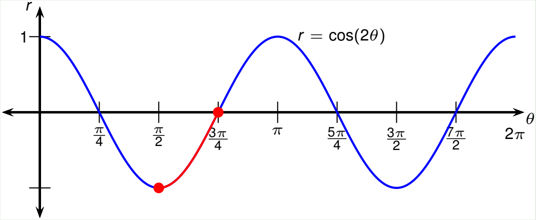

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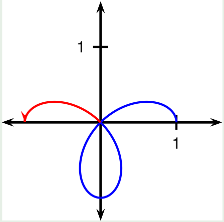

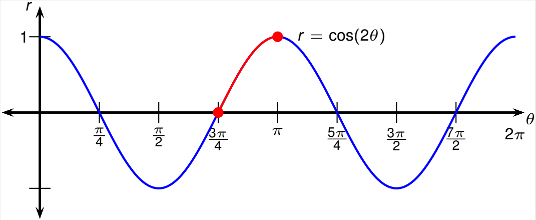

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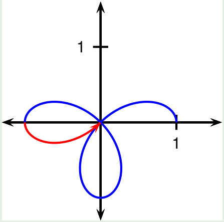

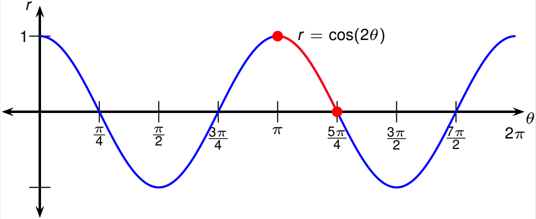

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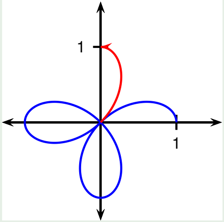

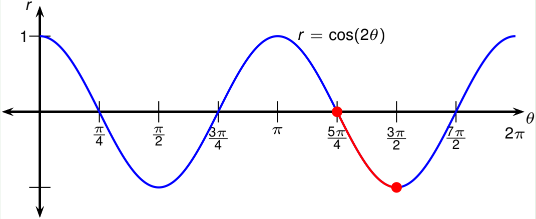

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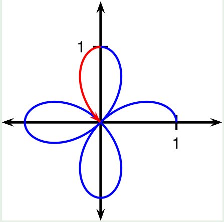

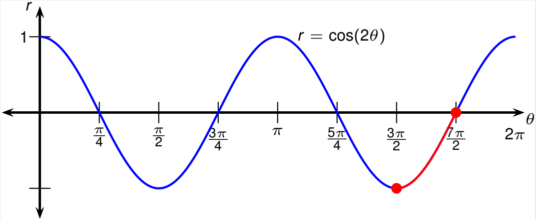

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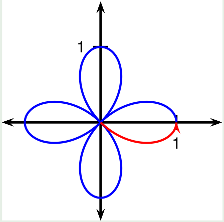

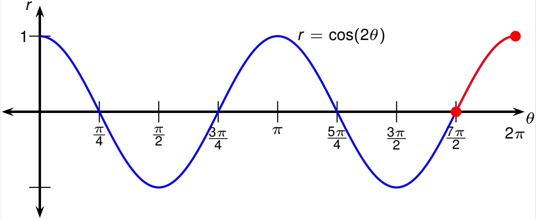

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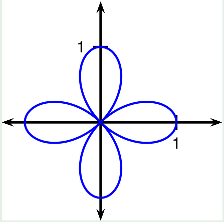
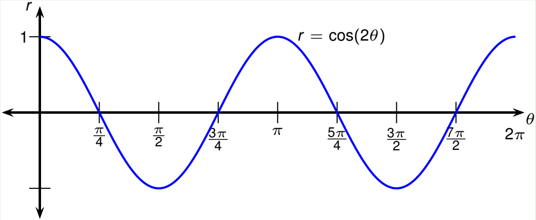
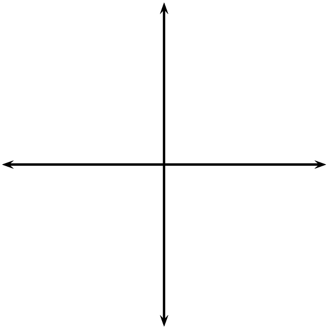




Symmetry

- If the polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis.
- If the equation is unchanged when θ is replaced by $\pi + \theta$, the curve is symmetric under rotation about the pole.
- If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \frac{\pi}{2}$.



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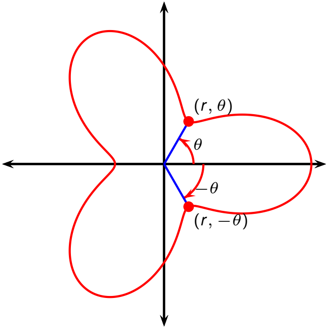

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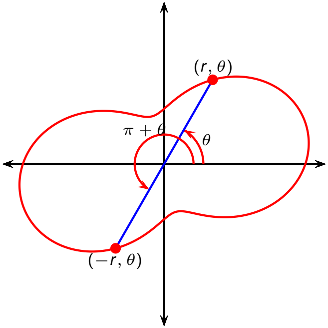
- If the equation is unchanged when θ is replaced by $\pi + \theta$, the curve is symmetric under rotation about the pole.
- If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \frac{\pi}{2}$.



Symmetry

- If the polar equation is unchanged when θ is replaced by $-\theta$, the curve is symmetric about the polar axis.
- If the equation is unchanged when θ is replaced by $\pi + \theta$, the curve is symmetric under rotation about the pole.
- If the equation is unchanged when θ is replaced by $\pi - \theta$, the curve is symmetric about the vertical line $\theta = \frac{\pi}{2}$.