Dijkstra's algorithm: another example

Step	N'	D(v),p(v)	D(w),p(w)	D(x),p(x)	D(y),p(y)	D(z),p(z)
0	u	2,u	5,u	1,u	∞	∞
1	ux	2,u	4,x		2,x	∞
2	uxy	2,u	3,y			4,y
3	uxyv		3,y			4,y
4	uxyvw					4,y
5	uxyvwz					

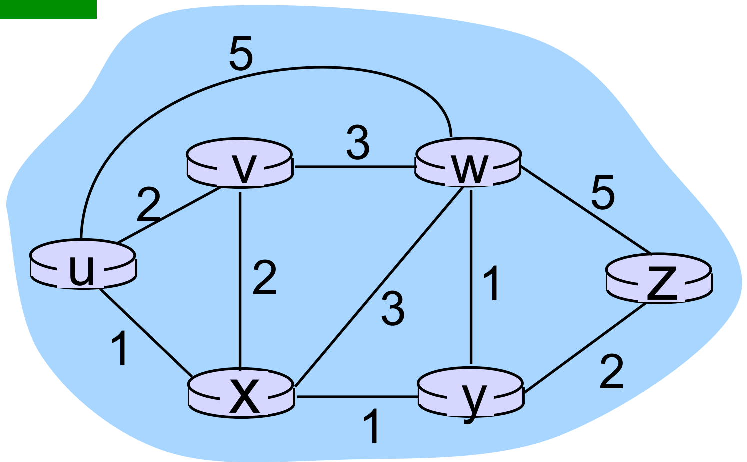
Node u's routing table: Shortest path from u to w according to above result

A. 5 through v

B. 4 through x

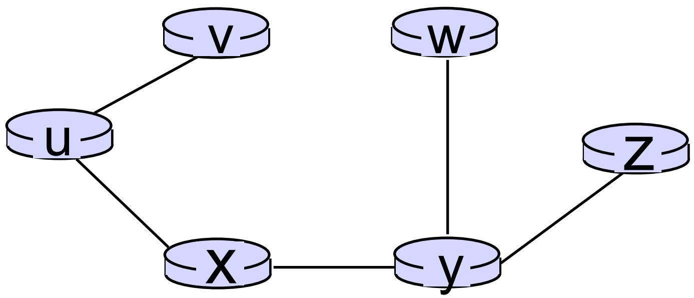
C. 3 through x

D. 3 through y



Dijkstra's algorithm: example (2)

resulting shortest-path tree from u:



resulting forwarding table in u:

destination	link
v	a
x	b
y	(u,x)
w	(u,x)
z	(u,x)

A. a = (u,v) ; b = (u,x)

B. a = (u,u) ; b = (u,u)

C. a = (u,x) ; b = (u,v)

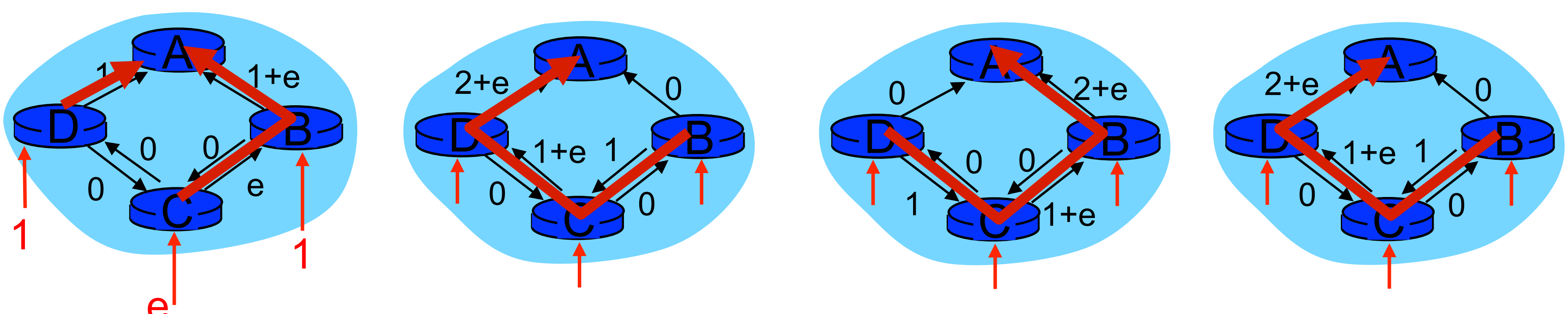
Dijkstra's algorithm, discussion

algorithm complexity: n nodes

- each iteration: need to check all nodes, w, not in N
- n(n+1)/2 comparisons: $O(n^2)$
- more efficient implementations possible: O(nlogn)

oscillations possible:

- e.g., support link cost equals amount of carried traffic:



initially

given these costs, find new routing…. resulting in new costs

given these costs, find new routing…. resulting in new costs

given these costs, find new routing…. resulting in new costs

Distance vector algorithm

Bellman-Ford equation (dynamic programming)

let

$d_x(y)$:= cost of least-cost path from x to y

then

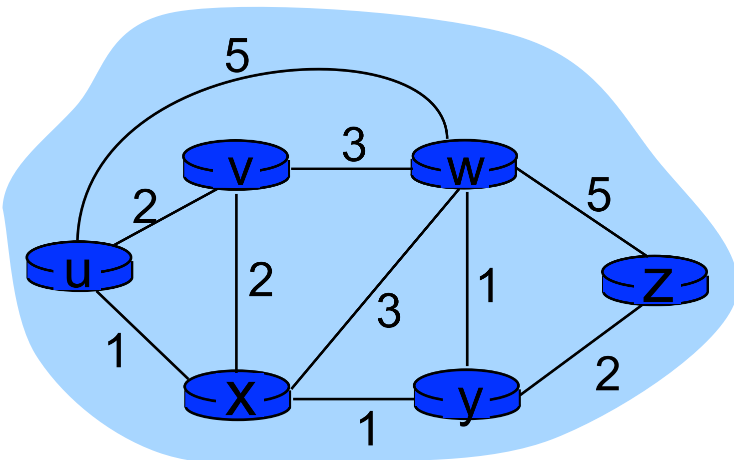
$$d_x(y) = \min_v \{c(x,v) + d_v(y)\}$$

cost from neighbor v to destination y

cost to neighbor v

min taken over all neighbors v of x

Bellman-Ford example



clearly, $d_v(z) = 5$, $d_x(z) = 3$, $d_w(z) = 3$

B-F equation says:

$$
\begin{aligned}
d_u(z) = \min \{ & c(u,v) + d_v(z), \\
& c(u,x) + d_x(z), \\
& c(u,w) + d_w(z) \} \\
= \min \{ & 2 + 5, \\
& 1 + 3, \\
& 5 + 3\} = 4
\end{aligned}
$$

node achieving minimum is next
hop in shortest path, used in forwarding table

Distance vector algorithm

- $D_x(y)$ = estimate of least cost from x to y
  - x maintains distance vector $\mathbf{D}_x = [D_x(y): y \in N]$
- node x:
  - knows cost to each neighbor v: $c(x,v)$
  - maintains its neighbors' distance vectors. For each neighbor v, x maintains
    $\mathbf{D}_v = [D_v(y): y \in N]$

Distance vector algorithm

key idea:

- from time-to-time, each node sends its own distance vector estimate to neighbors
- when x receives new DV estimate from neighbor, it updates its own DV using B-F equation:

$$D_x(y) \leftarrow \min_v\{c(x,v) + D_v(y)\} \quad \textit{for each node } y \in N$$

- under minor, natural conditions, the estimate $D_x(y)$ *converge to the actual least cost* $d_x(y)$

Distance vector algorithm

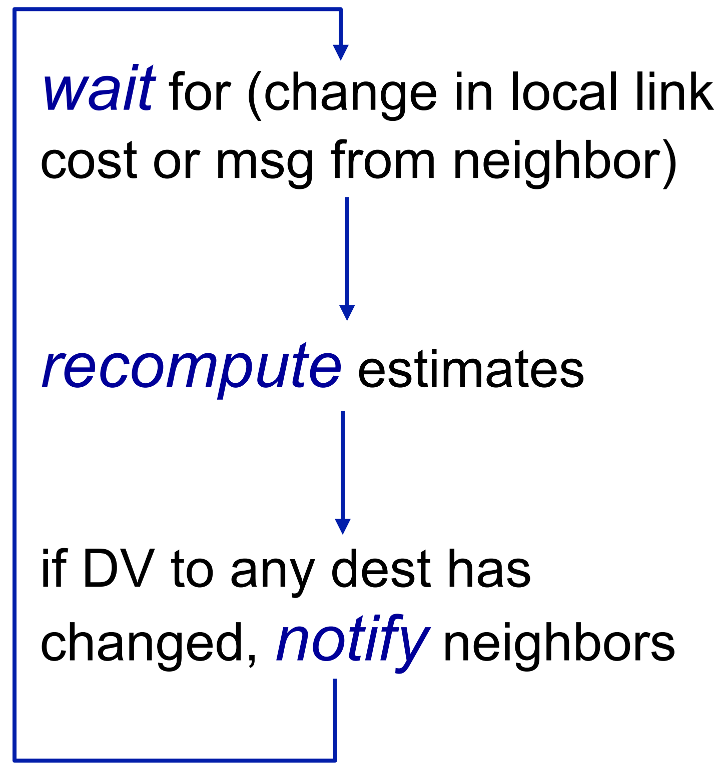
iterative, asynchronous:
each local iteration caused by:

- local link cost change
- DV update message from neighbor

distributed:

- each node notifies neighbors *only* when its DV changes
  - neighbors then notify their neighbors if necessary

each node:



$D_x(y) = \min\{c(x,y) + D_y(y), c(x,z) + D_z(y)\}$
$= \min\{2+0 , 7+1\} = 2$

$D_x(z) = \min\{c(x,y) + D_y(z), c(x,z) + D_z(z)\}$
$= \min\{2+1 , 7+0\} = 3$

node x table

from \ cost to	x	y	z
x	0	2	7
y	∞	∞	∞
z	∞	∞	∞

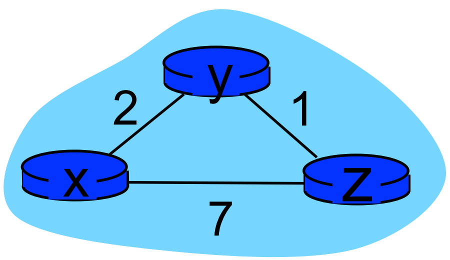
from \ cost to	x	y	z
x	0	2	3
y	2	0	1
z	7	1	0

node y table

from \ cost to	x	y	z
x	∞	∞	∞
y	2	0	1
z	∞	∞	∞

node z table

from \ cost to	x	y	z
x	∞	∞	∞
y	∞	∞	∞
z	7	1	0



time

$$D_x(y) = \min\{c(x,y) + D_y(y),\ c(x,z) + D_z(y)\} = \min\{2+0\ ,\ 7+1\} = 2$$

$$D_x(z) = \min\{c(x,y) + D_y(z),\ c(x,z) + D_z(z)\} = \min\{2+1\ ,\ 7+0\} = 3$$

node x table

from \ cost to	x	y	z
x	0	2	7
y	∞	∞	∞
z	∞	∞	∞

from \ cost to	x	y	z
x	0	2	3
y	2	0	1
z	7	1	0

from \ cost to	x	y	z
x	0	2	3
y	2	0	1
z	3	1	0

node y table

from \ cost to	x	y	z
x	∞	∞	∞
y	2	0	1
z	∞	∞	∞

from \ cost to	x	y	z
x	0	2	7
y	2	0	1
z	7	1	0

from \ cost to	x	y	z
x	0	2	3
y	2	0	1
z	3	1	0

node z table

from \ cost to	x	y	z
x	∞	∞	∞
y	∞	∞	∞
z	7	1	0

from \ cost to	x	y	z
x	0	2	7
y	2	0	1
z	3	1	0

from \ cost to	x	y	z
x	0	2	3
y	2	0	1
z	3	1	0

time →