

Chapter 1: Fundamental Principles

Roadmap for 1st chapter:

- Why AE need fluid mechanics?
- Introductory concepts
- Forces & moments
- Center of pressure
- Types of flow
- Dimensional analysis
- Flow similarity
- Fluid statics

Why study Fluid Mechanics?

Fluid Mechanics is key to many problems aerospace engineers face.

Aerospace engineers need to predict and control

- Forces & moments (acting on bodies moving relative to a fluid)
- Rates of heat transfer (between a body and fluid)

Historical examples

- evaluation of ship resistance
- Wright brothers's experiments to design a new wing
- blunt re-entry body

Fluid Mechanics in Aerospace System Design

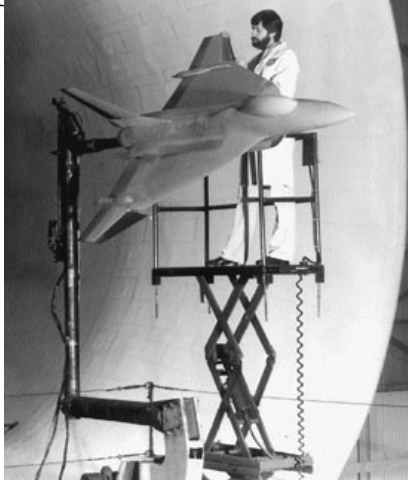
What are two main equations of aerospace engineering?

Aircraft:

Rockets:

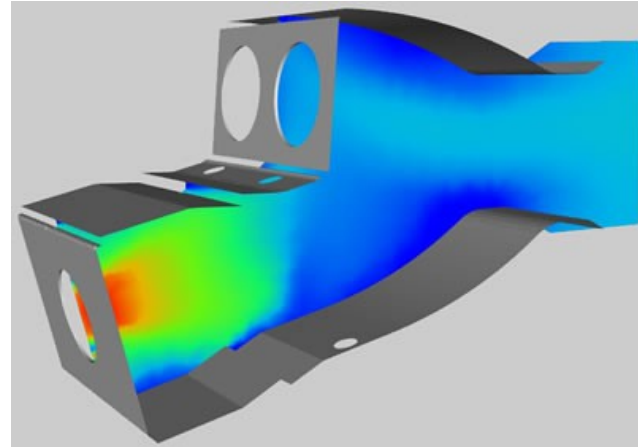
Other examples:

airplane lift and drag

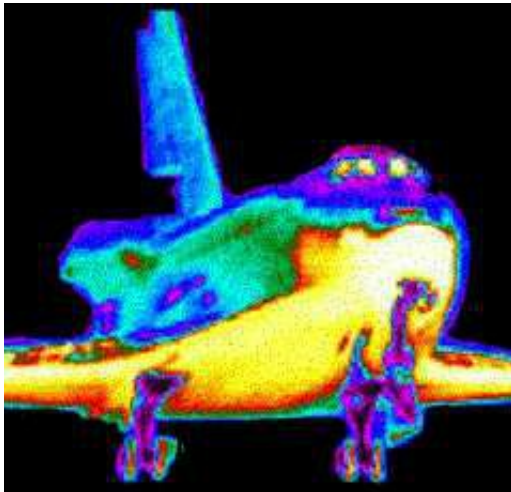


Scale model of fighter aircraft in wind tunnel.
(NASA)

combustion

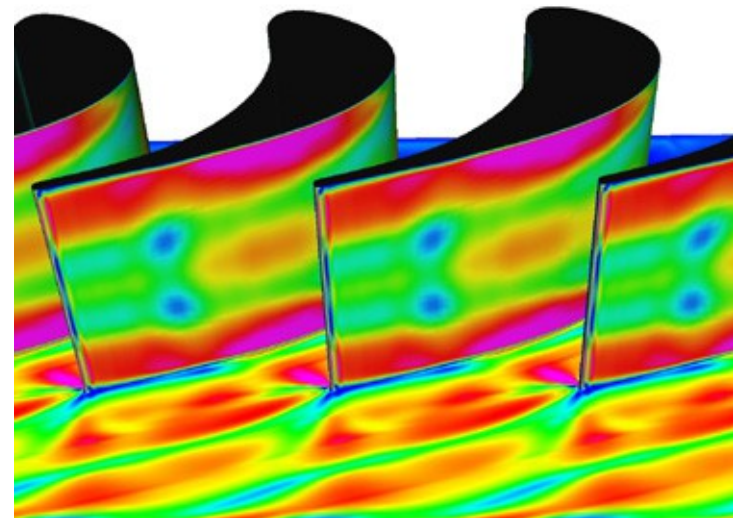


Temperature distribution within the combustion chamber. Two successive combustions followed by a quick dilution reduce significantly NO_x emissions
(ONERA)



IR image of a landing space shuttle
(Inframetrics)

heating



Temperature contours on a transonic turbine rotor (NASA)

Interdisciplinary problems

- fluid & structures

- Tail buffet
- Flutter
- Cavity noise (acoustic loading)

- fluid & propulsion

- Scramjet inlet
- Rocket nozzles
- Powered lift systems (over-the-wing blowing/ flaps) (high lift, low noise) for V/STOL aircraft

- fluids & controls

- Control stall of compressors
- Vortex generators (avoid stall, tail buffet)

- 1-5 • Reduce drag due to turbulence by active control

Introductory Concepts

Fluid: A substance that continuously deforms when a shear force is applied.

Solid: A substance that does not deform under an applied shear force by static deformation.

2 types of fluids:

Liquid: A fluid in which the molecules are relatively close together and interact strongly; it will take the shape of container but maintains a constant volume.

Gas: A fluid in which the molecules are widely spaced and interact only briefly. A gas will take the shape of a container and expand to fill the entire volume.

In air at standard atmosphere conditions the spacing between molecules is 10 times larger than the molecular diameter ($d \sim 4 \times 10^{-10}$ m).

Hydrodynamics: flow of

Gas dynamics: flow of

Aerodynamics: flow of

- external
- internal

Continuum Hypothesis

Treat the fluid as a continuous medium rather than being made up of molecules separated by void.

Continuum hypothesis is valid when mean free path of molecules is negligible compared to the characteristic size of the flow problem.
Continuum hypothesis is about fluid flows not fluids!

For air at 1 atmosphere and room temperature: $\text{mfp} \sim 60 \text{ nm}$.

For most aerodynamic flows of interest, characteristic size $\gg \text{mfp}$.

Continuum hypothesis fails for:

- Low-pressure (vacuum) systems
- High-altitude (above 100 km) flight
- Flows in micro/nanodevices

We will only consider continuum flows in AAE333. Non-continuum flows: AAE519, AAE590D.

For continuum fluid flows, we assume that fluid properties vary continuously in space.

Q: What are the most important fluid properties?

Fundamental Variables:

Pressure(p): normal force per unit area due to molecular motion

Let:

dA = surface element

dF = normal force on dA

$p =$

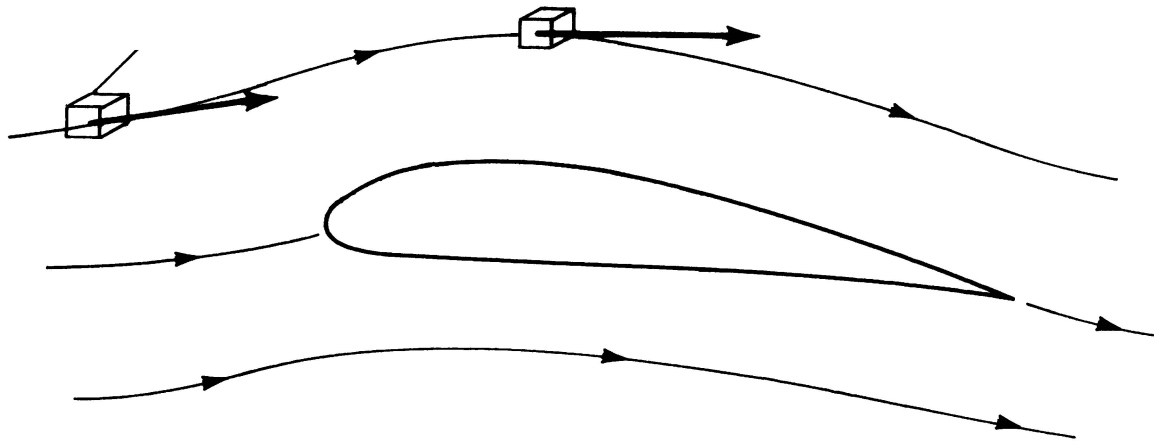
density(ρ): =

where ΔV = volume element, Δm = mass of

Temperature (T) = directly proportional to kinetic energy of fluid molecules

velocity ($\vec{v}$) for a fluid element can vary from point to point;
velocity it is a vector

streamline: the path of a fluid element (fixed for steady flow)



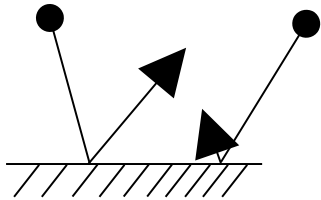
Forces & Moments

Pressure is a scalar

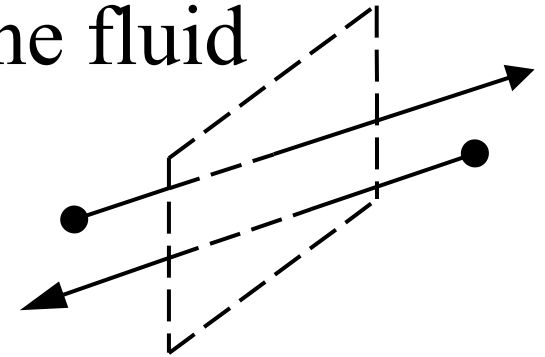
Pressure force acts perpendicular to any given surface

Pressure has no direction – surface has

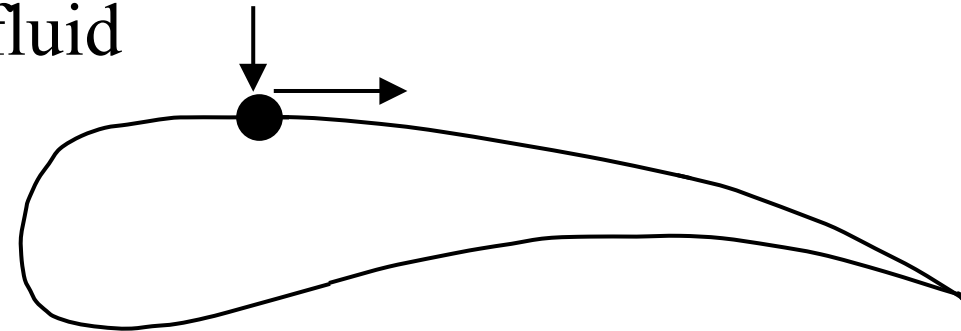
solid surface



in the fluid

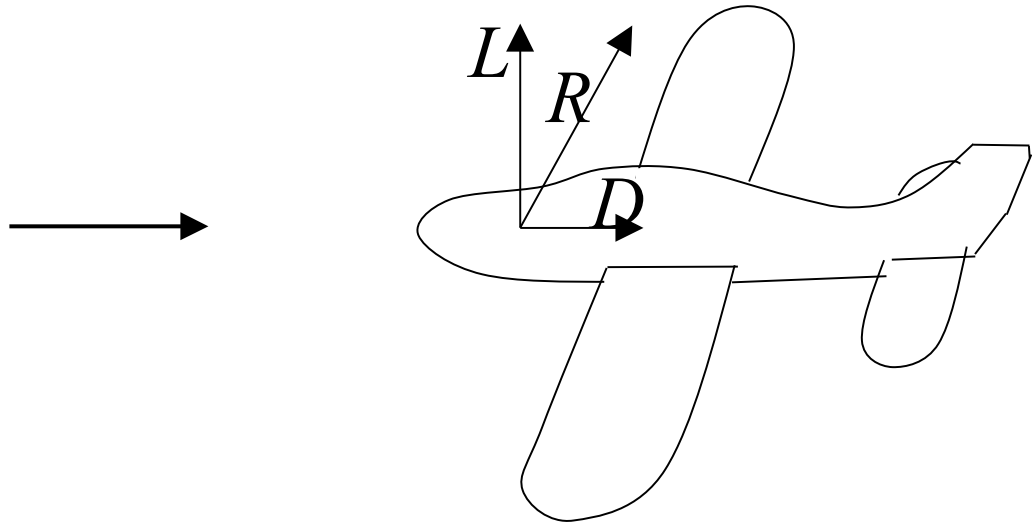


Assume a body moving relative to a fluid



1. Pressure force (to surface)
2. Viscous force (to surface) due to transfer of momentum
due to molecular motion in the presence of a velocity gradient
=viscosity coefficient

Forces on a Body



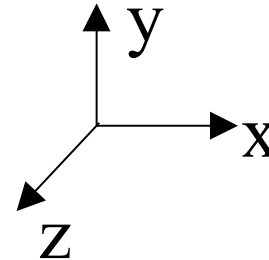
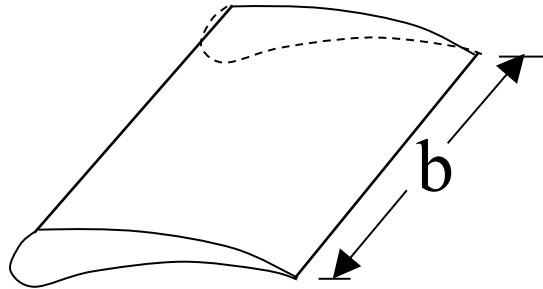
L =Lift: force

D = Drag: force

R = Resultant (total) force

R =

2-D Flow:



If: 1) no variation in the z direction

2) no velocity in the z direction

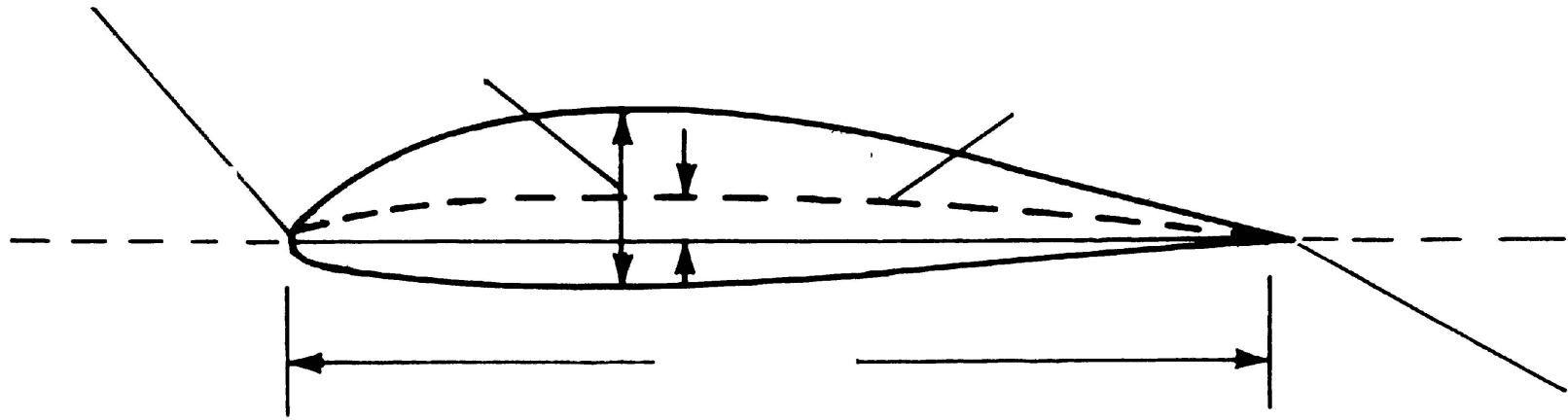
2-D flow corresponds to a wing of infinite span;

3-D effects at the tips

Denote

L', D', R' : forces/unit span (N/m)

Airfoil nomenclature (Anderson, Ch. 4)



Leading edge (LE)

Trailing edge (TE)

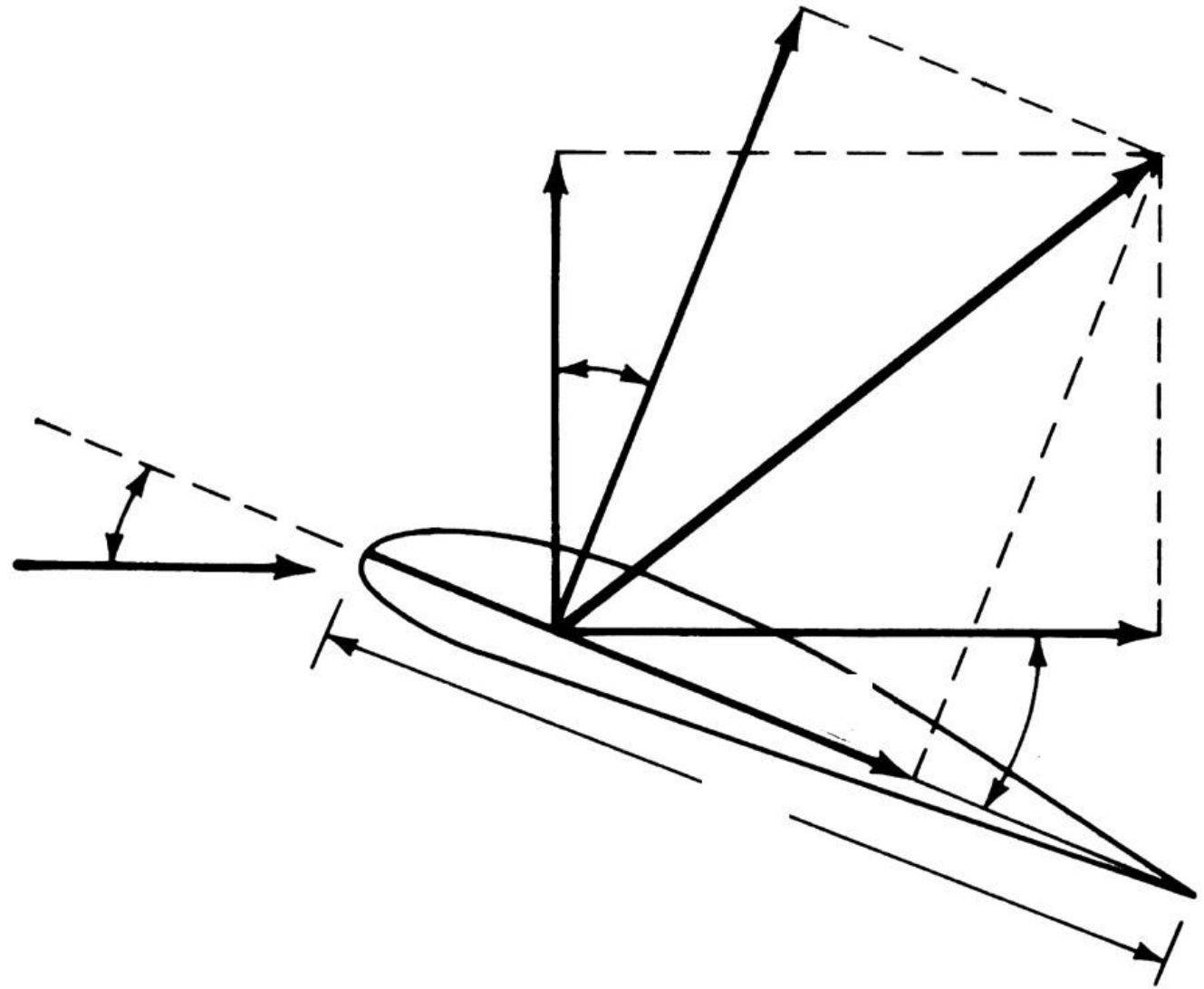
Chord (c)

Thickness (t)

Mean camber line

1-14 Camber

Forces on airfoil



TE= trailing edge, LE=leading edge

c = chord length, α = angle of attack

Forces:	$R' =$	force/unit span
	$L' =$	force/unit span
	$N' =$	force/unit span
	$A' =$	force/unit span
	$D' =$	force/unit span

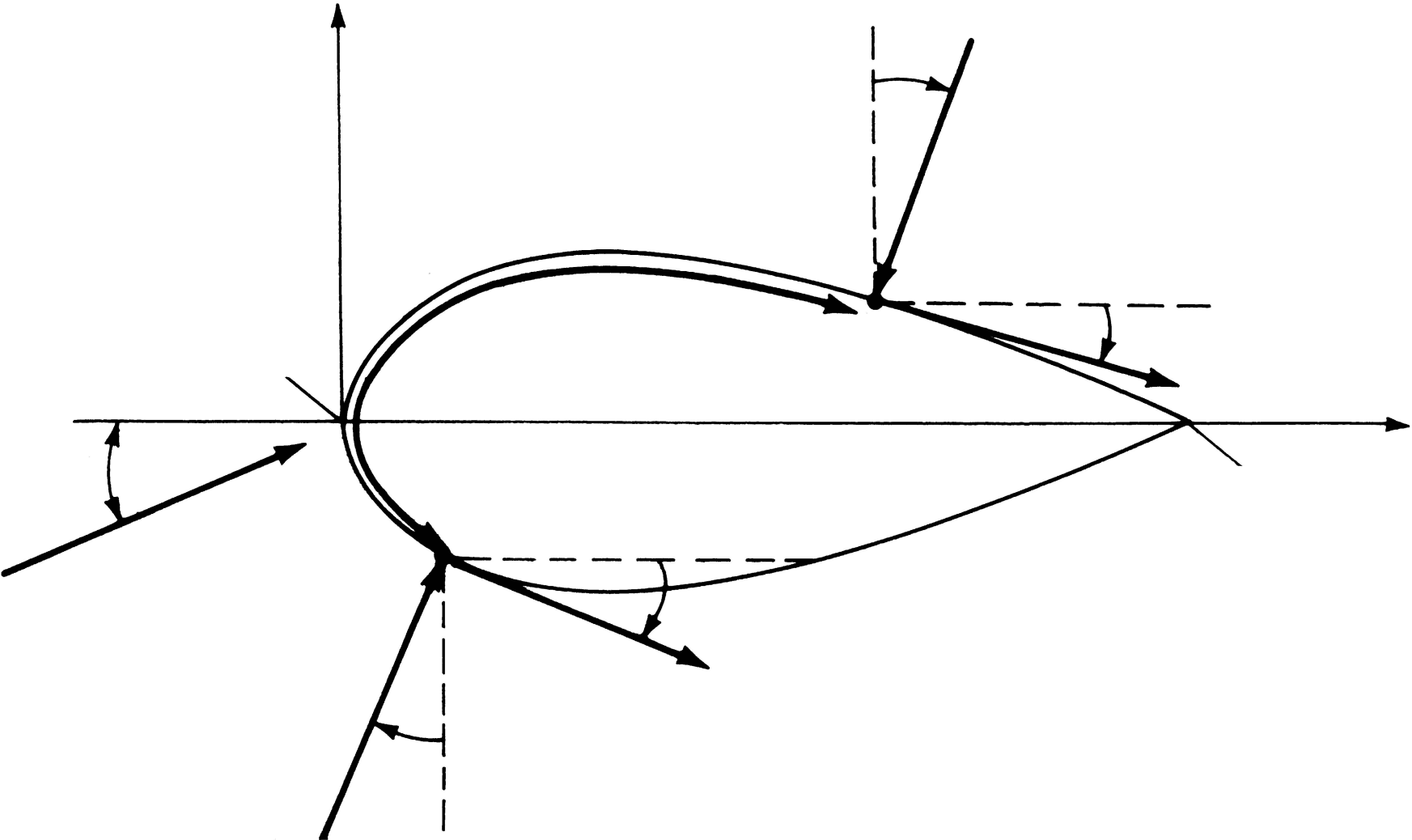
$$\begin{pmatrix} D' \\ L' \end{pmatrix} = \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} A' \\ N' \end{pmatrix}$$

rotation matrix

$$L' =$$

$$D' =$$

Calculation of N' and A' from pressure and shear stress distributions.



: upper surface

: lower surface

: outward normal

: surface inclination angle

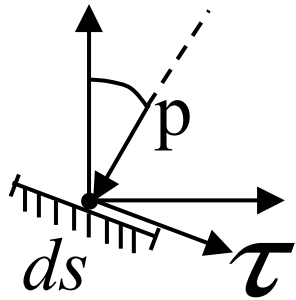
1-17 : arclength

: differential s

upper surface:

$$dA_u' = (\quad) ds_u$$

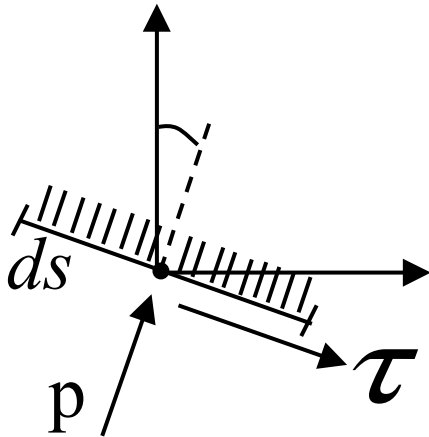
$$dN_u' = (-p_u \cos \theta - \tau_u \sin \theta) ds_u$$

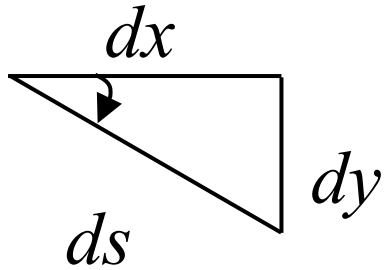


lower surface:

$$dA_l' = (p_l \sin \theta + \tau_l \cos \theta) ds_l$$

$$dN_l' = (\quad) ds_l$$





Express ds through dx , dy and θ :

$$dx = \cos \theta ds$$

$$dy = -\sin \theta ds$$

$$\frac{dy}{dx} =$$

$$dN'_u = (-p_u \cos \theta - \tau_u \sin \theta) ds_u =$$

$$= (-p_u + \tau_u \frac{dy_u}{dx}) dx$$

Similarly: $dN'_l = (p_l + \tau_l \frac{dy_l}{dx}) dx$

$$dA'_u = (p_u \frac{dy_u}{dx} + \tau_u) dx$$

$$dA'_l = (-p_l \frac{dy_l}{dx} + \tau_l) dx$$

Integrate over the chord from LE ($x=0$) to TE ($x=c$):

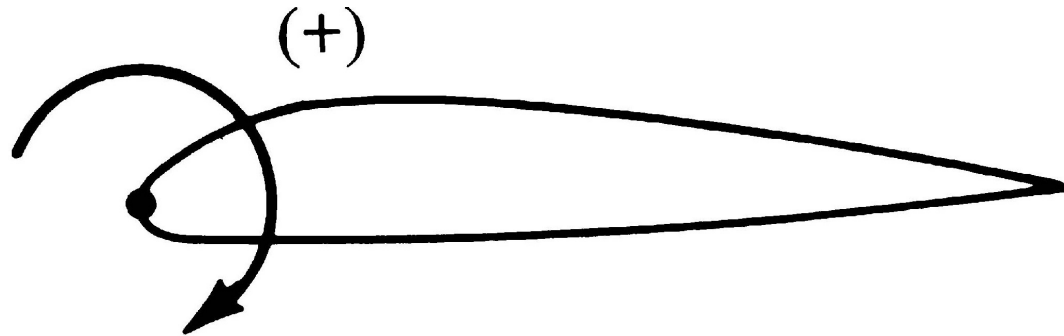
$$N' = \int_0^c dN'_u + \int_0^c dN'_l = \int_0^c (p_l - p_u) dx + \int_0^c \left[\tau_u \frac{dy_u}{dx} + \tau_l \frac{dy_l}{dx} \right] dx$$

$$A' = \int_0^c dA'_u + \int_0^c dA'_l = \int_0^c \left[p_u \frac{dy_u}{dx} - p_l \frac{dy_l}{dx} \right] dx + \int_0^c (\tau_u + \tau_l) dx$$

From p and τ -----> N' , A' -----> L' , D' .

Aerodynamic moment – moment of the resultant force on the airfoil (or wing) around a reference axis parallel to spanwise direction

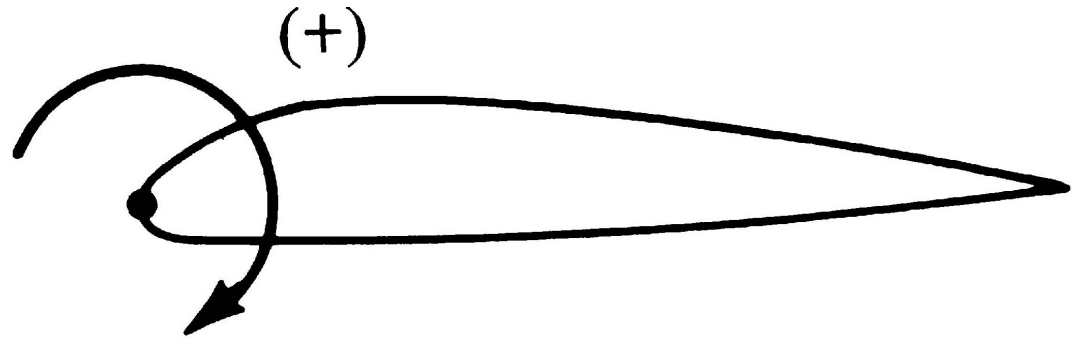
The moment tends to change the angle of attack. Also called pitching moment.



Sign convention: positive moment is pitch up (increases angle of attack)

Reference axis can be placed in different locations, e.g. LE, quarter-chord, half-chord, and so on. Notations:

Moment around LE:



upper surface: $d M'_u = -x d N'_u + y d A'_u =$

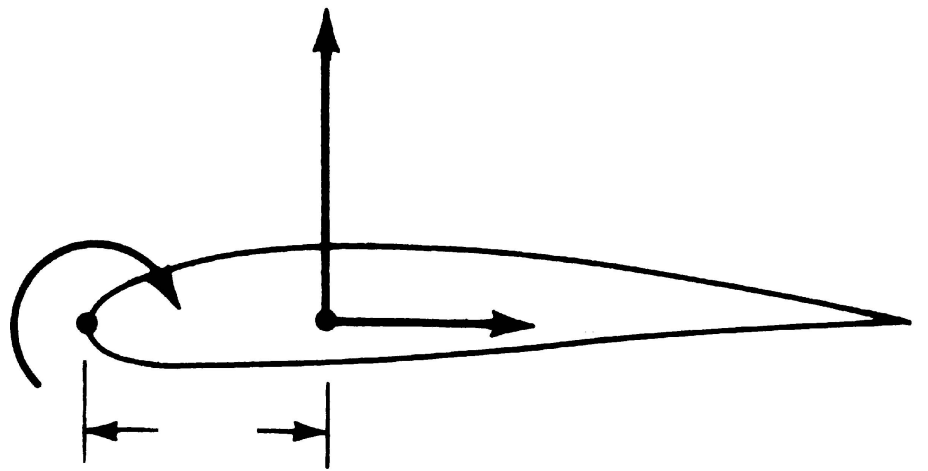
lower surface: $d M'_l = -x d N'_l + y d A'_l =$

Integrate:

$$M'_{LE} = \int_0^c (p_u - p_l) x dx - \int_0^c \left[\tau_u \frac{dy_u}{dx} + \tau_l \frac{dy_l}{dx} \right] x dx$$

$$+ \int_0^c \left[p_u \frac{y_u}{dx} + \tau_u \right] y_u dx + \int_0^c \left[-p_l \frac{y_l}{dx} + \tau_l \right] y_l dx$$

Center of Pressure



= location along the chord where the resultant force can be applied to produce the same moment as the distributed load.

for small α :

Moment can be taken about any point e.g, $c/4$

(for thin symmetric airfoils $x_{cp} = c/4$)

Example: (Anderson, Problem 1.3)

Consider an infinitely thin flat plate of chord c at an angle of attack α in a supersonic flow. The pressure on the upper and lower surfaces are different but constant over each surface; that is $p_u(s) = c_1$ and $p_l(s) = c_2$ where c_1 and c_2 are constants and $c_2 > c_1$. Ignoring the shear stress, calculate the location of the center of pressure.

Non-dimensionalization

for comparison of different designs

for scaling of wind tunnel tests to full scale

dynamic pressure: (force/area)

(specific kinetic energy of free stream)

Let S = reference area, l = reference length

Define: $C_L = \frac{L}{q_\infty S}$ - lift coefficient

$C_D = \frac{D}{q_\infty S}$ - drag coefficient

$C_M = \frac{M}{q_\infty S l}$ - moment coefficient

$$C_N = \frac{N}{q_\infty S} \quad \text{- normal force coefficient}$$

$$C_A = \frac{A}{q_\infty S l} \quad \text{- axial force coefficient}$$

2-D force coefficients: $(S = c \cdot 1 = c)$

$$c_l = \frac{L'}{q_\infty c} \quad c_d = \frac{D'}{q_\infty c} \quad c_m = \frac{M'}{q_\infty c^2}$$

Similarly we can define c_n and c_a

$$c_p = \frac{p - p_\infty}{q}$$

$$c_f = \frac{\tau}{q}$$

Previous eqs for N' and A' can be non-dimensionalized to obtain c_n , c_a and c_m in terms of C_p , c_f .

$$c_n = \frac{1}{c} \left[\int_0^c (c_{p,l} - c_{p,u}) dx + \int_0^c \left(c_{f,u} \frac{dy_u}{dx} + c_{f,l} \frac{dy_l}{dx} \right) dx \right]$$

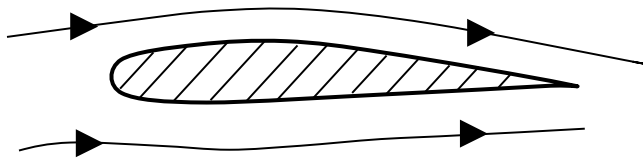
$$c_a = \frac{1}{c} \left[\int_0^c \left(c_{p,u} \frac{dy_l}{dx} - c_{p,l} \frac{dy_u}{dx} \right) dx + \int_0^c (c_{f,u} + c_{f,l}) dx \right]$$

$$c_{m_{LE}} = \frac{1}{c^2} \int_0^c (c_{p,u} - c_{p,l}) x dx - \int_0^c \left[c_{f,u} \frac{dy_u}{dx} + c_{f,l} \frac{dy_l}{dx} \right] x dx$$

$$+ \int_0^c \left[c_{p,u} \frac{y_u}{dx} + c_{f,u} \right] y_u dx + \int_0^c \left[-c_{p,l} \frac{y_l}{dx} + c_{f,l} \right] y_l dx$$

Specifying reference areas and lengths

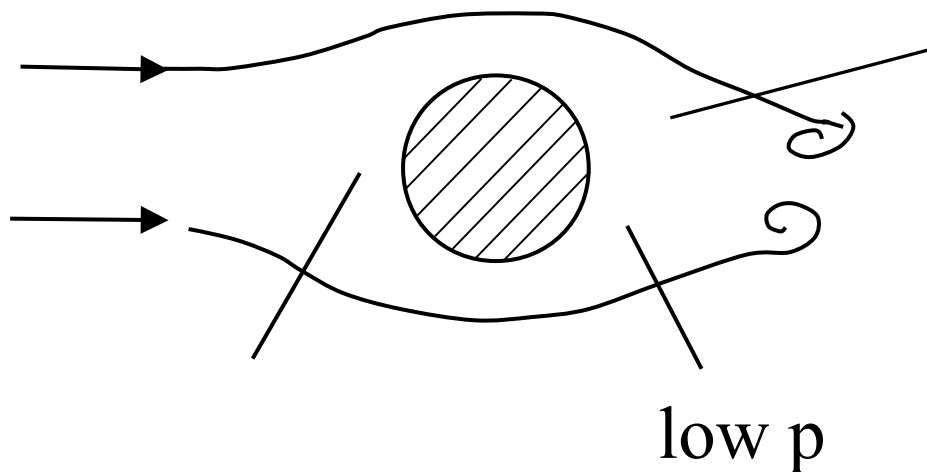
Streamline bodies: drag due mostly to



High/low drag

Bluff bodies=

drag due mostly to



High/low drag

For streamlined bodies:

3-D $S = \text{planform area}, \quad l = \text{mean chord}$

2-D $S = \text{chord} \cdot (1), \quad l = \text{chord}$

for bluff bodies

3-D $S =$

2-D $S =$

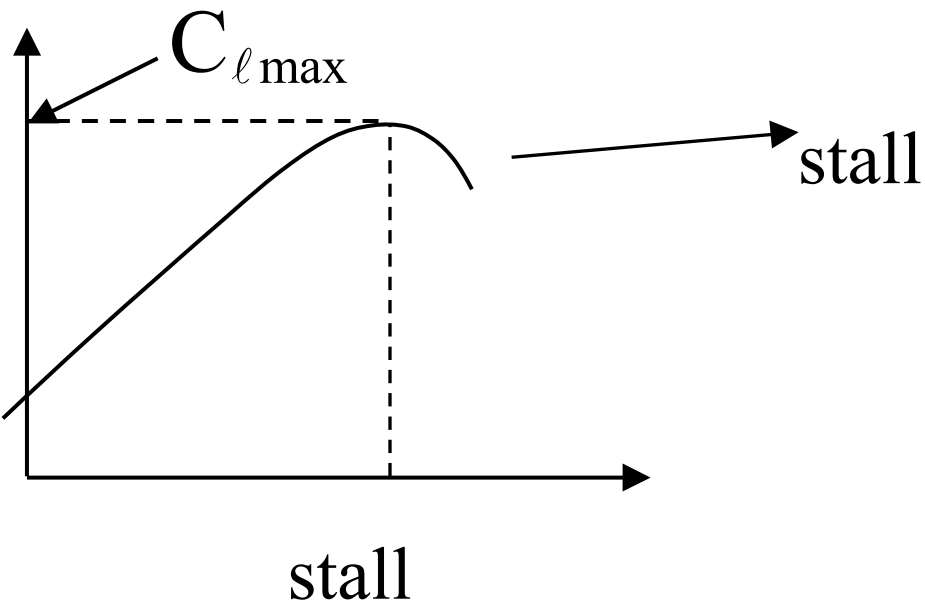
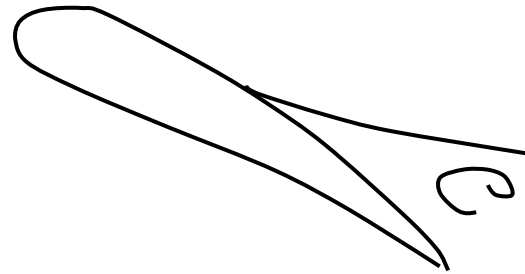
$S =$

Typical values of c_l , c_d

Airfoils:

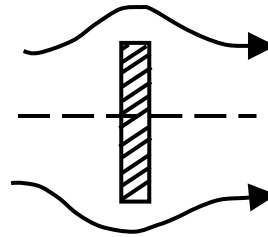
$$c_{l \max} = 1.5 - 2.0 \text{ (single element)}$$

$$= 4 \text{ (multi-element)}$$



Bluff bodies:

flat plate



cylinder

Two important non-dimensional numbers:

Reynolds number: $Re =$

ρ =density V =velocity L =characteristic length

μ = viscosity coefficient

Mach number : $M =$

a = speed of sound

Types of Flow

Continuum vs free molecular flow

mean free path

at sea level

at 100 km

Knudsen number:

$Kn =$

[AAE 519, AAE590D]

if $Kn < 0.01$

continuum flow

$0.01 < Kn < 10$

rarefied

$Kn > 10$

free molecule

Incompressible vs compressible flow

if $M < 0.3$ then ρ is nearly constant

incompressible flow (AAE 333)

for compressible flow, Mach # regimes:

$$M < 1 \quad \text{everywhere} \quad M_{\infty} < 0.8$$

$$\begin{array}{l} M < 1 \\ M > 1 \end{array} \quad \text{regions} \quad 0.8 < M_{\infty} < 1.2$$

$$M > 1 \quad \text{everywhere} \quad M_{\infty} > 1.2$$

$$M_{\infty} > 5$$

Inviscid vs viscous flow

inviscid flow is ideal

for high Re viscous effects can be limited to thin regions near a body (boundary layers), except when separation occurs

inviscid flow assumption predicts $drag = 0$.

Some Practical Issues

Remember:

Blunt body= body where most of the drag is

Streamlined body = body where most of the drag is

Blunt bodies have a much higher C_D than streamlined bodies.

The skin friction coefficient C_f is higher for turbulent flows than laminar flows because velocity profile is flatter:

Explain: golf ball dimples, old baseball have smaller drag

Buckingham Pi Theorem

The name Pi comes from the mathematical notation Π , meaning a product of variables.

Principle of Dimensional Homogeneity (PDH):

Any equation describing a physical situation will only be true if both sides have the same units.

The Buckingham Pi Theorem states:

If a physical process satisfies PDH and involves N dimensional variables, it can be reduced to $N-K$ non-dimensional variables or Π s. The reduction K is equal to the maximum number of variables that do not form a Π among themselves and is always less than or equal to the number of fundamental units (mass, length, time, etc) describing the variables.

Method of repeating variables is based on the application of Buckingham Pi theorem to determine the non-dimensional parameters that can be formed from a group of dimensional variables describing a physical process.

Step 1) Come up with a list of N dimensional variables P_j that describe the process and are related by some unknown functional relation: $f_1(P_1, P_2, \dots, P_N) = 0$

Step 2) Let K be the number of fundamental units needed to describe the N dimensional variables. The fundamental units are mass, length, time, temperature and so on.

Step 3) Choose K of the dimensional variables to be repeating variables such that the K fundamental units can be all formed by some combination of the repeating variables and no non-dimensional parameters can be formed from the set of repeating variables.

Step 4) Form $N-K$ dimensionless Pi products by multiplying the K repeating variable, as follows

$$\Pi_1 = P_1^{a_{11}} P_2^{a_{12}} \dots P_K^{a_{1K}} P_{K+1}$$

The exponents a_{ij} are determined so that Π_i are non-dimensional.

The Buckingham Pi theorem then states that the $N-K$ Pi products are related by some functional relationship:

$$f_2(\Pi_1, \Pi_2, \dots, \Pi_{N-K}) = 0$$

The advantage of using the Buckingham Pi theorem is that it can greatly reduce the number of parameters one has to deal with.

Example: Force coefficient (2-D)

Based on physical intuition we expect the resultant force vector for a given 2-D shape to depend on:

Fluid density

Freestream velocity

Body size (chord)

Viscosity

Speed of sound

Therefore the magnitude of the resultant force per unit span is

$$R' = f(\rho_{\infty}, V_{\infty}, c, \mu_{\infty}, a_{\infty})$$

or as

The Buckingham Pi theorem will tell us how many parameters we can form from this set of variables.

Follow the steps outlined above to find the non-dimensional parameters Π_i

Step 1: The dimensional variables are

$$R', \rho_{\infty}, V_{\infty}, c, \mu_{\infty}, a_{\infty}$$

So $N=$

Step 2: What are the fundamental units needed?

Denote m =mass, l =length, t =time.

$$[R'] =$$

$$[\mu_{\infty}] =$$

$$[\rho_{\infty}] =$$

$$[a_{\infty}] =$$

$$[V_{\infty}] =$$

So $K=$

Step 3: Choose K *repeating variables* from among the N dimensional variables. We don't want to choose R' because it is the variable of interest. We don't want to choose both a_{∞} and V_{∞} because their ratio is a non-dimensional parameter.

Choose the following as repeating variables: $\rho_{\infty}, V_{\infty}, c$.
Check that we can represent each of the fundamental units with this set of variables:

mass:

time:

length:

Step 4: Form the non-dimensional products.

Let
$$\Pi_1 = \rho_{\infty}^{a_{11}} V_{\infty}^{a_{12}} c^{a_{13}} R'$$

Π_1 is dimensionless, so each exponent must be zero:

This is almost the same as the force coefficient except for a factor of 2. Since multiplying by a number doesn't change the units of Π , let

$$\Pi_1 = \frac{R'}{\frac{1}{2} \rho_{\infty} V_{\infty}^2 c}$$

Let $\Pi_2 = \rho_{\infty}^{a_{21}} V_{\infty}^{a_{22}} c^{a_{23}} \mu_{\infty}$

For Π_2 to be non-dimensional we need:

Thus, $\Pi_2 = \frac{\mu_{\infty}}{\rho_{\infty} V_{\infty} c}$ Similarly, we obtain $\Pi_3 = \frac{a_{\infty}}{V_{\infty}}$

The statement that we started with

can be rewritten as

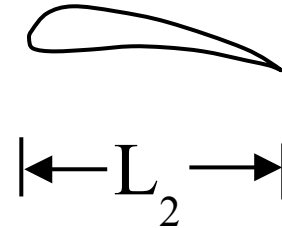
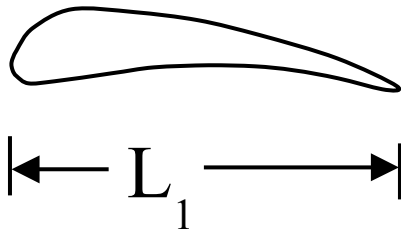
$$R' = f(\rho_{\infty}, V_{\infty}, c, \mu_{\infty}, a_{\infty})$$

$$c_{R'} = f_1\left(\frac{1}{\text{Re}_{\infty}}, \frac{1}{M_{\infty}}\right)$$

Dynamic Similarity

Approaches for experimental testing:

- Flight testing [expensive]
- Full scale wind tunnel testing
(80' x 120' at NASA Ames; $V_{max} = 150$ mph)
[still expensive]
- Scaled model testing

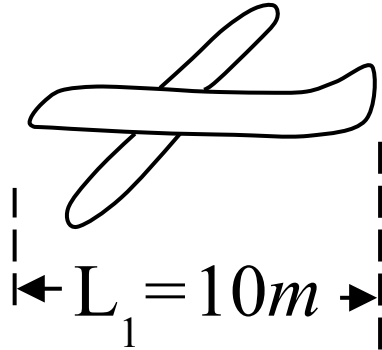


Conditions for dynamic similarity

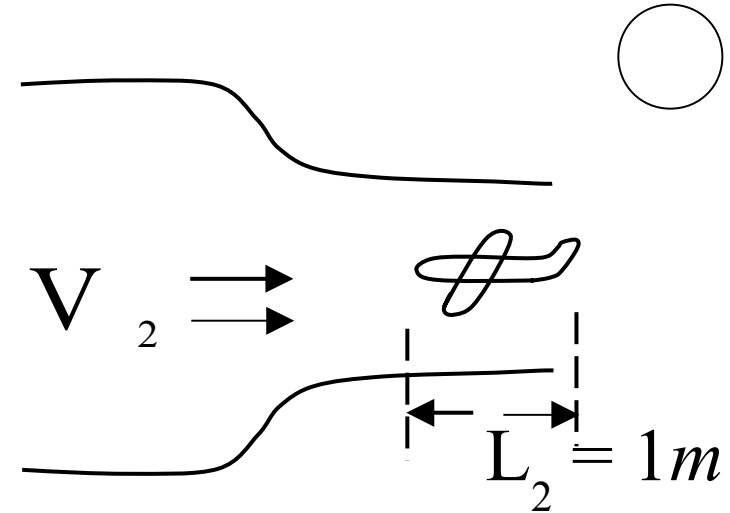
- 1) Bodies are similar (scale model)
- 2) Governing parameters are the same

$$c_D = f(Re, M)$$

Example: c_D on a sailplane.



1 atm



1 atm

Find $V_{\infty 2}$

$$c_D = f(\text{Re}, M)$$

$$M_{\infty 1} = \frac{V_{\infty 1}}{a_{\infty 1}} = \frac{10 \text{ m/s}}{340 \text{ m/s}} = 0.03$$

For incompressible flow $c_D = f(Re)$.

$$\text{Re}_{\infty 1} = \text{Re}_{\infty 2} \Rightarrow \left(\frac{\rho_1}{\rho_2} \right) \left(\frac{V_{\infty 1}}{V_{\infty 2}} \right) \left(\frac{L_1}{L_2} \right) \left(\frac{\mu_2}{\mu_1} \right) = 1.$$

$$\Rightarrow V_{\infty 2} = 10 V_{\infty 1} = 100 \text{ m/s}.$$

Check Mach number for the model:

Example: c_D on an airliner.

Prototype:

$$L_1 = 50 \text{ m}$$

Altitude is 10 km

$$M_{\infty 1} = 0.8 \text{ (transonic)}$$

$$\rho_{\infty 1} = 0.413 \text{ kg/m}^3$$

$$\mu_{\infty 1} = 1.447 \times 10^{-5} \text{ kg/m s}$$

$$a_{\infty 1} = 299 \text{ m/s}$$

Model:

$$L_2 = 5 \text{ m}$$

At sea level

$$\rho_{\infty 2} = 1.226 \text{ kg/m}^3$$

$$\mu_{\infty 2} = 1.78 \times 10^{-5} \text{ kg/m s}$$

$$a_{\infty 2} = 340 \text{ m/s}$$

First, find freestream velocity of the prototype:

$$\begin{aligned} V_{\infty 1} &= M_{\infty 1} a_{\infty 1} = 239 \text{ m/s.} \quad \text{Need to match Mach numbers:} \\ V_{\infty 2} &= \quad \quad \quad = 272 \text{ m/s.} \end{aligned}$$

$$\text{Re}_{\infty 1} = \frac{\rho_{\infty 1} V_{\infty 1} L_1}{\mu_{\infty 1}} = 3.4 \times 10^8$$

$$\text{Re}_{\infty 2} = \frac{\rho_{\infty 2} V_{\infty 2} L_2}{\mu_{\infty 2}} = 9.4 \times 10^7$$

How can we increase the Re ?

Increase L : No, too expensive

Increase No, need to match

Increase

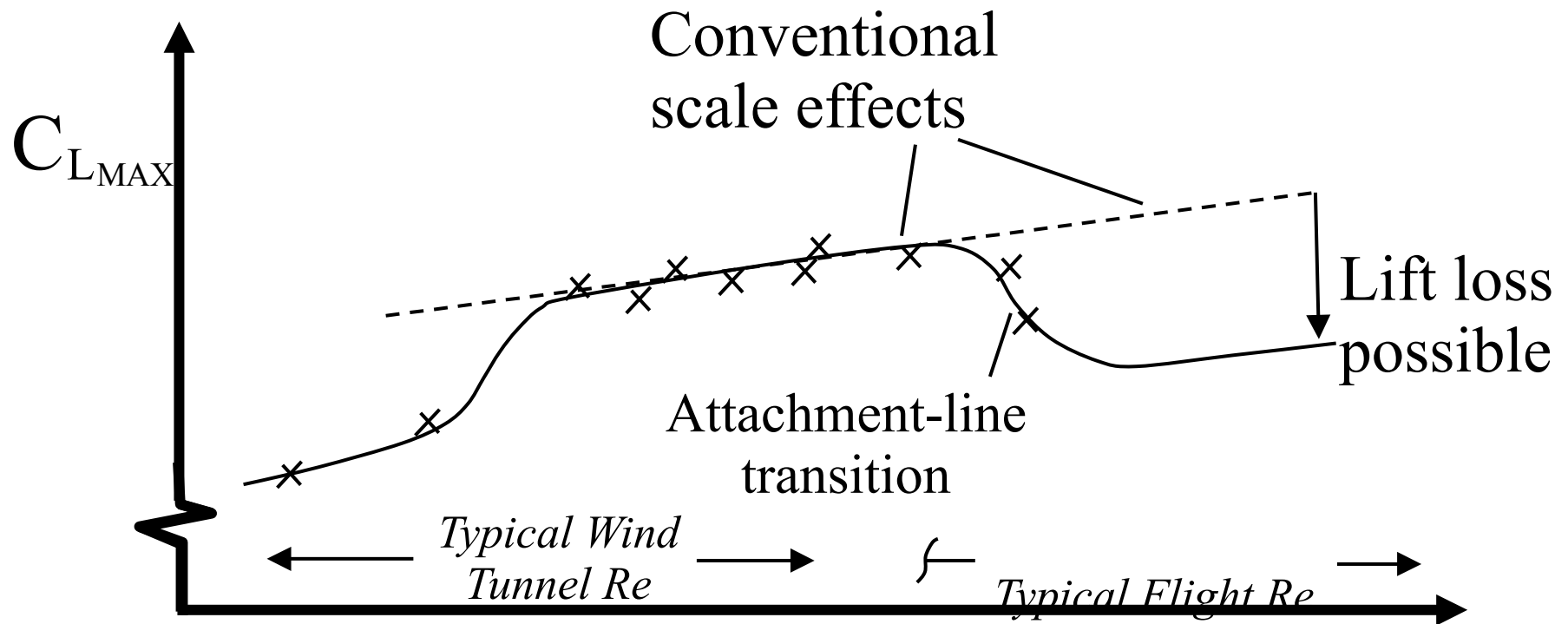
- increase p -(see example 1.5 in Anderson)
(pressurize the wind tunnel) expensive, dangerous
- reduce T (cryogenic facility)
expensive, difficult to instrument
- heavy gas

Increase μ :

No clean answer what to do. Preferred approach:

$M\#$ simulation in one tunnel & $Re\#$ simulation in other.

Source: National Facilities Study, Volume 2; Subsonic and Transonic Wind Tunnel Complex, 1994.



Fluid Statics describes fluids at rest or in rigid-body motion, i.e. no relative motion between adjacent fluid layers.

Some aerostatics applications:

- blimps (non-rigid airships)
- science balloons

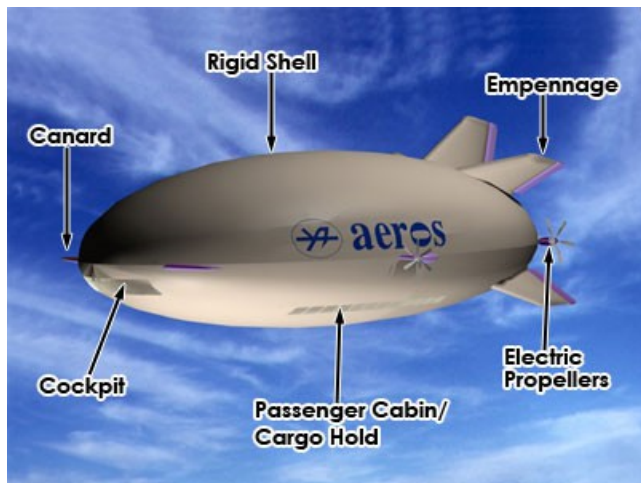


Purdue blimp (Prof. John Sullivan and team)



Constellation of stratospheric balloons (NASA/Global Aerospace Corp.)

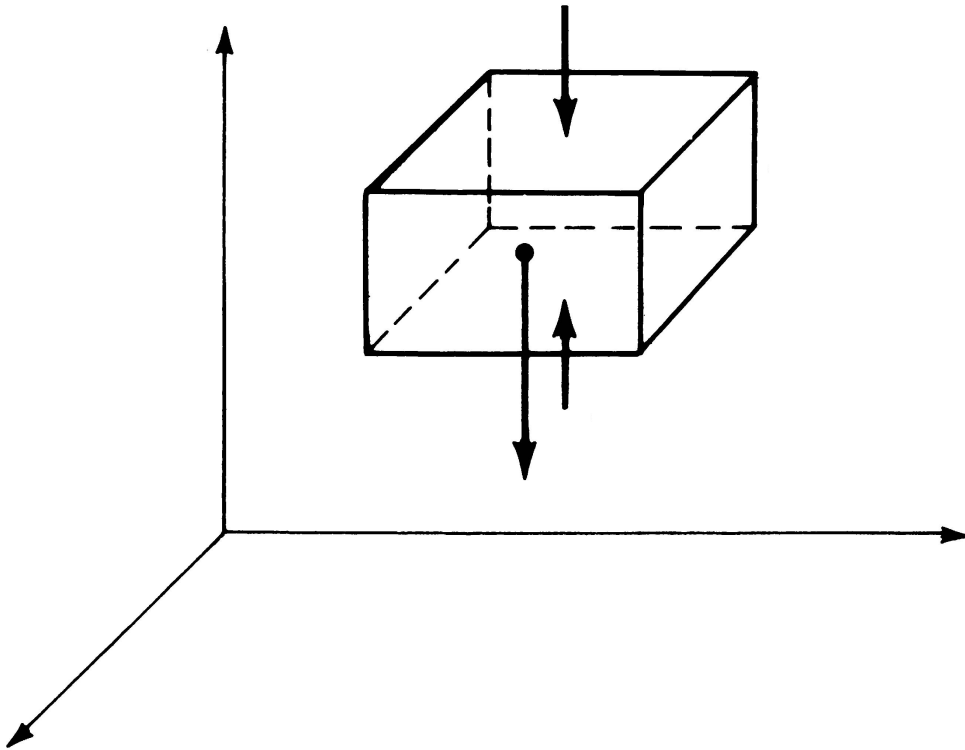
- hybrid airships (aerostatic/aerodynamic lift)



Aeroscraft is an aircraft that utilizes a combination of buoyant and dynamic lift and is designed to fly further (~10,000 miles) and lift more(~ 500 tons) than any other craft today (Worldwide Aeros Corp)

Hydrostatic equation.

Fluid is static: no relative motion between adjacent fluid layers.



Force balance:

$$p \, dx \, dz - (p + dp) \, dx \, dz - (\rho \cdot dx \, dy \, dz) \, g = 0$$

$$\frac{dp}{dy} = -\rho \, g \quad \text{Hydrostatic equation}$$

For liquids $\rho = \text{const}$. Let's integrate with respect to y :

$$dp = -\rho \, g \, dy \quad \Rightarrow \quad \int_{p_0}^p dp = - \int_{y_0}^y \rho \, g \, dy$$

$$p = p_0 - \rho \, g \, (y - y_0) \quad \text{Hydrostatic pressure}$$

Example 1: At what depth does $p=2$ atm?

$$p = p_a - \rho g y \quad (p = p_a \text{ at } y = 0)$$

$$y = -\frac{p - p_a}{\rho g} = \frac{-1.01 \times 10^5 \text{ N/m}^2}{(1 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)} = -10.3 \text{ m}$$

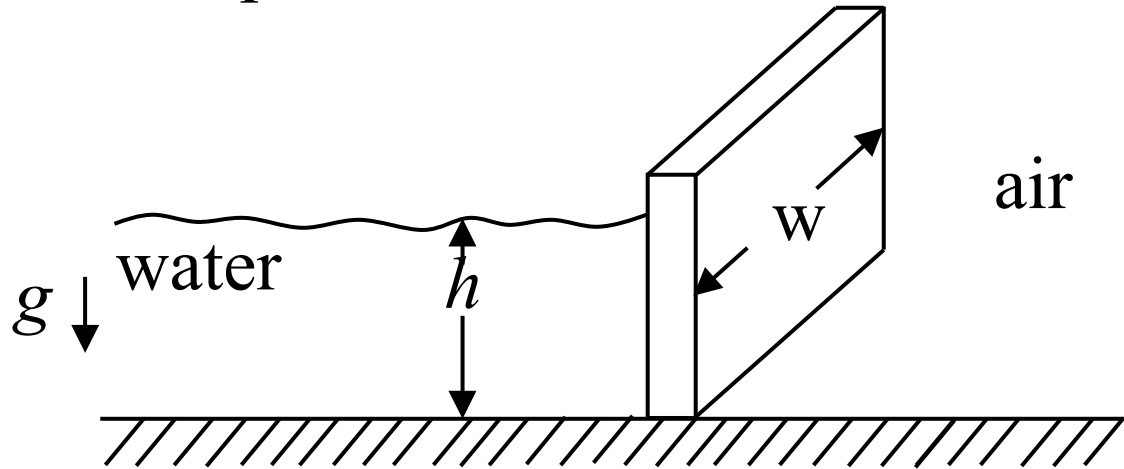
Pressure on a diver at a depth of 100 ft is 3.95 atm.

$$p_1 V_1 = p_2 V_2$$

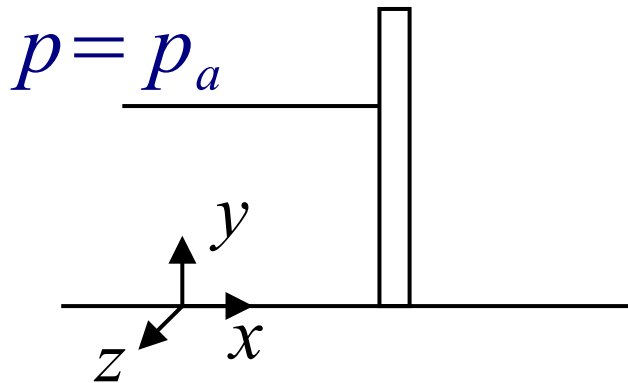
$$\frac{V_1}{V_2} = \frac{p_2}{p_1} = 3.95$$

If the diver holds breath on ascent, the lung volume would increase by a factor of 4.

Example 2: Find forces and moments on a dam.



Choose a coordinate system:



$$p = p_0 - \rho_w g (y - y_0) \qquad y_0 = h, p_0 = p_a \qquad \Rightarrow$$

$$1-55 \quad p = p_a + \rho_w g (h - y)$$

Static equilibrium: $\sum F_x = 0, \quad \sum M_z = 0.$

$$\sum F_x = R_x + \int_0^h (p(y) - p_a) W dy = 0 \Rightarrow$$

$$R_x = -W \int_0^h \rho_w g (h - y) dy =$$

$$\sum M_z = M_{R,z} - \int_0^h (p(y) - p_a) y W dy = 0 \Rightarrow$$

$$M_{R,z} = \int_0^h [-\rho_w g (y - h) y] W dy = -\rho_w g W \left[\frac{y^3}{3} - h \frac{y^2}{2} \right]_0^h = \frac{\rho_w g W h^3}{6}$$

What if fluid is a gas not a liquid? ($\rho \neq \text{const}$)

$$\frac{dp}{dy} = -\rho g$$

Ideal gas equation of state: $p = \rho R T$

Assume $T = T_0$ (isothermal).

Substitute $\rho = \frac{p}{R T_0}$ into hydrostatic equation:

$$\frac{dp}{dy} = -\frac{g}{R T_0} p$$

$$\frac{dp}{p} = -\frac{g}{R T_0} dy \quad \text{Integrate:} \quad \int_{p_1}^p \frac{dp}{p} = -\frac{g}{R T_0} \int_{y_1}^y dy \quad \Rightarrow$$

$$\ln(p/p_1) = -\frac{g}{RT_0}(y-y_1) \quad \Rightarrow$$

Lower stratosphere: $T_0 = 216.7 \text{ }^0\text{K}$ ($-56.5 \text{ }^0\text{C} = -69.6 \text{ }^0\text{F}$)

$$c_0 = \frac{g}{RT_0} = \frac{9.8 \text{ m/s}^2}{296 \text{ J/(kg}\cdot\text{K)} 216.7 \text{ K}} = 1.53 \times 10^{-4} \frac{1}{\text{m}}$$

$$p = p_1 e^{-c_0 y}$$

pressure drops to a half every 4.5 km
(2.8 miles)

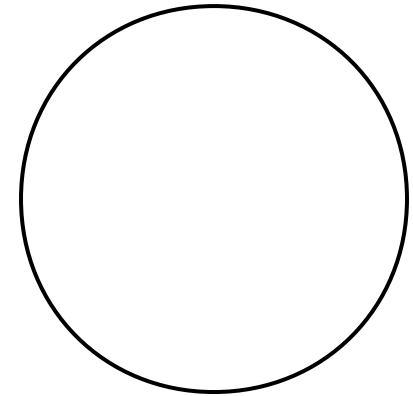
Force on a submerged body

Archimedes principle: Buoyancy force = weight of fluid displaced by the body.

Example: an immersed sphere.

Net force:

$$F_y = \text{Buoyancy force} + \text{gravitational force} = \\ = \frac{4}{3} \pi R^3 \rho_f g - \frac{4}{3} \pi R^3 \rho_s g$$



If $\rho_s < \rho_f \Rightarrow$ *sphere rises*
 $\rho_s > \rho_f \Rightarrow$ *sphere sinks*

ρ_f = fluid density, ρ_s = sphere density

Fluid rigid-body motion with constant acceleration.

Let $\vec{a}$ acceleration of fluid volume, $\vec{g}$ accel of gravity. We need to add mass x accel. term in the force balance for a fluid element. Hydrostatic equation takes form:

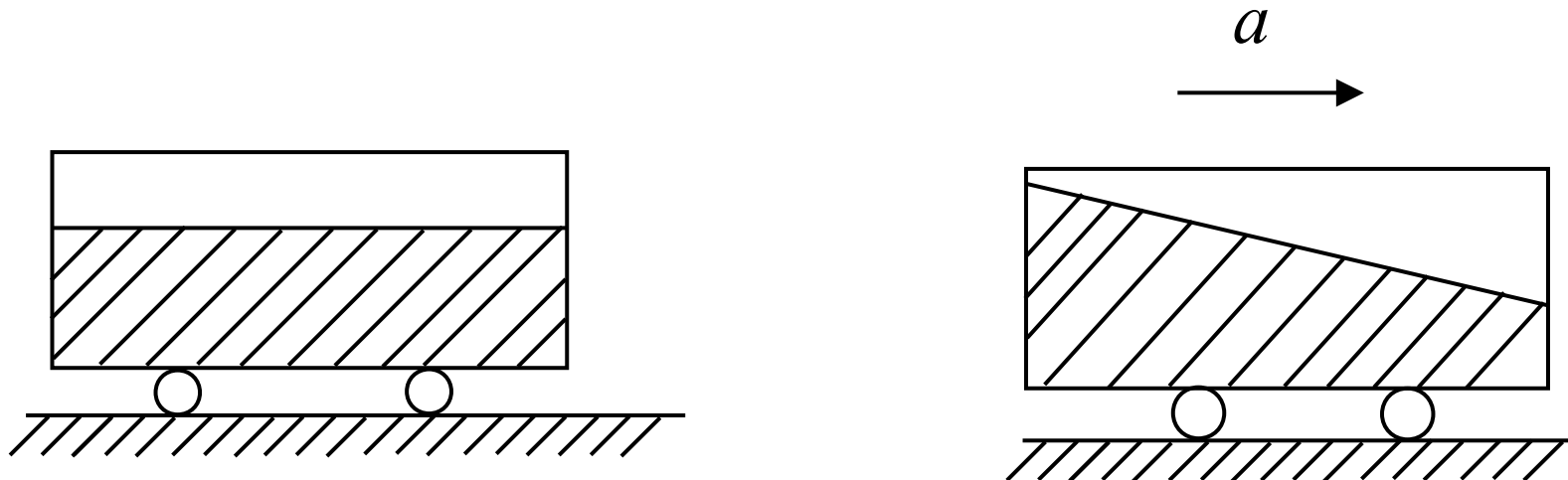
$$\frac{\partial p}{\partial x} = \rho g_x - \rho a_x$$

or in vector form:

$$\frac{\partial p}{\partial y} = \rho g_y - \rho a_y$$

$$\nabla p = \rho \vec{g} - \rho \vec{a}$$

Example: Fluid in an accelerating box car:



(1) y-direction: $\frac{\partial p}{\partial y} = -\rho g$ (gravity in neg. y-dir as before)

(2) x-direction: $\frac{\partial p}{\partial x} = -\rho a$ (inertial force in neg. x dir)

Integrate (1): $p = -\rho g y + c_1(x)$

Then substitute p into (2):

$$\frac{dc_1}{dx} = -\rho a$$

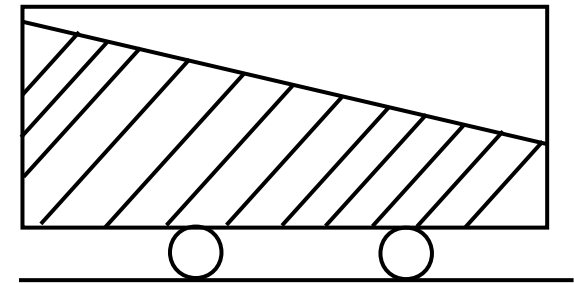
$\Rightarrow p(x, y) = -\rho a x - \rho g y + c_0$ c_0 depends on coordinate system

$p = p_a$ for the water free surface

$$p_a = -\rho a x - \rho g y + c_0$$

$\Rightarrow y = -\frac{a}{g} x + \frac{c_0 - p_a}{\rho g}$

Find slope: $\frac{dy}{dx} = -\frac{a}{g}$



Free surface is a straight line

Fluid in solid-body rotation. Consider a spinning cylindrical bucket with fluid.

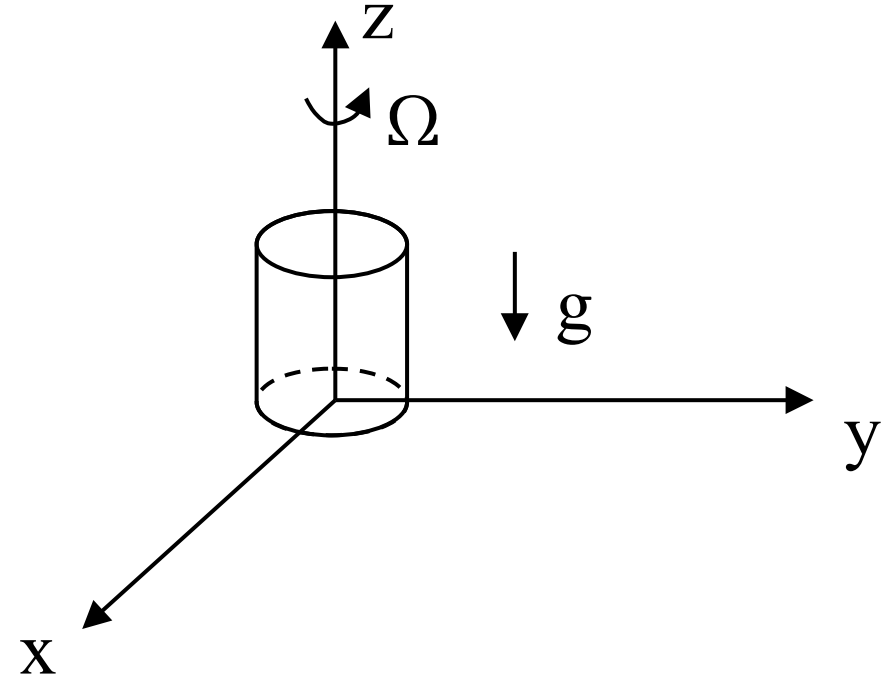
$$\nabla p = -\rho \vec{g} - \rho \vec{a} \quad \Rightarrow$$

In cylindrical coordinates:

$$z\text{-direction:} \quad \frac{\partial p}{\partial z} = -\rho g$$

$$r\text{-direction:} \quad \frac{\partial p}{\partial r} = \rho r \Omega^2$$

Integrate to find $p = p(r, z)$:



$$\frac{\partial p}{\partial z} = -\rho g \quad \Rightarrow \quad p = -\rho g z + c_1(r)$$

$$\frac{\partial p}{\partial r} = \rho r \Omega^2 \quad \Rightarrow \quad \frac{\partial c_1(r)}{\partial r} = \rho r \Omega^2$$

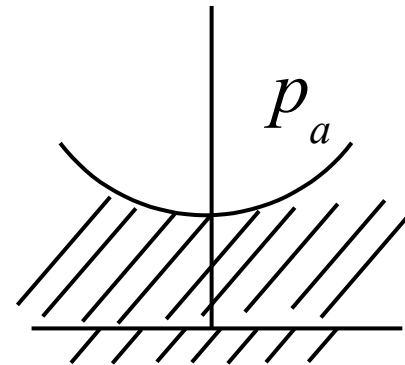
$$p = -\rho g z + \rho \frac{r^2}{2} \Omega^2 + c_0$$

Find free surface: $p = p_a$

$$\Rightarrow z = \frac{\Omega^2}{2g} r^2 + \frac{(c_0 - p_a)}{\rho g}$$

Free surface is a parabola.

Note: surface tension effects are neglected.



Summary of Chapter 1

$$c_n = \frac{1}{c} \left[\int_0^c (c_{p,l} - c_{p,u}) dx + \int_0^c \left(c_{f,u} \frac{dy_u}{dx} + c_{f,l} \frac{dy_l}{dx} \right) dx \right]$$

$$c_a = \frac{1}{c} \left[\int_0^c \left(c_{p,u} \frac{dy_l}{dx} - c_{p,l} \frac{dy_u}{dx} \right) dx + \int_0^c (c_{f,u} + c_{f,l}) dx \right]$$

$$c_d = c_a \cos \alpha + c_n \sin \alpha \quad c_l = -c_a \sin \alpha + c_n \cos \alpha$$

$$\begin{aligned} c_{m_{LE}} = & \frac{1}{c^2} \int_0^c (c_{p,u} - c_{p,l}) x dx - \int_0^c \left[c_{f,u} \frac{dy_u}{dx} + c_{f,l} \frac{dy_l}{dx} \right] x dx \\ & + \int_0^c \left[c_{p,u} \frac{dy_u}{dx} + c_{f,u} \right] y_u dx + \int_0^c \left[-c_{p,l} \frac{dy_l}{dx} + c_{f,l} \right] y_l dx \end{aligned}$$

$$\text{Center of pressure } x_{cp} = -\frac{M_{LE}}{N} \approx -\frac{M_{LE}}{L}$$

For dynamic similarity

- bodies must be geometrically similar
- non-dimensional similarity parameters must be equal (usually Re , M).

c_l , c_d are equal

Hydrostatic equation: $\frac{dp}{dy} = -\rho g$

For $\rho = \text{const}$: $p = \text{constant} - \rho g y$

- Acceleration a in x -direction: $p = \text{constant} - \rho g y - \rho a x$

surface ($p = p_a$): $y = -\frac{a}{g}x + \frac{\text{constant} - p_a}{\rho g}$ (straight line)

- Solid body rotation with angular speed Ω :

$p = \text{constant} - \rho g y + \rho \frac{r^2}{2} \Omega^2$
surface ($p = p_a$): $y = \frac{1}{2} \frac{\Omega^2}{g} r^2 + \frac{\text{constant} - p_a}{\rho g}$ (parabola)