Algorithm Design, Anal. & Imp., Homework 4

Out: April 23, 2015, Due: May 04, 2015 (beginning of the class), Total: 90

Note:

- This homework will carry 9 points towards your final score. It is due at the beginning of the class on the due date. No late homeworks will be accepted.
- For any solution to an algorithm problem, you always need to provide correctness proof and complexity analysis, by default.
- Homeworks are individual work, please do not collaborate with others inside or outside of the class.
- Please use piazza forum, email the instructor or TAs, or use the office hours for understanding the questions or for getting hints for solution.

Questions

1: Solve problem 34.5-2 by reducing from vertex cover problem (CLRS page 1100) (12)

2: Solve 34-1 (a, b) (CLRS page 1102) (5+10)

3. Source-Sink-Hamiltonian-path is a known $\mathcal{NP}$-complete problem. It is defined as below: $\textsc{SourceSinkHamPath}(H, s, t)$: Given an undirected graph H, does there exists a Hamiltonian-path $s - \ldots - t$ in the graph H. Also Hamiltonial-path problem is defined as below: $\textsc{HamPath}(G)$: Given an undirected graph G, does there exists a Hamiltonian-path in G. Prove that $\textsc{HamPath}(G)$ is $\mathcal{NP}$-complete by using the fact that $\textsc{SourceSinkHamPath}(H, s, t)$ is $\mathcal{NP}$-complete. (13)

4. Let G be a directed graph with positive weights on the edges. Let's consider a problem in which we find paths that may go backwards along some of the edges. Define a *no-cons-back path* in G to be a path in which some edges in G may be used in the backwards direction, but no two consecutive edges in the path are backwards. Thus on a path from v_i to v_j you could go from v_q to v_r backwards along edge (v_r, v_q) at a cost of $l(v_r, v_q)$. But, of course, the immediately preceding and following edges (if any) on the path must be used in a forward direction.

Design an efficient algorithm to find a shortest no-cons-back path from v_i to v_j, for all pairs v_i, v_j of vertices in G. Analyze the time and space of your algorithm. (15)

5: A small airline, Midwest Air, flies between three cities: Indianapolis, Columbus, and Philadelphia. They offer several flights but, for this problem, let us focus on the Friday afternoon flight that departs from Indianapolis, stops in Columbus, and continues to Philadelphia. There are three types of passengers: (i). Those traveling from Indianapolis to Columbus; (ii). Those traveling from Columbus to Philadelphia; (iii). Those traveling from Indianapolis to Philadelphia.

The aircraft is a small commuter plane that seats 30 passengers. The airline offers three fare classes: (i). Y class: full coach; (ii). B class: nonrefundable; (iii). M class: nonrefundable, 3-week advanced purchase.

Ticket prices, which are largely determined by external influences (i.e., competitors), have been set and advertised as follows:

	Indpls-Columbus	Columbus-Philadelphia	Indpls-Philadelphia
Y	300	160	360
B	220	130	280
M	100	80	140

Based on past experience, demand forecasters at Midwest Air have determined the following upper bounds on the number of potential customers in each of the 9 possible origin-destination/fare-class combinations:

	Indpls-Columbus	Columbus-Philadelphia	Indpls-Philadelphia
Y	4	8	3
B	8	13	10
M	22	20	18

The goal is to decide how many tickets from each of the 9 origin/destination/fareclass combinations to sell. The constraints are that the plane cannot be overbooked on either off the two legs of the flight and that the number of tickets made available cannot exceed the forecasted maximum demand. The objective is to maximize the revenue.

a. Formulate this problem as a linear programming problem (8)

b. Use AMPL (`www.ampl.com`) to solve this problem. Submit the model/data file, and your answer (number of tickets made available for each of possible 9 origin/destination/fareclass combinations. (7)

6: Consider the following LP:

$$\begin{array}{lrcrcl} \text{maximize} & 2x_1 & + & 5x_2 & & \\ \text{subject to} & & & & & \\ & 2x_1 & - & x_2 & \leq & 4 \\ & x_1 & + & 2x_2 & \leq & 9 \\ & -x_1 & + & x_2 & \leq & 3 \\ & & & x_1, x_2 & \geq & 0 \end{array}$$

Now answer the followings:

a. Solve the LP using Simplex algorithm, show all iterations (10)

b. Write the corresponding dual LP (3)

c. What is the primal and dual optimal solution? Show that the value of optimal solution for primal and dual are equal (3)

d. Show that complementary slackness holds for the optimum solution. (4)