PHY 450S Relativistic Electrodynamics: Midterm Exam **Thursday March 10, 2016**

One double-sided handwritten 8"x11" aid sheet and a calculator allowed. Make sure to READ each question first and answer all parts. The exam has four question and two pages. All questions are of equal value.

Time allowed: 2 hours

1. During the 2000's, the BaBar experiment at Stanford studied the properties of heavy subatomic particles called B-mesons. A beam of electrons collided with a beam of positrons travelling in the opposite direction. The electrons and positrons would annihilate to form a heavy particle called an $\Upsilon(4S)$, which subsequently decayed to B-mesons which were studied. You don't need to know any particle physics to solve this problem - just relativistic kinematics.

   In this problem, the units of energy are GeV (giga-electron-volts, the preferred unit for this kind of particle physics experiment), and the corresponding units of mass are GeV/c^2 and for momentum are GeV/c. Since c is absorbed into the units of this problem, you will not need the numerical value of c. Since the mass of the electron (and positron) (0.511×10^{-6} GeV/c^2) is much less than its energy, treat the electrons and positrons as MASSLESS in this problem.

   (a) If an electron of energy 9.0 GeV travelling in the $\hat{z}$ direction and a positron with energy 3.1 GeV travelling in the $-\hat{z}$ direction (in the lab frame) annihilate to form an $\Upsilon(4S)$, find
      i. the energy and momentum of the $\Upsilon(4S)$ in the laboratory frame in units of GeV and GeV/c respectively,
      ii. the mass of the $\Upsilon(4S)$ in GeV/c^2, and
      iii. the 3-velocity of the $\Upsilon(4S)$ in the laboratory frame (as a fraction of c).

   (b) The $\Upsilon(4S)$ then decays to a B-meson and an anti-B-Meson, each with mass 5.27 GeV/c^2. Find the speeds of the two mesons (again, as a fraction of c) in the rest frame of the $\Upsilon(4S)$ and show that they are nonrelativistic (v/c not close to 1).

2. (a) In nonrelativistic mechanics, $\vec{F} = m\vec{a}$. The relativistic equation, $\vec{F} = d\vec{p}/dt$, cannot be so simply expressed: show that, in relativistic mechanics,

   $$\vec{F} = \frac{m}{\sqrt{1 - v^2/c^2}} \left(\vec{a} + \frac{\vec{v}(\vec{v} \cdot \vec{a})}{c^2 - v^2} \right)$$

   where $\vec{a} = d\vec{v}/dt$ is the ordinary acceleration.

   (b) Consider a particle with four-momentum p^μ and an observer with four-velocity u^μ. Show that the magnitude of the particle's 3-momentum as measured by the observer is

   $$|\vec{p}| = \sqrt{(p_\mu u^\mu)^2 - (p_\mu p^\mu)}.$$

3. (a) Using the fact that $F^{\mu\nu}$ is a 2-index tensor, derive the transformation laws for the components of the electric and magnetic fields $\vec{E}$ and $\vec{B}$ between a reference frame S and a reference frame S' moving at a velocity $\vec{v} = v_0\hat{x}$ relative to S.

   (b) Consider a particle of charge e at rest in reference frame S in a constant electric field, $\vec{E} = E_x\hat{x} + E_y\hat{y}$ and a constant magnetic field $B = B\hat{z}$. Find the force on the particle. Now consider the same situation as viewed in a reference frame S' moving with velocity v_0 in the $\hat{x}$ direction. Find the electric and magnetic fields in S', and use them to find the force on the particle in S' . Are your results consistent with each other? (Recall your results on the second problem set for the Lorentz transformation of forces).

4. Consider a hypothetical particle called a *tachyon* that always moves FASTER than the velocity of light.

(a) Show that if the speed of the particle is greater than c in one reference frame, it is greater than c all reference frames.

(b) A tachyon is moving with constant speed $\vec{v} = v_x \hat{x}$ in a frame S, where $v_x > c$. Draw a space-time diagram indicating the worldline of the tachyon in S. Show using the same diagram that there exist reference frames in which the tachyon could be used to send signals backwards in time. What is the minimum velocity $\vec{v}_0 = v_0 \hat{x}$ of an inertial frame S' with respect to S in which the tachyon is moving backwards in time?

(c) Define the interval $d\tilde{s}^2 = -ds^2$, so that $d\tilde{s}^2$ is positive and hence $d\tilde{s}$ is real along the worldline of the tachyon. If we define the four-velocity of the tachyon to be $u^\mu = dx^\mu / d\tilde{s}$, show that the four-velocity of the tachyon obeys $u^\mu u_\mu = -1$.

(d) Find the components of u^μ in terms of the 3-velocity $\vec{v} = d\vec{x}/dt$.

(e) Define the four-momentum by $p^\mu = m u^\mu$ and find the relation between energy and momentum for a tachyon.

(f) Show that there always exist inertial frames where the energy of any tachyon is negative. If the tachyon is moving at a speed $\vec{v} = v_x \hat{x}$ in frame S, what is the minimum boost in the x direction of a frame S' in which the energy of the tachyon is negative?