Phy 450 Midterm: Abbreviated Solutions

1. This question largely deals with conservation of energy/momentum.

a) i) energy of e^- is E_1
" " " e^+ " E_2

Then 4-momentum of e^- is $(E_1, E_1\hat{z}) \equiv p_1^\mu$
" " " " e^+ is $(E_2, -E_2\hat{z}) \equiv p_2^\mu$

(since $p_\mu p^\mu = 0$ for massless particles)

$\therefore$ 4-momentum of Υ is $p_1^\mu + p_2^\mu = (E_1+E_2, (E_1-E_2)\hat{z})$ in lab frame.

$\therefore$ $E_\Upsilon = E_1 + E_2 = 12.1$ GeV
$\vec{p}_\Upsilon = (E_1 - E_2)\hat{z} = 5.9$ GeV/c

(NB I am setting $c=1$ here for simplicity; you can restore factors of c)

ii) ~~say~~ again setting $c=1$, we have $p^\mu p_\mu = m^2$

So $M_\Upsilon^2 = (12.1)^2 - (5.9)^2 = 111.6$ GeV^2

or $M_\Upsilon = 10.56$ GeV (or GeV/c^2 if we restore factors of c)

iii) $p^\mu = mc\,u^\mu$ where $u^\mu = (\gamma, \vec{v}/c)$

$\therefore p_z = m\gamma v_z = \frac{mc}{\sqrt{1 - v_z^2/c^2}}\left(\frac{v_z}{c}\right) \to$ solve for $\frac{v_z}{c}$

2. a) This problem was done on a previous problem set.

b) The easiest way to do this is to work in the rest frame of the observer. Then $u^\mu = (1, \vec{0})$ and the RHS is $\sqrt{(p_0)^2 - p_\mu p^\mu}$ $= \sqrt{|\vec{p}|^2} = |\vec{p}|$. Since the RHS is a Lorentz scalar, this must hold in any frame.

3. a) This is done in both L&L and Griffiths.

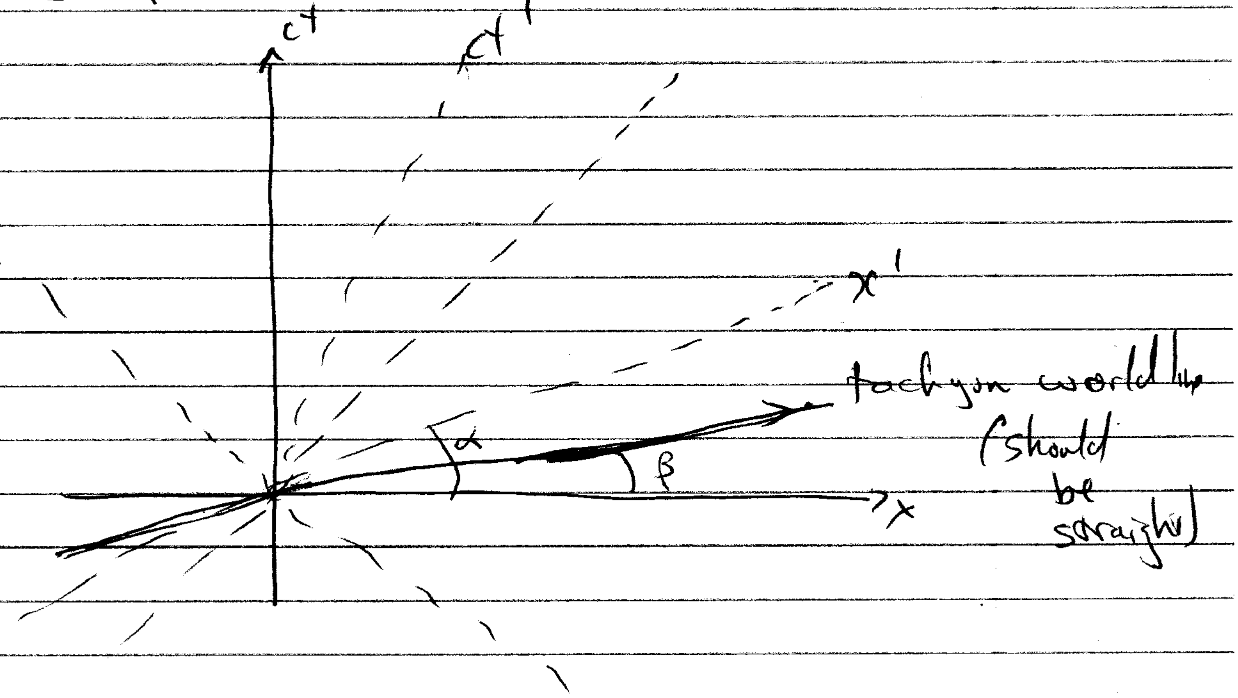
b) This follows straightforwardly from the transformation properties of $\vec{E}$ & $\vec{B}$ fields, and the transformation of the $\parallel$ and $\perp$ components of $\vec{F}$ from Problem Set #2.

4. a) the spacetime interval between 2 infinitesimally close points on the tachyon's worldline is

$$ds^2 = c^2 dt^2 - d\vec{r}^2 = c^2 dt'^2 - d\vec{r}'^2$$

If $v > c$, then $ds^2 < 0$. Since this is true in any frame, must have $v > c$ in any frame.

b)



In S', the tachyon is moving backwards in time.

$\tan\alpha = \frac{u_x}{c}$, where u_x = boost of S' frame

$\tan\beta = \frac{c\Delta t}{\Delta x} = \frac{c}{v_x}$ where v_x = speed of tachyon

is moving back in time if $\alpha > \beta \Rightarrow u_x > \frac{c^2}{v_x}$

(NB, since $v_x > c$, $u_x < c$)

c) $d\tilde{s}^2 = -ds^2$, $u^\mu \equiv \dfrac{dx^\mu}{d\tilde{s}}$

then $u^\mu u_\mu = \dfrac{dx^\mu}{d\tilde{s}} \dfrac{dx_\mu}{d\tilde{s}} = \dfrac{ds^2}{d\tilde{s}^2} = -1$

d) $d\tilde{s}^2 = d\vec{r}^2 - c^2 dt^2$

$$= c^2 dt^2 \left[\frac{d\vec{r}^2}{c^2 dt^2} - 1 \right] = c^2 dt^2 \left[\frac{v^2}{c^2} - 1 \right]$$

$\therefore d\tilde{s} = c\,dt \sqrt{\dfrac{v^2}{c^2} - 1}$

so $u^0 = \dfrac{d(ct)}{\sqrt{\frac{v^2}{c^2} - 1}\, c\,dt} = \dfrac{1}{\sqrt{\frac{v^2}{c^2} - 1}}$

$$\vec{u} = \frac{d\vec{x}}{\sqrt{\frac{v^2}{c^2} - 1}\, c\,dt} = \frac{\vec{v}}{\sqrt{\frac{v^2}{c^2} - 1}\, c}$$

e) Apologies for missing a factor of c; should be

$p^\mu = m c u^\mu$

$\therefore p^\mu p_\mu = m^2 c^2 u^\mu u_\mu = -m^2 c^2 = \dfrac{E^2}{c^2} - \vec{p}^{\,2} = -m^2 c^2$

or $E^2 - c^2 \vec{p}^{\,2} = -m^2 c^4$

f) $p^\mu = \left(\frac{\varepsilon}{c}, \vec{p}\right)$

~~the~~ ~~Bos~~ Boost in $\hat{x}$ axis by u:

$$p'^0 = \gamma_u \left(p^0 - \frac{u}{c} p^1\right)$$

$\therefore p'^0 < 0$ if $p^0 < \frac{u}{c} p^1$

or $\frac{u}{c} > \frac{p^0}{p^1} = \frac{u^0}{u^1} = \frac{c}{v_x}$

$\therefore$ need $u v_x > c^2$, or $u > \frac{c^2}{v_x}$

Note that since $v_x > c$, $\underline{\underline{u < c}}$. This is the same as in part (b).