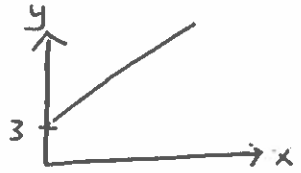
Problem Set 1 Solutions

Q1)

1. $y = 6x + 3$

Slope is 6, y-intersection 3

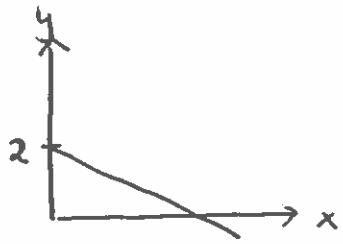


2. $4 = 3x + 2y$

Rewrite in terms of y:

$$2y = 4 - 3x$$
$$y = 2 - \frac{3}{2}x$$
$$y = -\frac{3}{2}x + 2$$

Slope is $-\frac{3}{2}$, y-intersection 2



3 $y = 7$

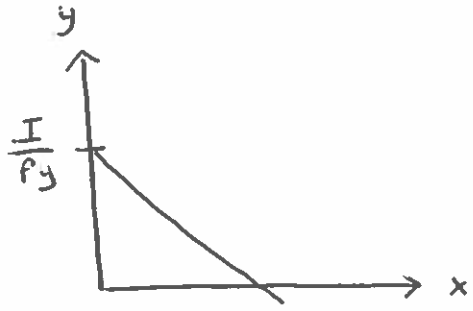
Slope is 0, y-intersection of 7 (horizontal line)

4. $I = P_x x + P_y y$

Rewrite in terms of y

$$P_y y = I - P_x x$$
$$y = \frac{I}{P_y} - \frac{P_x}{P_y}x = \frac{-P_x}{P_y}x + \frac{I}{P_y}$$

Slope is $\frac{-P_x}{P_y}$, y-intersection is $\frac{I}{P_y}$



Q2)

1. $A=(0,10)$, $B=(5,20)$

$$m=\frac{20-10}{5-0}=\frac{10}{5}=2$$ upward line - sloping

2. $A=(5,3)$, $B=(6,1)$

$$m=\frac{1-3}{6-5}=\frac{-2}{1}=-2$$ downward-sloping line

Q3)

1. $y=5x$, $x=3y+4$

Substituting the 1st equation into the second:

$$x=3(5x)+4$$
$$\Rightarrow x=15x+4$$
$$\Rightarrow -14x=4$$
$$\Rightarrow x=\frac{-4}{14}=-\frac{2}{7}$$

Substitute $x=-\frac{2}{7}$ into $y=5x$

$$y=5\left(-\frac{2}{7}\right)=\frac{-10}{7}$$

$\left(-\frac{2}{7}, \frac{-10}{7}\right)$

2. $Q=23-5P$, $Q=4+2P$

Setting the two equations equal and solve for P.

$$23-5P=4+2P$$
$$\Rightarrow 19=7P$$
$$\Rightarrow P=\frac{19}{7}$$

Substitute into first equation

$$Q=23-5\left(\frac{19}{7}\right)=23-\frac{95}{7}=\frac{161}{7}-\frac{95}{7}=\frac{66}{7}$$

(2)

$P=\frac{19}{7}$, $Q=\frac{66}{7}$

3 $y = 24, \quad x = 14 - 0.5y$

Substitute the first equation into the second:

$X = 14 - 0.5(24)$

$\Rightarrow X = 14 - 12 = 2$

So we have $x = 2, y = 24$.

4. $Q = A - BP$

$Q = C + DP$

Setting them equal, we have

$$A - BP = C + DP$$

$$\Rightarrow A - C = DP + BP$$

$$\Rightarrow A - C = P(D + B)$$

$$\Rightarrow P = \frac{A - C}{B + D}$$

Sub into first equation:

$$Q = A - B\left(\frac{A-C}{B+D}\right) = A - \frac{AB - BC}{B+D} = \frac{A(B+D)}{(B+D)} - \frac{AB - BC}{B+D}$$

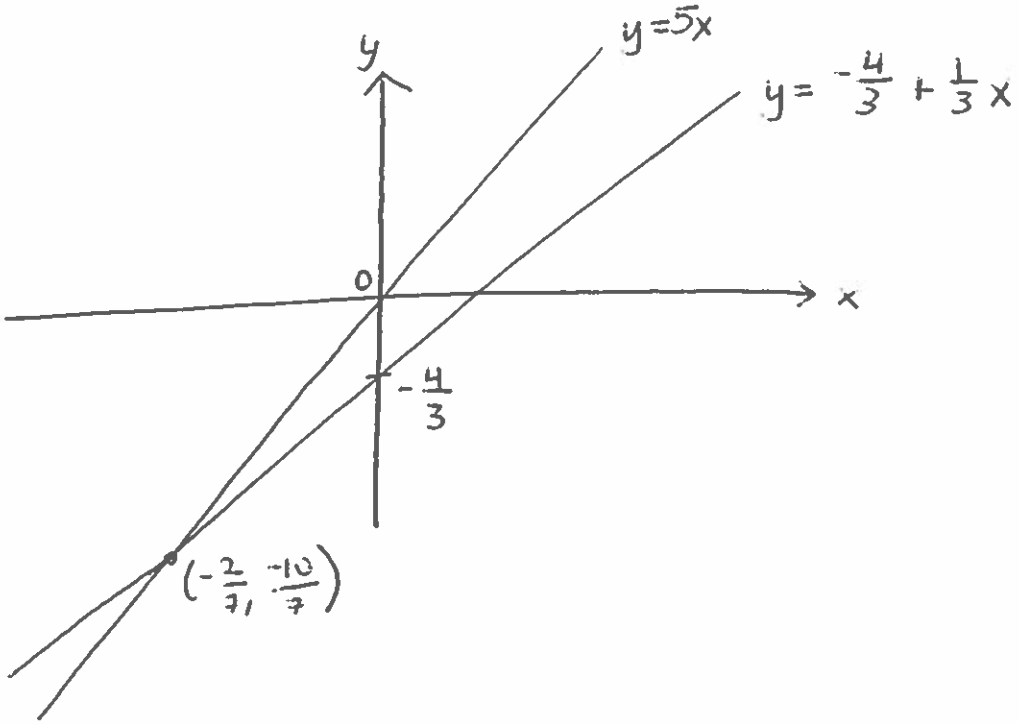
$$= \frac{AB + AD - (AB - BC)}{B+D}$$

$$= \frac{AD + BC}{B+D}$$

So $P = \frac{A-C}{B+D}$

$Q = \frac{AD+BC}{B+D}$

(Q4)



Rewrite $x = 3y + 4$ in terms of y:

$$3y = x - 4$$
$$\Rightarrow y = \frac{x}{3} - \frac{4}{3}$$
$$\Rightarrow y = \frac{1}{3}x - \frac{4}{3}$$

Solve for point of intersection:

$$y = 5x = \frac{1}{3}x - \frac{4}{3}$$
$$\Rightarrow 5x - \frac{1}{3}x = -\frac{4}{3}$$
$$\Rightarrow \frac{15}{3}x - \frac{1}{3}x = -\frac{4}{3}$$
$$\Rightarrow \frac{14}{3}x = -\frac{4}{3}$$
$$\Rightarrow x = \frac{-4}{14} = -\frac{2}{7}$$

Substitute into $y = 5x$:

$$y = 5\left(-\frac{2}{7}\right) = \frac{-10}{7}$$

Point of intersection.

$$\left(-\frac{2}{7}, \frac{-10}{7}\right)$$