Suppose that firm A and B both produce 80 units of pollution. The federal government wants to reduce pollution levels. The marginal costs associated with pollution reduction are

$$MC_A = 50 + 3Q_A$$

for firm A and

$$MC_B = 20 + 6Q_B$$

for firm B, where Q_A and Q_B are the quantities of pollution *reduced* by each firm. Society's marginal benefit from pollution reduction is given by

$$MB = 590 - 3Q_T$$

where Q_T is the total reduction in pollution.

1. What is the socially optimal level of each firm's pollution reduction?
2. How much total pollution is there in the social optimum?
3. Explain why it is inefficient to give each firm an *equal* number of pollution permits (if they are not allowed to trade them).
4. Explain how the social optimum can be achieved if firms are given equal numbers of pollution permits but *are* allowed to trade them.
5. Can the social optimum be achieved using a tax on pollution?

Solutions:

1. The social optimum requires that $MC = MC_A = MC_B$. Setting $MC_A = MC_B$ and solving for Q_A in terms of Q_B:

$$\begin{aligned} 50 + 3Q_A &= 20 + 6Q_B \\ \implies 3Q_A &= -30 + 6Q_B \\ \implies Q_A &= -10 + 2Q_B \end{aligned}$$

Now set $MC_B = MB$, substitute in for Q_A, and solve for Q_B:

$$\begin{aligned} 20 + 6Q_B &= 590 - 3Q_T \\ 20 + 6Q_B &= 590 - 3(Q_A + Q_B) \\ 20 + 6Q_B &= 590 - 3Q_Q - 3Q_B \\ 20 + 6Q_B &= 590 - 3(-10 + 2Q_B) - 3Q_B \\ 6Q_B &= 570 + 30 - 6Q_B - 3Q_B \\ 15Q_B &= 600 \\ \implies Q_B &= 40 \end{aligned}$$

Having solved for Q_B, plug back into the equation for Q_A to obtain:

$$Q_A = -10 + 2(40) = 70$$

So the social optimum level of pollution reduction is $Q_A = 70, Q_B = 40$.

2. The question states that the firms were both producing 80 units of pollution each, for a total of 160 units. Social optimum has $40 + 70 = 110$ pollution *reduction*, so social optimum has $160 - 110 = 50$ units of pollution.

3. The government would like to achieve the social optimum and allow only 50 units of pollution, meaning it will only issue 50 pollution permits. By dividing them equally, each firm gets 25 permits, which means that they will have to reduce pollution by $80 - 25 = 55$ units. Note that the costs to A and B are different here:

$$MC_{A,55} = 50 + 3(55) = 215$$

$$MC_{B,55} = 20 + 6(55) = 350$$

Thus, it costs less for firm A to reduce pollution at this level than it does for firm B. In particular, by forcing the firms to both reduce pollution by 55 units, the government is not taking into account that it is more cost-effective to have A reduce more than B. Thus, it is inefficient.

4. If they can trade pollution permits, B will have an incentive to buy pollution permits (i.e. buy the *right* to pollute) since it costs much more for B to reduce pollution than A. By compensating (paying) A at least $MC_{A,55}$ and at most $MC_{B,55}$ for a pollution permit, B can make both himself and A better off. In particular, these trades will happen up till the point where $MB = MC_A = MC_B$, at which point there are no more gains from trading permits and the socially optimum level of pollution reduction has been attained.

5. Yes. To find the amount of tax needed to achieve the socially optimum amount of pollution reduction, we must find T such that

$$MC_{B,\text{social optimum}} = T = MC_{A,\text{social optimum}}$$

We see that

$$MC_{A,\text{social optimum}} = 50 + 3Q_A = 50 + 3(70) = 260$$

and

$$MC_{B,\text{social optimum}} = 20 + 6Q_B = 20 + 6(40) = 260$$

So a tax of $T = 260$ would achieve the social optimum.