TELE302
Queueing Analysis: V

Jeremiah Deng

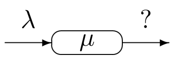
University of Otago

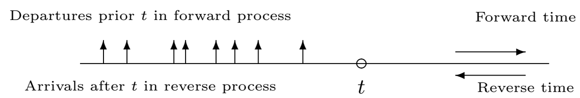
10 August 2015

Lecture Outline

Burke's Theorem

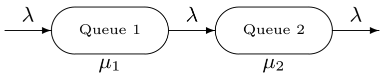
- Consider an $M/M/1$ system (λ, μ).
- What is the departure rate?
- Burke's Theorem states
  - The departure is of a Poisson process (λ)
  - Number of jobs in system at time t is independent of the number of departures prior to t
- Also applies to $M/M/c$ systems.
- Can we prove it?





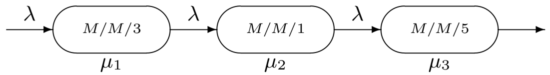
- Imagine a time-reversed $M/M/1$ system, whose arrival process is the departure process in the forward system.
- It is proved that the reversed has the same stationary distribution as the forward system.
- For a fixed time t, the departures prior to t in the forward process are also the arrivals after t in the reversed process, which is independent Poisson.
- Future arrivals in the reversed system does not depend on the current number in system.
- That means in the forward system the past departures are independent of the current number in system.

Two $M/M/1$ in tandem



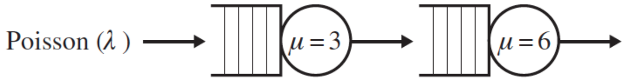
- Two queues in tandem
- $\rho_1 = \lambda/\mu_1$, $\rho_2 = \lambda/\mu_2$
- Number of customers in each queue is independent at a given time
- $P(n \text{ at queue } 1, m \text{ at queue } 2) = \rho_1^n(1-\rho_1)\rho_2^m(1-\rho_2)$

Tandem Queueing

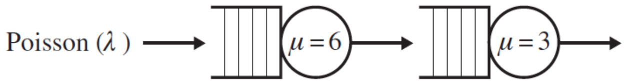


- We can tandemize $M/M/c$ and $M/M/1$
- Arrival and departure at each service center are Poisson with **the same** arrival rate (Burke's Theorem): $\lambda_1 = \lambda_2 = \lambda_3 = ...$
- ☺ Network at steady state can be analyzed as three **independent** queueing systems.
- Condition for reaching stationary states: $\lambda < \min(3\mu_1, \mu_2, 5\mu_3)$

Quiz 1

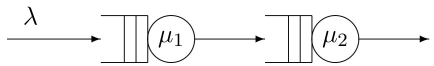


versus



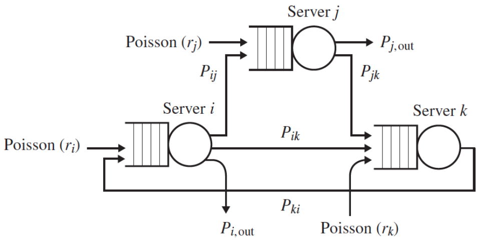
- Harcol-Balter, p.293
- Which one is faster?
- Can you work out the total time in system / number of jobs in system?

Quiz 2



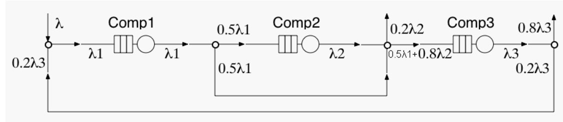
- Suppose $\mu_1 + \mu_2 = 9$, λ small enough
- What values of μ_1 and μ_2 will give the optimal setting?
- Can you work out the shortest total time in system?

Jackson Network



- Jackson, *Management Sciences*, 1963
- K servers each with unbounded FCFS queue
- Each server receives job arrivals from inside and outside of the network, service rate μ_i
- Outside arrivals are of Poisson process, with rate λ_i
- Routing of jobs is probabilistic

Jackson Network - Solution

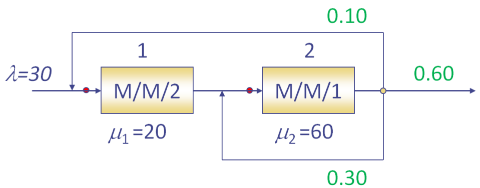


- Effective arrival rates at each node: made of external inflows (λ_i) and interconnecting inflows with probabilities $p_{j,k}$:

$$\Lambda_k = \lambda_k + \sum_{k=1}^{K} p_{j,k}\Lambda_j$$

- Each node behaves as an independent $M/M/c$ queueing system with mean arrival rate .

An Example



- $\lambda_1 = 30, \lambda_2 = 0$
- Work out the effective arrival rates:

$$\Lambda_1 = \lambda + 0.1\Lambda_2, \Lambda 2 = \Lambda_1 + 0.3\Lambda 2$$

- $\Lambda_1 = 35, \Lambda_2 = 50.$
- Is steady state reachable?

Recap

- Burke's Theorem (for $M/M/c$)
- Tandem & Jackson network
- Readings: Harchol-Balter, Chapter 16
- Assignment 1 due in a week
- NO Lab this week