PH320: Condensed Matter Physics II

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PROBLEM SET 0, DEADLINE: AUG. 13, 2012

A request: Please try to work out the problems, as far as possible, by yourself (discuss with friends, if necessary). Please do not give in to your urge to "google out the answer". Furthermore, the work you submit in the exam must be your own.

A note on the problems: Problems are marked by C, T, C/T or E. C means that the problem is *conceptual*, i. e., it helps you to understand and use the conceptual ideas of the subject. Such problems, usually, will not involve long calculations. On the other hand, problems marked with a T have primarily *technical* content. They should help you familiarise with a calculational technique that has been either discussed in class, or is newly introduced in the problem. Such problems may also involve the use of MATHEMATICA etc. A suggestion to approach the T problems is to reason out what the answers *ought to be* by conceptual arguments, i. e., always do the physics before you do the math. C/T problems involve both. E stands for *exploratory*, i. e., that will point you to material elsewhere or ask you a question to ponder and think about.

Reading: Ashcroft and Mermin, Chapters 1 and 2.

0/1. **(C) Warming up the Fermi sea:** Consider a collection of chargeless non-interacting fermions of mass m (equal number of $\uparrow$ and $\downarrow$ fermions) at a density ρ in 3D free space – this system will be called the free Fermi gas.

   (a) What is(are) the characteristic energy scale(s) of this system?

   (b) *Estimate* the ground state energy density, pressure and bulk modulus by *physical arguments*.

   (c) Now suppose you raise the temperature of the system from $T = 0$ by a "little bit" to a temperature T. What would "a little bit" mean, i. e., what is the condition on T so that it can be taken as "small"?

   (d) *Estimate* the temperature dependence of the specific heat by *physical arguments*.

   (e) *Estimate* the temperature dependence of the magnetic susceptibility by *physical arguments*.

0/2. **(T) Fermi sea, again:** In this problem, you will obtain precise answers for the qualitative results obtained in last problem.

   (a) Derive expressions (with all numerical factors) for the ground state energy, pressure and bulk modulus of the fermion gas discussed in the last problem.

   (b) Derive an expression for the chemical potential as a function of temperature T for "small" T – by now, you know what this means. Before you actually do this problem, ask yourself: Should the chemical potential increase or decrease as a function of temperature?

(c) Derive expressions for the specific heat and magnetic susceptibility as a function of temperature.

These calculations involve some technical manipulations which go under the name of "Sommerfeld expansion".

0/3. **(C) Fashionable dimension:** Consider the Fermi gas discussed in the first problem of this set, but now in 26 (twenty six) spatial dimensions again at a given density ρ.

(a) What is the characteristic energy scale in the system?

(b) How does the specific heat depend on temperature for "small" T?

(c) State precisely what you have learnt from this problem.

0/4. **(?C/?T/?E) To go up..or down?** Prove or disprove the statement: The chemical potential of a free Fermi gas in *any spatial dimension* always decreases with increasing temperature.

0/5. **(C/T) Bose in 1D:** Consider a collection of *bosons* in one spatial dimension with a density ρ (number of bosons per unit length), with *linear dispersion* $\varepsilon(k) = c|k|$, where c is a "velocity".

(a) What is(are) the characteristic energy scale(s) of the system?

(b) What is the ground state of this system?

(c) How does the energy of the system depend of the temperature? Identify the key regimes of temperature based on what you found in part 0/5a, and give *physical* arguments for the temperature dependence of the specific heat.

(d) Perform the full calculation of specific heat over all the relevant temperature regimes.

(e) Compare the "low temperature" specific heat of this Bose gas with that of the Fermi gas. Are you surprised? Is your finding merely an accident, or is there something deeper lurking underneath?

(f) Why on earth would someone worry about Bosons in 1D?

0/6. **(C/T) Flatland:** Consider electrons (mass m, charge $-e$) in a two dimensional plane of area A with a total areal density ρ. Use a coordinate system such that the plane is described by the x and y axes, with the z axis perpendicular to the plane. A magnetic field $\mathbf{B} = B\boldsymbol{e}_z$ is applied to the system. Answer the following questions for two separate cases, one where the electrons are spinless, and second when the electrons have spin $\frac{1}{2}$.

(a) Find the allowed energy eigenvalues and corresponding eigenstates.

(b) Find the density of states of the system.

(c) Find the specific heat of the system as a function of temperature T and magnetic field B.

0/7. **(E) Arxiv:** Go to `http://arxiv.org/list/cond-mat/new`. If you scroll down, you will see a huge list of papers – this the typical output of the condensed matter community on a given day! Find a paper that is of interest to you. Read it carefully, and write a two hundred word summary of the paper *in your own words*, focussing on the *bigger picture*. What is the problem the authors solve? What is its importance? What is their solution? Please *do not* copy down verbatim what the authors say in the paper. Make a list of key things that you *do not understand* in the paper, and prepare a study plan. That is, (a) What are the things you need to learn to understand the problems/results paper? (b) Of these which are conceptual, and which are technical? (c) What are the key references?

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