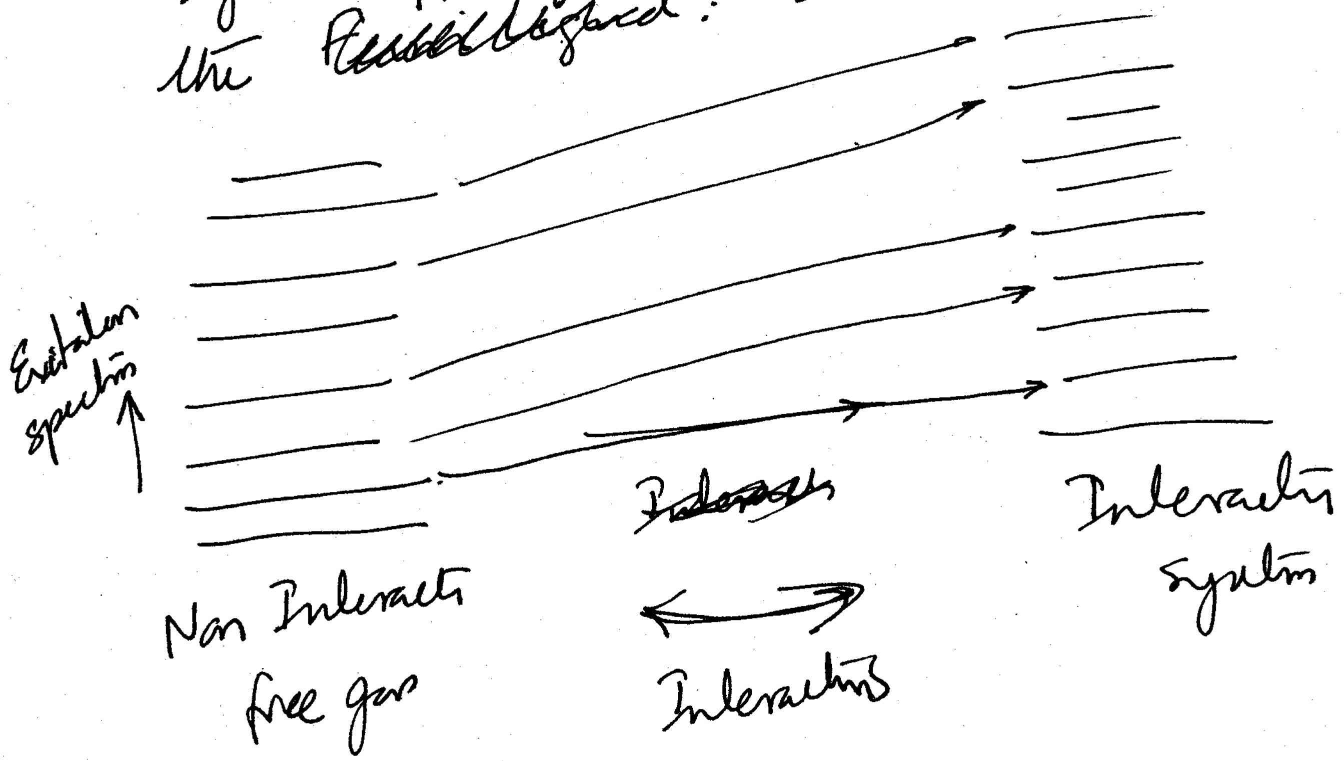
# LANDAU FERMI LIQUID THEORY

What we have seen thus far is that in a fermionic systems (with weak interactions), the identity of the Fermi surface is kept, and the excitations of the particle type — quasiparticle excitations are well defined. ~~[illegible]~~

This idea made Landau construct a phenomenological ~~theory~~ theory of interacting fermi system — now called Fermi Liquid Theory.

Landau's key idea is that the interacting system is adiabatically connected to the ~~[illegible]~~ free gas: Stated in a picture



Stated in words, ~~a~~ the key idea is that every excitation of the free fermi system has an "analog" in the interacting system. Thus, ~~in~~ an excitation with a given set of quantum numbers, is also present (with the same quantum numbers). We note ~~th~~ here that there are excitations of the ~~F~~ Interacting system which do not have an ~~and~~ analog in the free gas. Indeed, this is the key prediction of Landau theory.

Any phenomenological theory is based on ~~the~~ identifying a "fundamental" quantity, and then making a statement about the energetics etc of that quantity.

Let us take steps to make this more concrete. Consider a Fermion system of volume $V$, with a total density of fermions (spin 1/2) of $\rho$.

Landau's postulates

(0) — The "fundamental" quantity of the Fermi liquid is the quasiparticle occupation function ~~F~~ $n_\sigma(\vec{k})$.

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(1) — The quasi particles are Fermions
(this is "true" since in the absence
of interacting they are Fermions!)
This postulate allows us to write
down the total entropy of
the system as a functional of
$n_\sigma(\vec{k})$

$$S[n] = -\sum_{\vec{k}\sigma}{}' n_\sigma(\vec{k}) \ln n_\sigma(\vec{k}) + (1-n_\sigma(\vec{k}))\ln(1-n_\sigma(\vec{k}))$$

(2) # particles = numbr of occupied quasi excitations $N = \sum_{k\sigma}' n^{\circ}(k\sigma)$

(3) → There is an energy functional
$\mathcal{E}[n]$ which the system has,
and describes the interal energy of
the system.

~~Now let $n^{\circ}_\sigma(k)$ be the equilibrium value of the qp distribution function.~~

~~Then~~

$$\delta \mathcal{E} = \sum_k{}'$$

The key point is that the change in energy of the system when the qp distribution function ~~of~~ changes from $n_\sigma(\vec{k}) \to n_\sigma(\vec{k}) + \delta n_\sigma(\vec{k})$ is given by

$$\delta \mathcal{E} = \sum_{\vec{k}}' \varepsilon_\sigma(\vec{k})\, \delta n_\sigma(\vec{k})$$

where $\varepsilon_\sigma(\vec{k})$ is the quasi particle energy. ~~note~~ Thus $\varepsilon_\sigma(\vec{k})$ is a function of $n_\sigma(\vec{k})$ since the quasi particles interact.

Suppose $n^0_\sigma(\vec{k})$ is the equilibrium distribution function then

$$\left.\frac{\delta \mathcal{E}}{\delta n_\sigma(\vec{k})}\right|_{n^0_\sigma(\vec{k})} = \varepsilon^0_\sigma(\vec{k})$$

This is the equilibrium quasi particle energy for $n_\sigma(\vec{k}) = n^0_\sigma(\vec{k}) + \delta n_\sigma(\vec{k})$

$$\mathcal{E}[n] = \mathcal{E}^0 + \sum_{k}' \varepsilon^0_\sigma(\vec{k})\, \delta n_\sigma(\vec{k}) + \frac{1}{2!}\frac{1}{V}\sum' f_{\sigma\sigma'}(\vec{k},\vec{k}')\, \delta n_\sigma(\vec{k})\, \delta n_\sigma(\vec{k}')$$

where $f_{\sigma\sigma'}(\vec{k},\vec{k}')$ is the function that describes the interaction of quasiparticles

To see this, note that

$$\varepsilon_\sigma(\vec{k}) = \varepsilon^0_\sigma(\vec{k}) + \frac{1}{V}\sum_{\vec{k}'\sigma'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\,\delta n_{\sigma'}(\vec{k}')$$

$f_{\sigma\sigma'}(\vec{k},\vec{k})$ describes the interaction beetween qp with $(\vec{k}\sigma)$ and $(\vec{k}'\sigma')$.

If we assume that the system is spinrotation invariant, and that the ground state also has spin rotation invariance then $f_{\sigma\sigma'}(\vec{k},\vec{k})$ can be written in term of the interaction in the triplet and singlet channels

ie
$$f_{\sigma\sigma'}(\vec{k},\vec{k}') = \overbrace{\frac{1}{2}\left(f_t(\vec{k},\vec{k}') + f_s(\vec{k},\vec{k}')\right)}^{f^d(\vec{k},\vec{k}')} + \delta_{\sigma\sigma'}\underbrace{\frac{1}{2}\left(f_t(\vec{k},\vec{k}') - f_s(\vec{k},\vec{k}')\right)}_{f^e(\vec{k},\vec{k})}.$$

$$f_{\sigma\sigma'}(\vec{k},\vec{k}') = f^d(\vec{k},\vec{k}') + \delta_{\sigma\sigma'} f^e(\vec{k},\vec{k}')$$

Further in an isotropic system, with $\vec{k}$ near the fermi surface,

$$f^{d,e}(\vec{k},\vec{k}') = f^{de}(\theta)$$

⟺ angle between $\vec{k}$ and $\vec{k}'$.

Thus $f^{de}(\theta) = \sum_{l}' f_l^{d,e} \underbrace{P_L(\cos\theta)}_{\text{Legendre}}$.

$$f_l^{d,e} = \frac{2l+1}{4\pi} \int \underset{\substack{\uparrow \\ \text{solid angle}}}{d\Omega} \; f^{de}(\theta)\, P_L(\cos\theta)$$

Thus the interaction function is determined by a set of numbers $f_l^{d,e}$ called the Landau parameters)
(Later we will make a minor redefinition of these quantities).
This concludes the postulates of the Landau theory. Let us see what predictions it makes.

~~Occupation~~ ~~that~~. What is the equilibrium function $n_\sigma^0(\vec{k})$.
The ~~F~~ variation of free energy

$$\delta F = \delta E - T\delta S - \mu\, \delta N$$

$$= \Big[ \sum' \big[ \varepsilon_\sigma^0(\vec{k}) - T \ln\left(\frac{n_\sigma^0(\vec{k})}{(1-n_\sigma^0(\vec{k})}\right) - \mu\big] \delta n_\sigma(\vec{k})$$

$$= 0$$

This gives that $n^0_\sigma(\vec{k}) = \dfrac{1}{e^{\beta \xi^0_\sigma(\vec{k})}+1}$

$$\xi^0_\sigma(\vec{k}) = \varepsilon^0_\sigma(\vec{k}) - \mu$$

Note that this is really an interagal equation for $n^0_\sigma(\vec{k})$ to be solved for $n^0_\sigma(\vec{k})$ given $\varepsilon[n]$ and the condition $\sum'_{\vec{k}\sigma} n^0_\sigma(\vec{k}) = N$

which also determines $\mu$.

at $T=0$ $\quad n^0_\sigma(\vec{k}) = \theta(-\xi^0_\sigma(\vec{k}))$

the filled fermi see. Take $\varepsilon^0_\sigma(\vec{k}) = \mu$ as the Fermi surface, ie $\vec{k} = k_F$.

CA key point which we will prove later is that $k_F$ is not affected by the interaction and remains unchanged from the non interacting value.

Let us calculate some properties of the Fermi liquid.

Thermodynamic properties.

1. Specific heat.

For a free fermi gas the specific heat is linear in T (at small T) and is

$$C_V = \# \, g^0 T$$

where $g^0 = \frac{m k_F}{\pi^2}$, the density of states.

Let us calculate the density of states of our system, $g(\varepsilon)\, d\varepsilon = \frac{4\pi}{8\pi^3} k_F^2 \, dk$

Now $\frac{d\varepsilon^0_\sigma}{dk} = v_F^* \; dk$

↳ definition

$$\Rightarrow \quad g(\varepsilon_F) \equiv g = \frac{k_F^2}{\pi^2 v_F^*} = \frac{m^* k_F}{\pi^2}$$

where $m^* = \frac{k_F}{v_F^*}$ is the q.p. mass.

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We thus obtain that the ~~spe~~ $c_V = \# \, g T$ with $g = \frac{m^* k_F}{\pi^2}$ which is only a quantitative change. note $\frac{m^*}{m}$ can be quite large. For example, in heavy fermion system it can be $\gtrsim 1000$!

2. <u>Compressibility</u>

The bulk modulus ~~that~~ is defined as $B = -V \frac{dP}{dV}$ and $\chi = \frac{1}{B}$.

By Gibbs-Duhem relation, we have,

$$-V \frac{dP}{dV} = \rho^2 \frac{d\mu}{d\rho} = B. \quad \left[\rho^2 \frac{m k_F}{\pi^2} \text{ in free gas}\right]$$

or $$K = \frac{1}{\rho^2} \cancel{\frac{d\mu}{d\rho}} \frac{d\rho}{d\mu}$$

Let us evaluate the bulk modulus.

We need $\frac{d\mu}{d\rho}$.

Since we increase the density, ~~the~~ by $d\rho$, the size of the Fermi sea increases.

$$dp = \cancel{8\pi}\ \frac{2 \times \cancel{4}\pi}{8\pi^3} k_F^2\, dk_F$$

$$dk_F = \frac{\pi^2\, dp}{k_F^2}$$

Thus the change in the ~~number~~ q.p. occupancy is (at $T=0$) is

$$\delta n_\sigma(\vec{k}) = \theta(k_F + dk_F - k)\,\theta(k - k_F)$$

Now,

$$\varepsilon_\sigma(k + dk_F) = \mu + d\mu$$

$$\text{L.H.S} = \varepsilon^0 + \frac{d\varepsilon^0_\sigma}{dk} dk_F + \frac{1}{V} \sum'_{k'\sigma'} f_{\sigma\sigma'}(\vec{k}, \vec{k}')\,\delta n_{\sigma'}(\vec{k}')$$

$$\varepsilon^0 + v_F^* dk_F + \left[\frac{k_F^2}{8\pi^3} \int d\Omega \sum'_{\sigma'} f_{\sigma\sigma'}(\theta)\right] dk_F.$$

we get

$$\frac{d\mu}{d\rho} = \frac{\pi^2}{m^* k_F} + \frac{1}{2}\left[2 f_0^d + f_0^e\right]$$

$$= g^{-1}\underbrace{\left[1 + \frac{(2F_0^d + F_0^e)}{2}\right]}_{\text{determined by q.p. interaction}}.$$

Note that

$$\cancel{\frac{d\mu}{d\rho}} \quad \frac{d\rho}{d\mu} = \frac{g}{\left[1 + \frac{(2F_0^d + F_0^e)}{2}\right]}$$

is not siply $g$. (This is ~~called~~ an ~~Fermi~~ example of a Fermi liquid correction].

(3) Magnetic susceptibility.

Let us now look at the magnetic susceptibility

we have

$$\mu_\sigma = \mu + \sigma h.$$

$$M = \frac{1}{V}\sum_\sigma \sigma n_\sigma(k) \qquad \text{magnetization.}$$

Now ~~..~~ the change of chemical potential produces a change of Fermi sea

$$k_{F\sigma} = k_F + \sigma\, dk_F$$

$$dk_F \qquad M = 2 \times \frac{4\pi k_F^2}{8\pi^3}\, dk_F.$$

$$\delta n_\sigma(k) = \sigma\,\theta(\sigma(k_F + \sigma k_F - k))\,\theta(\sigma(k - k_F))$$

Now

$$\varepsilon_\sigma(k + \sigma dk_F) = \mu_0 + \sigma h$$

We get

$$\sigma h = v_F^* \sigma dk_F + \frac{1}{V} \sum_{k'\sigma'}' f_{\sigma\sigma'}(\vec{k},\vec{k}')\, \delta n_{\sigma'}$$

$$\Rightarrow \sigma h = v_F^* \sigma dk_F + \cancel{\frac{V}{V}} \frac{k_F^2}{8\pi^3} \int d\Omega \sum_{\sigma'}' f_{\sigma\sigma'}(\theta)\, \sigma'$$

$$\sigma h = v_F^* \sigma dk_F + \left[\frac{\sigma k_F^2}{8\pi^3} \int d\Omega\, f^e(\theta)\right] dk_F$$

$$h = \left[v^* + \frac{k_F^2}{8\pi^3} \int d\Omega\, f^e(\theta)\right] \frac{\pi^2}{k_F^2} M$$

$$= g^{-1} \left[1 + \frac{F_0^e}{2}\right] M$$

$$\Rightarrow \quad \chi_m = g \left[1 + \frac{F_0^e}{2}\right]^{-1} .$$

Again this is different from 14.
This is again a Fermi liquid correction!

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Now we will make one more connection; we will find $m^*$ in terms of the Landau parameters.

To do this ~~consider~~ note that our system is Galilean invariant. This, means that If we observe the ~~[illegible]~~ ~~for~~ system from a moving frame, ~~are~~ the physics should be invariant

The derivation uses two ideas,

~~(1) If we observe a quasiparticle on er~~

(1) The Landau energy functional is Galilean invariant.

(2) Suppose we have an excitation observed from the rest frame of the fluid with ~~[illegible]~~ quantum #s $|\vec{k},\sigma\rangle$, Its energy in the rest frame is $\varepsilon^0_\sigma(\vec{k})$.

What are the quantum numbers and ~~the~~ ~~exit~~ this excitation energies of when observed from a frame which is moving with velocity $\vec{V}$. w.r. to the fluid.

The momentum of the excitation will be $(\vec{k}-m\vec{v})$, spin $\sigma$

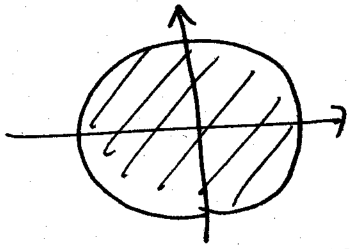
and its energy will be $\mathcal{E}_\sigma(\vec{k}-m\vec{v})$

We hence that

$$\mathcal{E}_\sigma(\vec{k}-m\vec{v}) = \mathcal{E}^0_\sigma(\vec{k}) - \vec{v}\cdot\vec{k}.$$

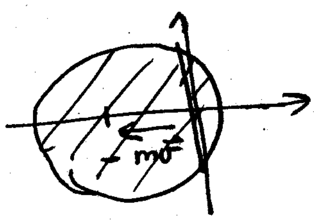
[This can be understood in the following way.

The ground state in the fluid frame is



$E^0_{GS}.$

and in the ~~Rest~~ moving frame is




The ~~Grand state~~ Energy of the ground state as viewed from moving frame

is $E^v_{GS} = E^0_{gs} + \dfrac{N m\vec{v}^2}{2}$

where N is the # of particles and m is the bare mass of the particles.

Suppose I create an excitation in the rest space with energy $\mathcal{E}^0_\sigma(\vec{k})$, what will be the energy of this excitation

$$\left[E^0_{gs} + \mathcal{E}^0_\sigma(\vec{k}) - \vec{k}\cdot\vec{v} + \frac{N m\vec{v}^2}{2}\right] - E^v_{GS}.$$

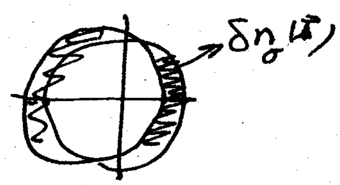
$$= \mathcal{E}^0_\sigma(\vec{k}) - \vec{k}\cdot\vec{v}.\,]$$

Now $\varepsilon_\sigma(k - m\vec{v})$ is a function of

$n_\sigma(\vec{k})$ in the moving frame

$$n_\sigma(\vec{k}) = n^0_\sigma(\vec{k}) + \delta n^*_\sigma(\vec{k})$$

$$\delta n_\sigma(\vec{k}) = \cancel{-mv\cos\theta} \sim -mv\cos\theta$$




$$-\cancel{\mathrm{sgn}[\cos\theta]\,\theta[}$$

Thus

$$\varepsilon_\sigma(k) = \varepsilon^0_\sigma(\hat{k}) + -\frac{d\varepsilon^0}{dk}mv + \left[\frac{k_F^2}{8\pi^3}\int d\Omega \sum_{\sigma'} f_{\sigma\sigma'}(\theta)\cos\theta\right] - mv$$

$\Rightarrow$ $\cancel{-\vec{k}\cdot\vec{v} =}$

$$-k_F v = -v_F^* mv - \frac{k_F^2}{\pi^2\, 2}\left[2f_1^d + f_1^e\right] mv$$

$\Rightarrow$ $$\frac{1}{m} = \frac{1}{m^*} + \frac{k_F}{\pi^2 \cancel{2}}\left[2f_1^d + f_1^e\right]$$

or $$\frac{1}{m} = \frac{1}{m^*}\left[1 + \frac{(2F_1^d + F_1^e)}{2}\right]$$

(using definitn, $gf = F$)

We have thus related the $m^*$ value to the Landau parameters. Note that in some systems $m^*$ can be quite large. In heavy fermion systems

$$\frac{m^*}{m} \simeq 1000 - 10000 \, ! \quad \text{(Hence heavy)}$$

~~to~~ The crux of the ~~mesa~~ story is that the Fermi liquid behaves qualitatively in exactly the same fashion as the free gas. The main difference is that everything in the free gas is determined by the density of states at the chemical potential. For example

$$\frac{C_V}{T \chi_m} = \text{cont}$$

wher cont is a universal constant in the free gas. On the other hand

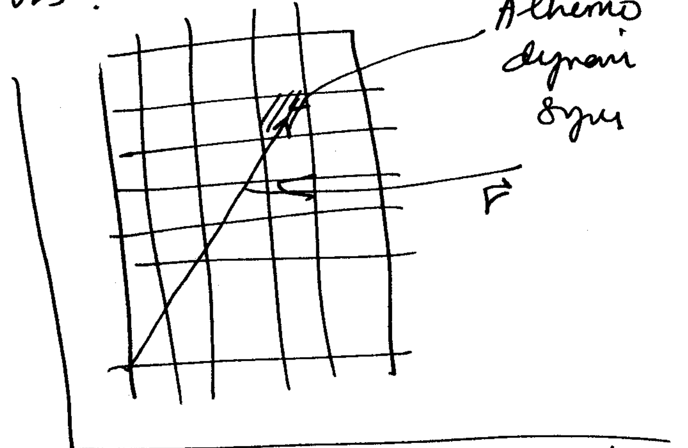
$$\frac{C_V}{T \chi_m} = \text{Function of Landau parameters}$$

and hence not universal.

An impotant point to be noted is that given that there are q.p. like excitates in our system, one can think of studying the excited states (including dynamics) via a Boltzmann theory (akin to what we did in the free gas).

Now what type of excited states are possible. For example, we can ask does sound propagate in the FL? The answer to this question at very low temperatures is NO! The reason is that quasi partill collisons ~~are~~ leave a life time of $\frac{1}{T^2}$ and at low enegh temperates the attainment of local equilibrium which is necessary for sound propagation is not possible. Any excitation at $T \approx 0$ must therefore be "collisionless", ie, should not depend on the relaxation process needed to attain equilibrium.

Let us develop a Boltzmann theory of this system. We now make the usual picture of a "thermodynamic system of thermodynamic systems".




Thus $n_\sigma(\vec{k},\vec{r})$ now describes the qp distribution at the thermodynamic system labelled $\vec{r}$.

One can also make this time dependent and have $n_\sigma(\vec{k},\vec{r},t)$.

We immediately obtain the transport equation

$$\frac{\partial n_\sigma}{\partial t} + \vec{v}_\sigma \cdot \frac{\partial n_\sigma}{\partial \vec{r}} + \vec{F}_\sigma \cdot \frac{\partial n_\sigma}{\partial \vec{k}} = \left(\frac{\partial f}{\partial t}\right)_{coll} \equiv I(n)$$

~~We went into the collisionless regime i.e. $I(n) = 0$.~~

$$\vec{v}_\sigma(\vec{k},\vec{r},t) = \frac{\partial \varepsilon_\sigma(\vec{k},\vec{r},t)}{\partial \vec{k}}.$$

$$\vec{F}_\sigma(\vec{k},\vec{r},t) = \vec{F}^{ext}_\sigma - \frac{\partial \varepsilon_\sigma(\vec{k},\vec{r},t)}{\partial \vec{r}}$$

↑ external force.

This is the general framework. We can use this to study collective excitations of the system in the <u>collisionless</u> regime. Thus $\vec{F}^{ext}_\sigma = \vec{0}$, as well as

$I(n) = 0$.

Consider the equilibrium system with

$$n^0_\sigma(\vec{k}, \vec{r}) = n^0_\sigma(\vec{k})$$

$$n_\sigma(\vec{k}, \vec{r}, t) = \delta n_\sigma(\vec{k}, \vec{r}, t) + n^0(\vec{k}).$$

$\delta n_\sigma(\vec{k}, \vec{r}, t)$ describes the collective mode.

~~Expanding in F~~. Linearizing the transport equation, ~~~~ requires

$$\varepsilon_\sigma(\vec{k}, \vec{r}, t) = \varepsilon^0(\vec{k}) + \frac{1}{V}\sum_{\vec{k}', \sigma'} f_{\sigma\sigma'}(\vec{k}, \vec{k}')\, \delta n_{\sigma'}(\vec{k}', \vec{r}, t).$$

Thus $$v_\sigma(\vec{k}, \vec{r}, t) = \overbrace{\frac{\partial \varepsilon^0(\vec{k})}{\partial \vec{k}}}^{v^0_\sigma(\vec{k})} + \frac{1}{V}\sum_{k'} \frac{\partial f_{\sigma\sigma'}}{\partial \vec{k}}\, \delta n_\sigma(k', \vec{r}, t)$$

$$\vec{F}_\sigma = -\frac{1}{V}\sum_{\vec{k}', \sigma'} f_{\sigma\sigma'}(\vec{k}, \vec{k}')\, \frac{\partial\, \delta n_{\sigma'}(\vec{k}', \vec{r}, t)}{\partial \vec{r}}.$$

We see that $\vec{F}_\sigma \sim O(\delta n)$.

The linearized transport equation is

$$\frac{\partial \delta n}{\partial t} + \underbrace{v^0_\sigma(\vec{k})}_{\substack{\uparrow \\ \text{drop} \\ \text{"0"}}} \frac{\partial \delta n}{\partial r} - \frac{1}{V}\sum_{k'\sigma'}{}' f_{\sigma\sigma'}(\vec{k},\vec{k}')\frac{\partial \delta n}{\partial \vec{r}} \cdot \frac{\partial n^0}{\partial \vec{k}}$$

Re writing

$$\frac{\partial \delta n}{\partial t} + v_\sigma(\vec{k}) \cdot \frac{\partial \delta n}{\partial \vec{r}} - \frac{1}{V}\sum_{\vec{k}'\sigma'}{}' f_{\sigma\sigma'}(\vec{k},\vec{k}')\frac{\partial \delta n}{\partial \vec{r}} \cdot \frac{\partial n^0}{\partial \vec{k}}$$

$$\frac{\partial \delta n}{\partial t} + \vec{v}_\sigma(\vec{k}) \cdot \frac{\partial \delta n}{\partial \vec{r}} - \frac{1}{V}\sum_{\vec{k}'\sigma'}{}' f_{\sigma\sigma'}(\vec{k},\vec{k}')\frac{\delta \delta n}{\delta \vec{r}} \cdot \vec{v}\,\frac{\partial n}{\partial \xi}$$

Re writing,

$$\frac{\partial \delta n_\sigma}{\partial t} + \vec{v}_\sigma(\vec{k}) \cdot \frac{\partial \delta n_\sigma}{\partial \vec{r}} - \frac{1}{V}\left(\sum_{\vec{k}'\vec{\sigma}'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\frac{\partial \delta n}{\partial \vec{r}}\right) \cdot \vec{v}\,\frac{\partial n_\sigma}{\partial \xi}.$$

@ T=0 $\quad \dfrac{\partial n}{\partial \xi} = -\delta(\xi^0_k).$

Thus we see that

$$\frac{\partial \delta n_\sigma}{\partial t} + \vec{v}_\sigma(\vec{k}) \cdot \frac{\partial \delta n_\sigma}{\partial \vec{r}} + \delta(\xi^0_k) \times \vec{v}_\sigma(\vec{k}) \cdot \frac{1}{V}\sum_{\vec{k}'\sigma'} f_{\sigma\sigma'}(\vec{k},\vec{k}')\frac{\partial n_\sigma(\vec{r}')}{\partial \vec{r}}$$

$$\delta n_\sigma(\vec{k},\vec{r},t) = \underbrace{\delta n_\sigma(\vec{k},\vec{q},\omega)}_{\hookrightarrow \text{ suppress these labels.}} e^{i(\vec{q}\cdot\vec{r}-\omega t)}$$

$$-i\omega\,\delta n_\sigma(\vec{k}) + i\vec{q}\cdot\vec{v}_\sigma(\vec{k})\,\delta n_\sigma(\vec{k})$$

$$+\ \delta(\xi^0_{\vec{k}}) \times \vec{v}_\sigma(\vec{k})\cdot i\vec{q}\ \frac{1}{V}\sum_{\vec{k}\sigma'}{}' f_{\sigma\sigma'}(\vec{k},\vec{k}')\,\delta n_{\sigma'}(\vec{k}).$$

$$= 0.$$

From the structure of this equation we see that $\delta n_\sigma(\vec{k}) \sim \delta(\xi^0_{\vec{k}})$, i.e., this represents the deformation of the ~~Fermi~~ Fermisurface

$$(\omega - \vec{v}_\sigma(\vec{k})\cdot\vec{q}\,)\,\delta n_\sigma(\vec{k})$$
$$= +\frac{k_*^2}{8\pi^3}\int d\Omega \sum_{\sigma'}{}' f_{\sigma\sigma'}(\hat{k},\hat{k}')\big)\,\delta n_\sigma(\vec{k}')$$

where $\vec{k}$ lies on the Fermi surface.

If we take $\vec{q}$ to be on the $z$ axis then

$$(\omega - \frac{k_F}{m^*}\cos\theta)\,\delta n_\sigma(\theta,\phi)$$
$$= \cos\theta\,\frac{k_F^2}{8\pi^3}\int d(\cos\theta')\,d\phi'\Big(\sum_{\sigma'}{}' f_{\sigma\sigma'}(\theta')\,\delta n_{\sigma'}(\theta',\phi')\Big)$$

Let us take $\delta n_{\sigma^*} = \delta n_\sigma$, and define

$$\sum_{\sigma'}{}' f_{\sigma\sigma'}(\theta) = 2f^d(\theta) + f^e(\theta) \equiv 2f(\theta)$$

With these definitions, and following Landau, we have, defining $\frac{m^* \omega}{k_F} = s$,

$$(s - \cos\theta)\, \delta n(\theta, \phi) = \frac{\cos\theta}{4\pi} \int d\Omega\, F(\theta)\, \delta n(\theta, \phi)$$

$$F = g f$$

Now one can obtain $s$ (the zero sound speed) by solving this eigenvalue problem. Note that this can be expressed in terms of $F_0$ and $F_1$, ~~and~~ These values can be obtained from experiment, and thus the theory predicts the speed of sound. Spin zero sound is also possible (see Landau 2)

The theory (details by Abrikosov & Khalatnikov) were verified in ~~[illegible]~~ experiments by Abel, Anderson and Wheatley. (See link on course page).

The theory can also be used to study the stability of the Fermi liquid.

At zero temperature, we require that the free energy be +ve definite w.r.t. variations about $n^0(\vec{k})$. $\mathcal{F} = E - \mu N$

Thus $\sum' \dfrac{\delta^2 \mathcal{F}}{\delta n_\sigma(\vec{k})\, \delta n_{\sigma'}(\vec{k}')} \delta n_\sigma(\vec{k})\, \delta n_{\sigma'}(\vec{k}') \geq 0.$

defining $f(\theta) = \sum_{\sigma'}' f_{\sigma\sigma'}(\theta)$,

and associated ~~to~~ Landau parameters, we obtain the condition of stability as

$$\left(1 + \frac{F_l}{2l+1}\right) > 0. \quad \forall\, l.$$

One can also study the magnetic stability by a similar analysis and obtain other relations.

When such a stability condition fails, the Fermi liquid is unstable and we say that we have a Pomeranchuk instability.