Cache Memory II

B&O Readings: 6.4-6.7

CSE 361: Introduction to Systems Software

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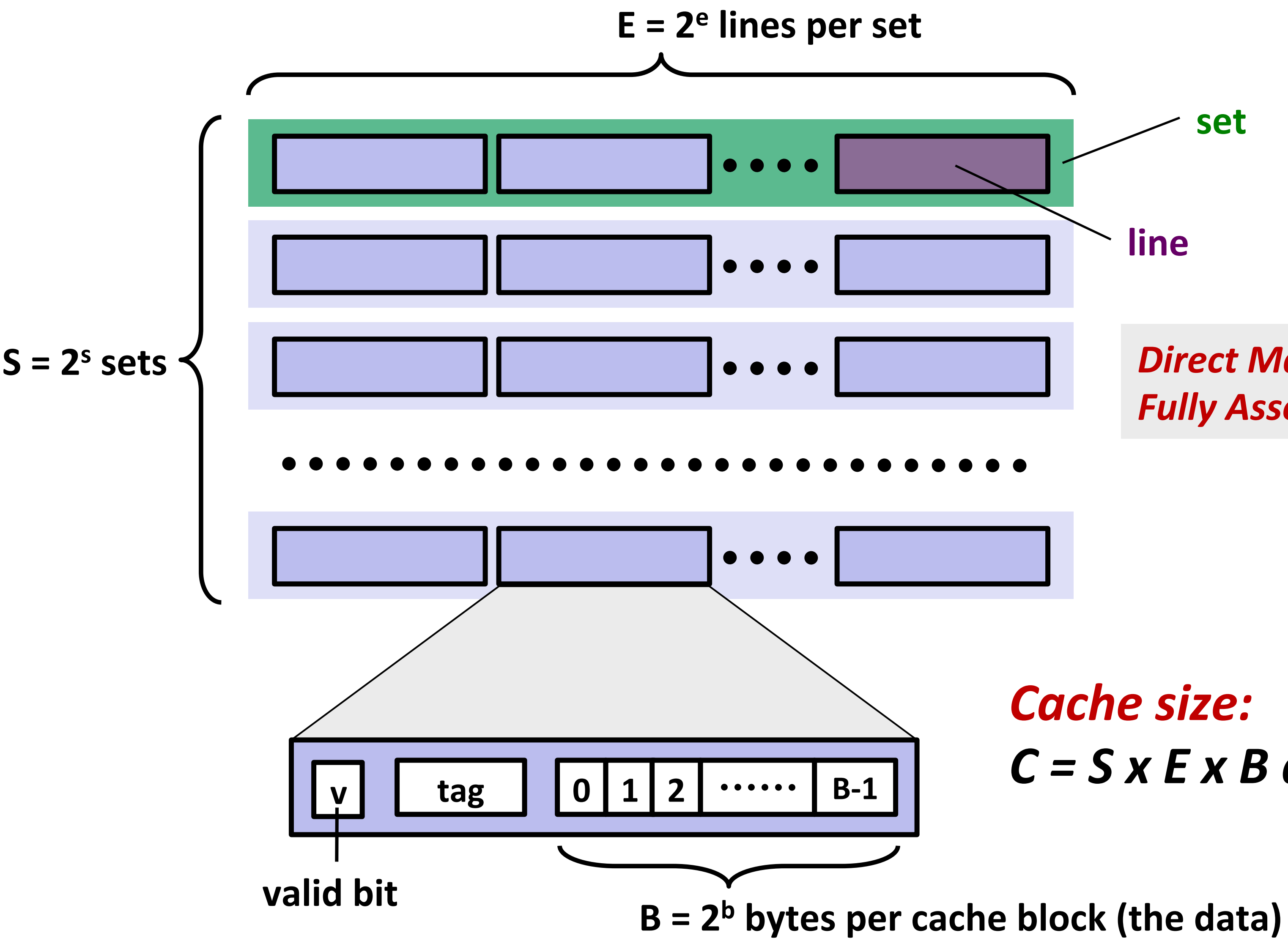
Note: these slides were originally created in parts by Markus Püschel at Carnegie Mellon University and in parts by Anne Bracy at Cornell

Today

- **Cache organization and operation (cont.)**
- **Performance impact of caches**
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

Recap: General Cache Organization (S, E, B)

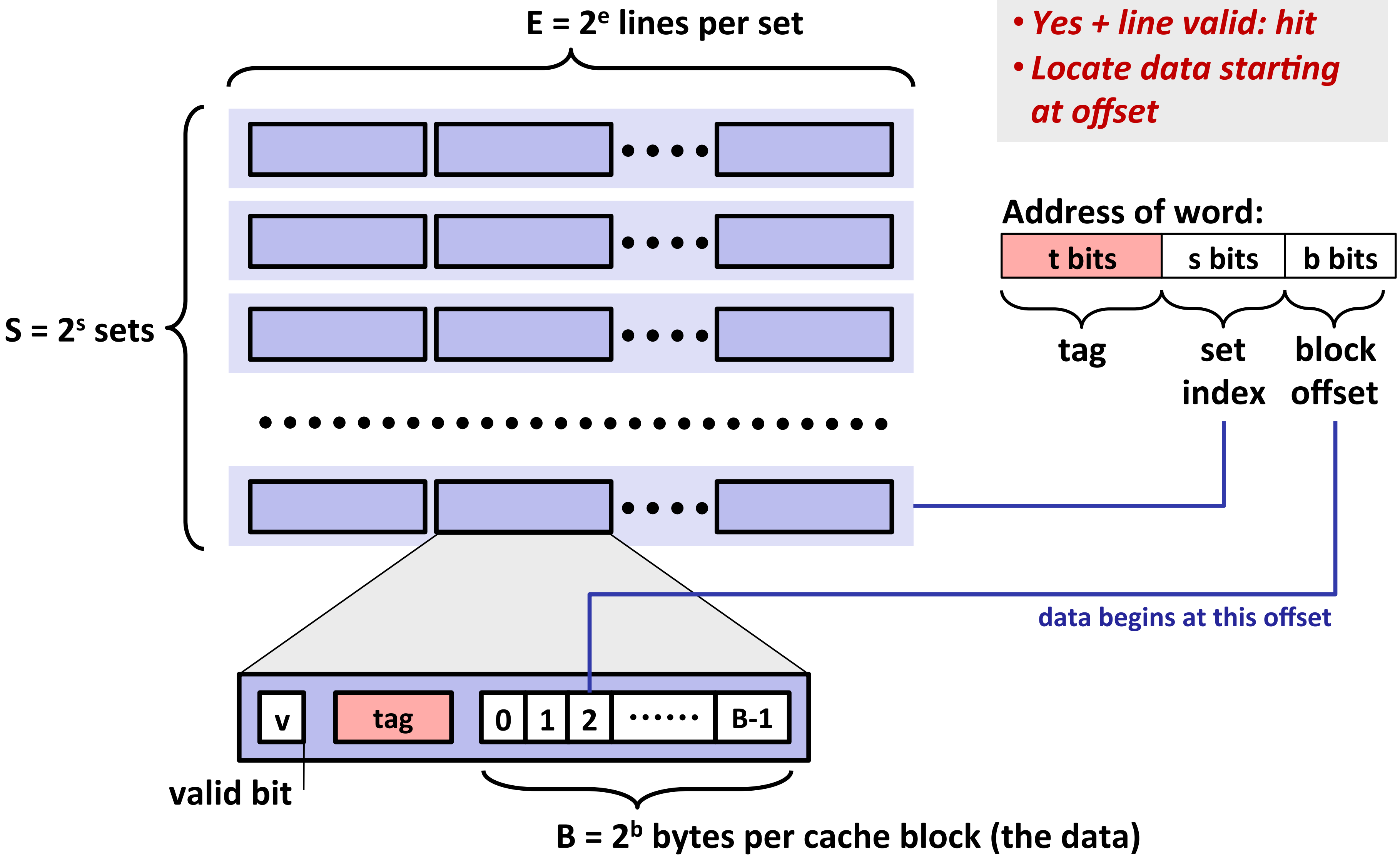


Direct Mapped: E=1
Fully Associative: S=1

Cache size:
C = S x E x B data bytes

Recap: Cache Read

- *Locate set*
- *Check if any line in set has matching tag*
- *Yes + line valid: hit*
- *Locate data starting at offset*



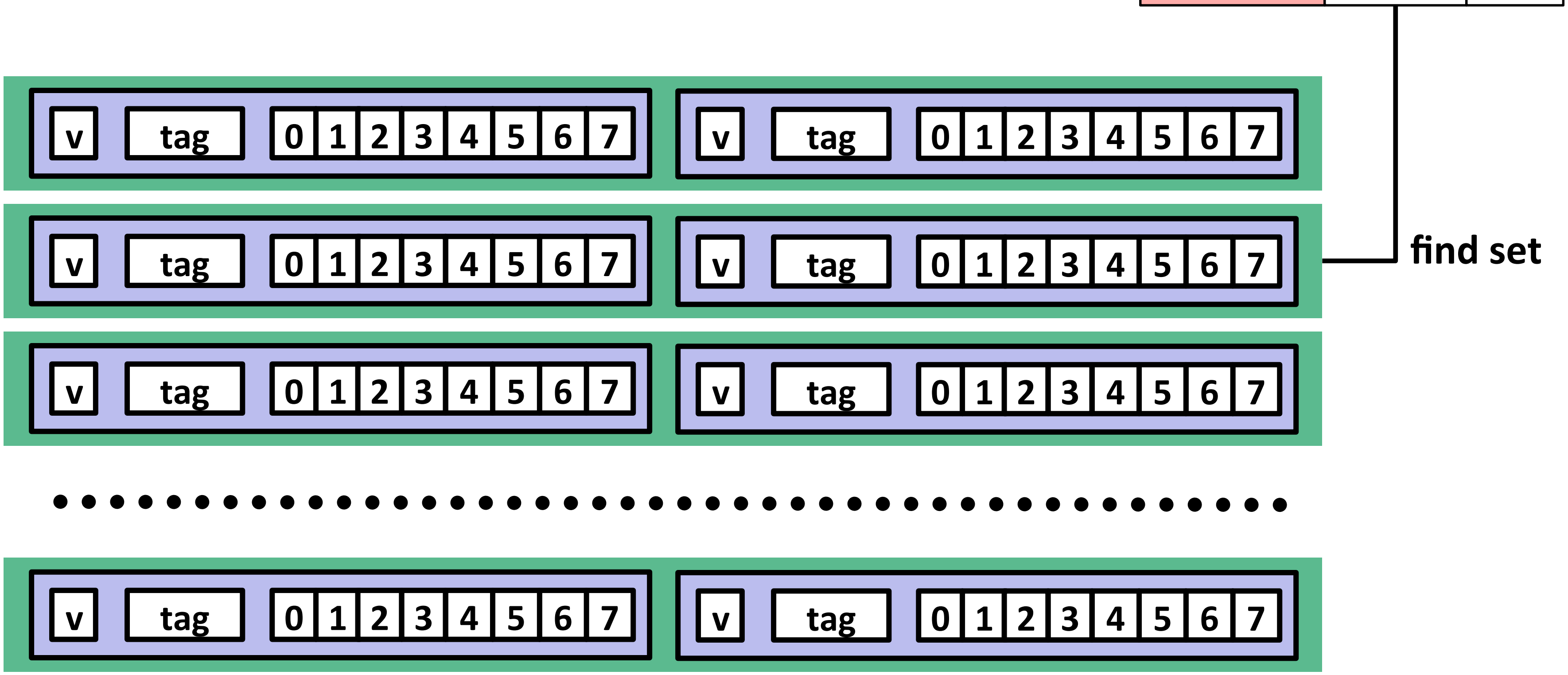
E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

Assume: cache block size 8 bytes

Address of short int:

t bits	0…01	100



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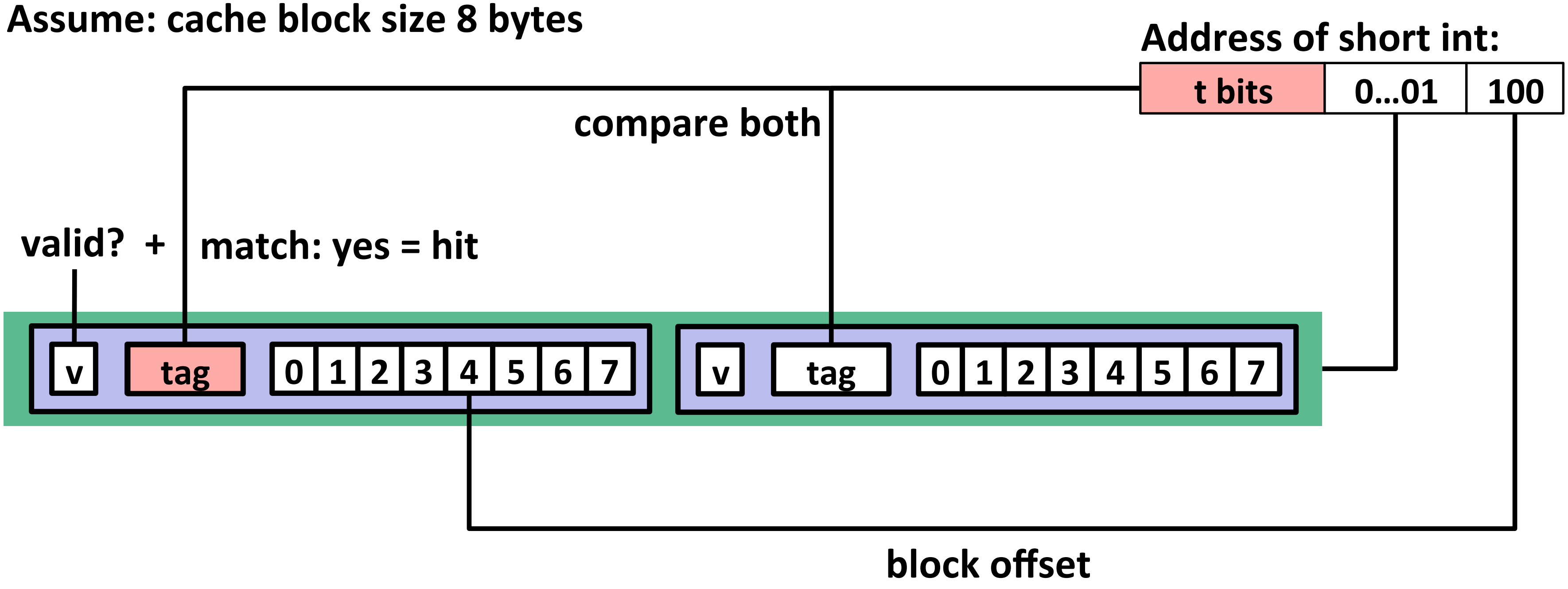

E-way Set Associative Cache (Here: E = 2)

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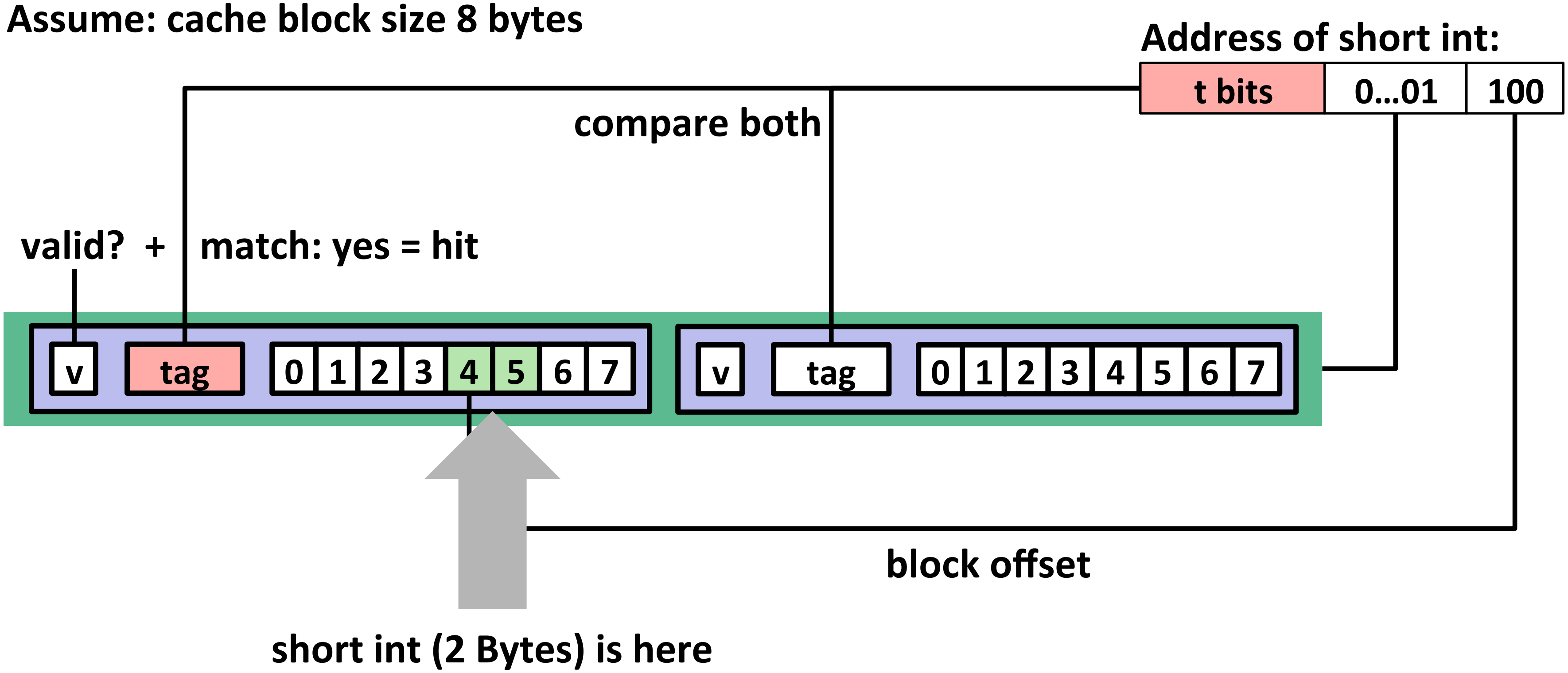
Assume: cache block size 8 bytes



No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), …

2-Way Set Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 blocks/set

Address trace (reads, one byte per read):

0	**$[00\underline{0}0_2]$,**	miss (cold)
1	**$[00\underline{0}1_2]$,**	hit
7	**$[01\underline{1}1_2]$,**	miss (cold)
8	**$[10\underline{0}0_2]$,**	miss (cold)
0	**$[00\underline{0}0_2]$**	hit

	v	**Tag**	**Block**
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	0		

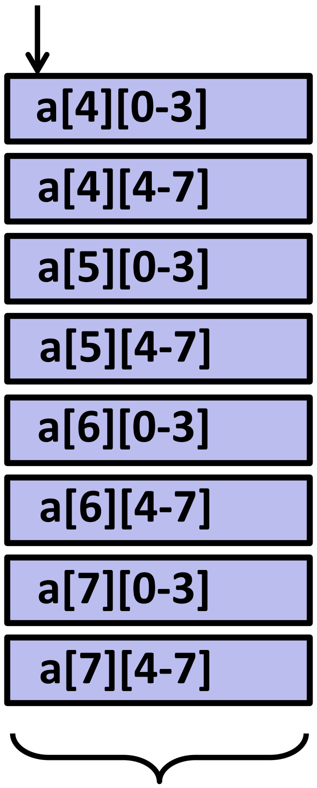
Ignore the variables sum, i, j

A Higher Level Example

```
int sum_array_rows(double a[8][8]) {
    int i, j;
    double sum = 0;

    for (i = 0; i < 8; i++)
        for (j = 0; j < 8; j++)
            sum += a[i][j];
    return sum;
}
```

assume: cold (empty) cache,
a[0][0] goes here



a[4][0-3]
a[4][4-7]
a[5][0-3]
a[5][4-7]
a[6][0-3]
a[6][4-7]
a[7][0-3]
a[7][4-7]

32 B = 4 doubles

Assume M = 2^{32} (32-bit addresses)

- **Capacity of this cache: 256 bytes**
- **Size of this array: 512 bytes**
- **Number of bits used for block offset: 5 bits**
- **Number of bits used for indexing sets: 3 bits**
- **Number of bits for tags: 24 bits**
- **Number of reads performed: 64**
- **Number of misses incurred: 16**

blackboard

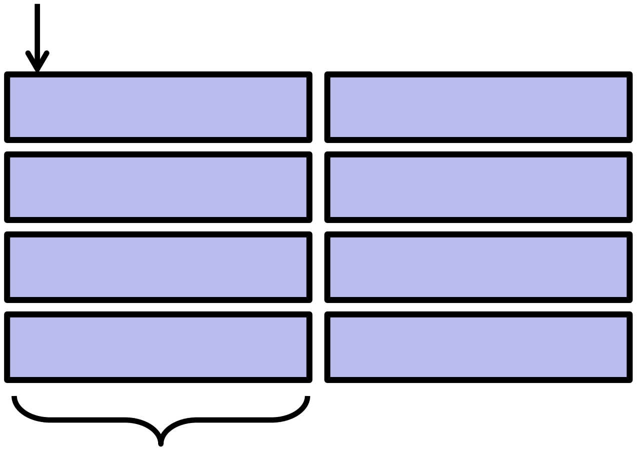
Ignore the variables sum, i, j

A Higher Level Example

```
int sum_array_rows(double a[8][8]) {
    int i, j;
    double sum = 0;

    for (j = 0; j < 8; j++)
        for (i = 0; i < 8; i++)
            sum += a[i][j];
    return sum;
}
```

assume: cold (empty) cache,
a[0][0] goes here



32 B = 4 doubles

Assume M = 2^{32} (32-bit addresses)

- Capacity of this cache:
- Size of this array:
- Number of bits used for block offset:
- Number of bits used for indexing sets:
- Number of bits for tags:
- Number of reads performed:
- Number of misses incurred:

blackboard

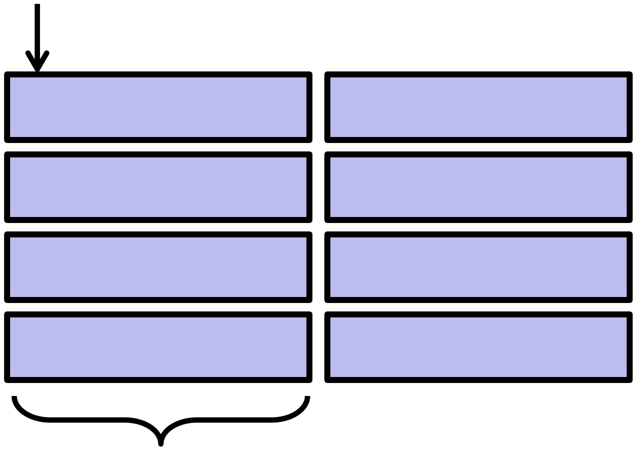
Ignore the variables sum, i, j

A Higher Level Example

```
int sum_array_rows(double a[8][12]) {
    int i, j;
    double sum = 0;

    for (j = 0; j < 12; j++)
        for (i = 0; i < 8; i++)
            sum += a[i][j];
    return sum;
}
```

assume: cold (empty) cache,
a[0][0] goes here



32 B = 4 doubles

Assume M = 2^{32} (32-bit addresses)

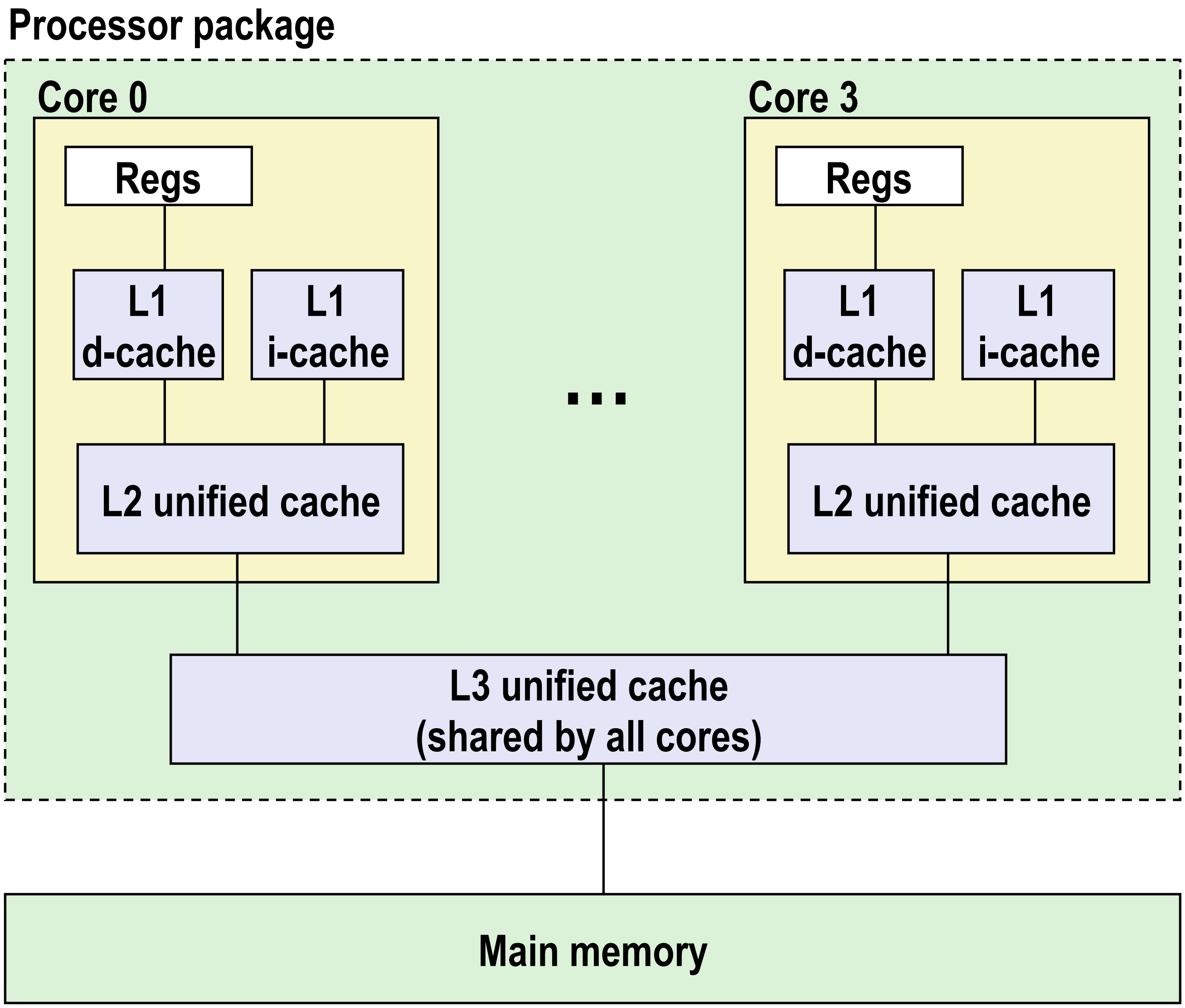
- Capacity of this cache: 256 bytes
- Size of this array: 768 bytes
- Number of bits used for block offset: 5 bits
- Number of bits used for indexing sets: 2 bits
- Number of bits for tags: 25 bits
- Number of reads performed: 96
- Number of misses incurred:

blackboard

What about writes?

- **Multiple copies of data exist:**
  - L1, L2, L3, Main Memory, Disk
- **What to do on a write-hit?**
  - Write-through (write immediately to memory)
  - Write-back (defer write to memory until replacement of line)
    - Need a dirty bit (line different from memory or not)
- **What to do on a write-miss?**
  - Write-allocate (load into cache, update line in cache)
    - Good if more writes to the location follow
  - No-write-allocate (writes straight to memory, does not load into cache)
- **Typical**
  - Write-through + No-write-allocate
  - **Write-back + Write-allocate**

Intel Core i7 Cache Hierarchy



L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 10 cycles

L3 unified cache:
8 MB, 16-way,
Access: 40-75 cycles

Block size: 64 bytes for all caches.

Cache Performance Metrics

- **Miss Rate**
  - Fraction of memory references not found in cache (misses / accesses) = 1 – hit rate
  - Typical numbers (in percentages):
    - 3-10% for L1
    - can be quite small (e.g., < 1%) for L2, depending on size, etc.
- **Hit Time**
  - Time to deliver a line in the cache to the processor
    - includes time to determine whether the line is in the cache
  - Typical numbers:
    - 4 clock cycle for L1
    - 10 clock cycles for L2
- **Miss Penalty**
  - Additional time required because of a miss
    - typically 50-200 cycles for main memory (Trend: increasing!)

Let’s think about those numbers

- **Huge difference between a hit and a miss**
  - Could be 100x, if just L1 and main memory

- **Would you believe 99% hits is twice as good as 97%?**
  - Consider:
    cache hit time of 1 cycle
    miss penalty of 100 cycles
  - Average access time:

    97% hits: 1 cycle + 0.03 * 100 cycles = **4 cycles**

    99% hits: 1 cycle + 0.01 * 100 cycles = **2 cycles**

- **This is why “miss rate” is used instead of “hit rate”**

Writing Cache Friendly Code

- **Make the common case go fast**
  - Focus on the inner loops of the core functions

- **Minimize the misses in the inner loops**
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories.

Today

- Cache organization and operation
- **Performance impact of caches**
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

The Memory Mountain

- **Read throughput (read bandwidth)**
  - Number of bytes read from memory per second (MB/s)

- **Memory mountain: Measured read throughput as a function of spatial and temporal locality.**
  - Compact way to characterize memory system performance.

Memory Mountain Test Function

```c
long data[MAXELEMS];  /* Global array to traverse */

/* test - Iterate over first "elems" elements of
 *        array “data” with stride of "stride", using
 *        using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}
```

mountain/mountain.c

Call test() with many combinations of elems and stride.

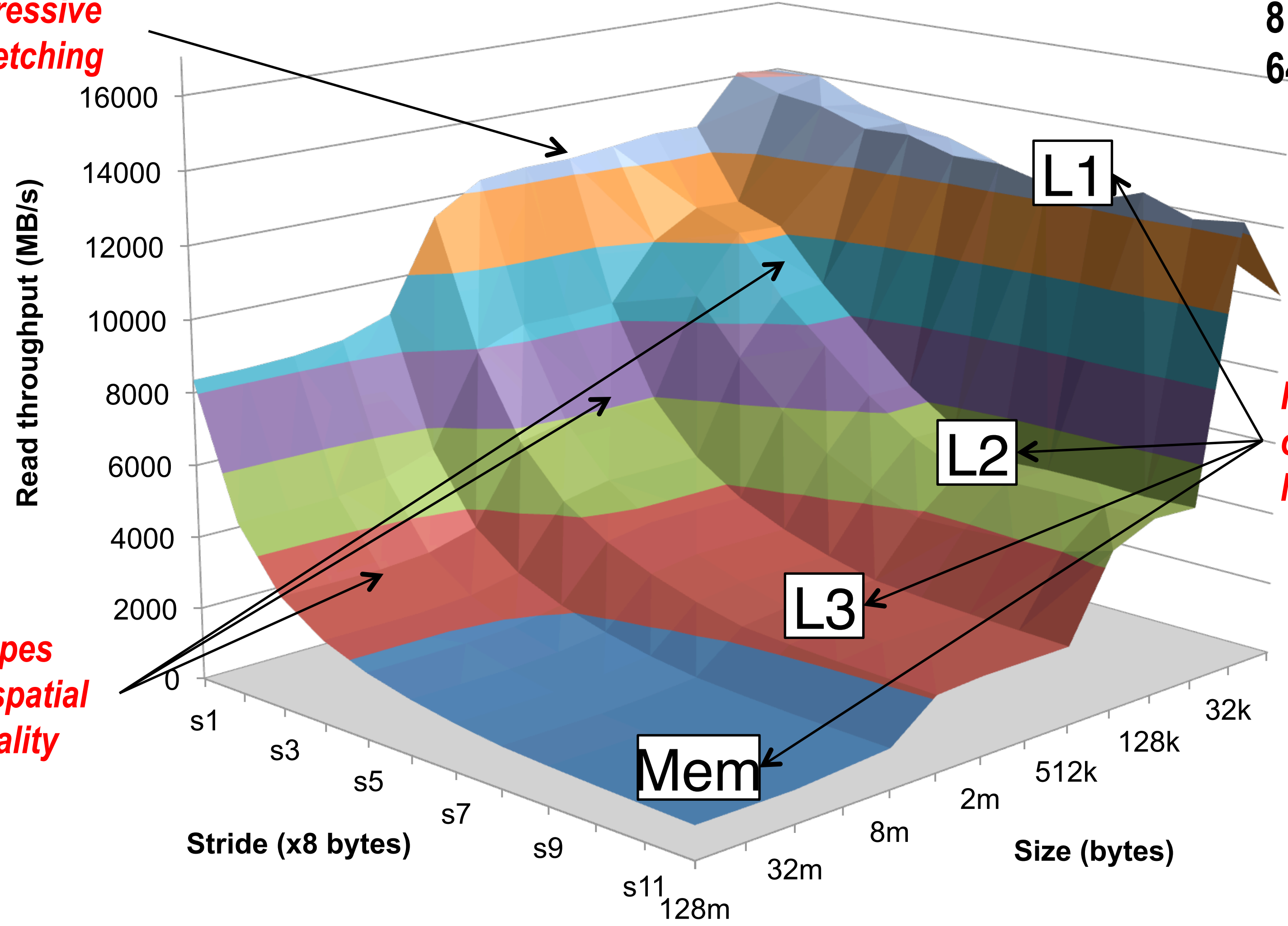
For each elems and stride:

1. Call test() once to warm up the caches.

2. Call test() again and measure the read throughput(MB/s)

The Memory Mountain

Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

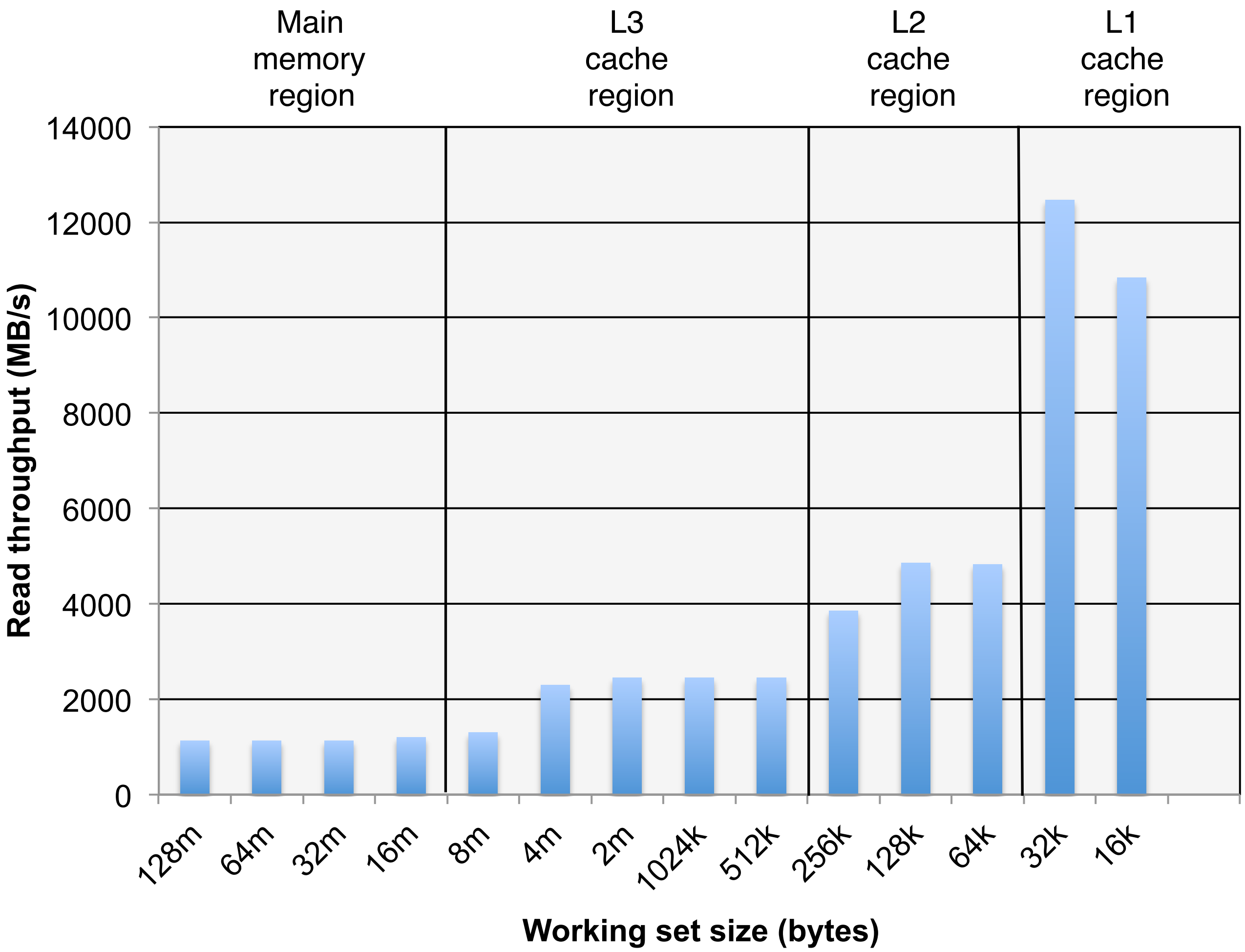


Aggressive prefetching

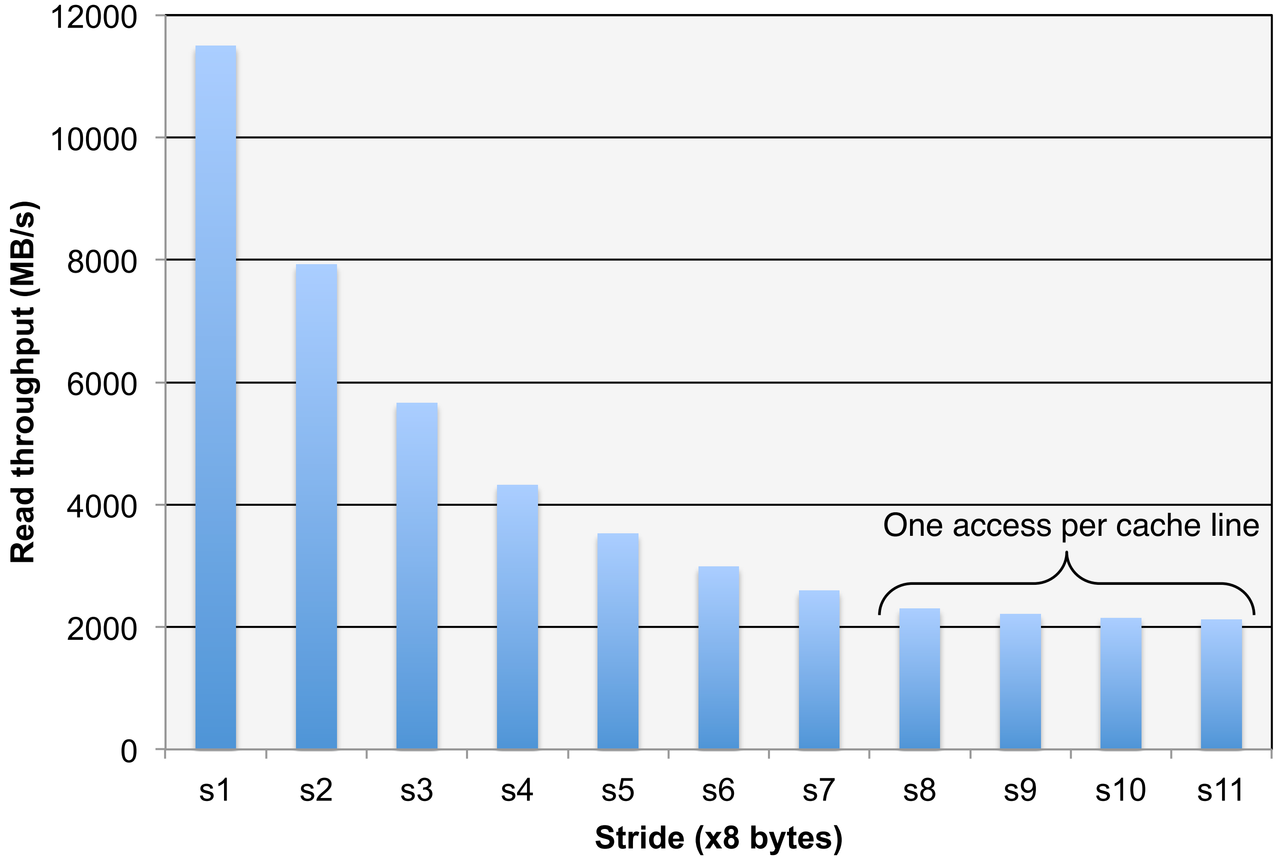
Ridges of temporal locality

Slopes of spatial locality

Ridges of Temporal Locality (stride=8)



A Slope of Spatial Locality



Today

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Matrix Multiplication Example

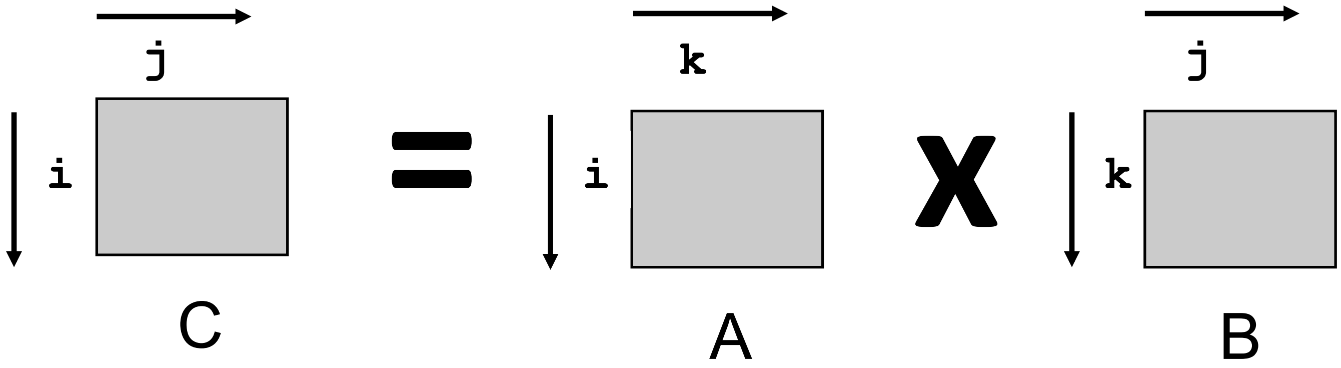
- **Description:**
  - Multiply N x N matrices
  - Matrix elements are doubles (8 bytes)
  - O(N^3) total operations
  - N reads per source element
  - N values summed per destination
    - but may be able to hold in register

Variable **sum** held in register

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}                        matmult/mm.c
```

Miss Rate Analysis for Matrix Multiply

- **Assume:**
  - Block size = 32B (big enough for four doubles)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - **Cache is not even big enough to hold multiple rows**
- **Analysis Method:**
  - Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

- **C arrays allocated in row-major order**
  - each row in contiguous memory locations
- **Stepping through columns in one row:**

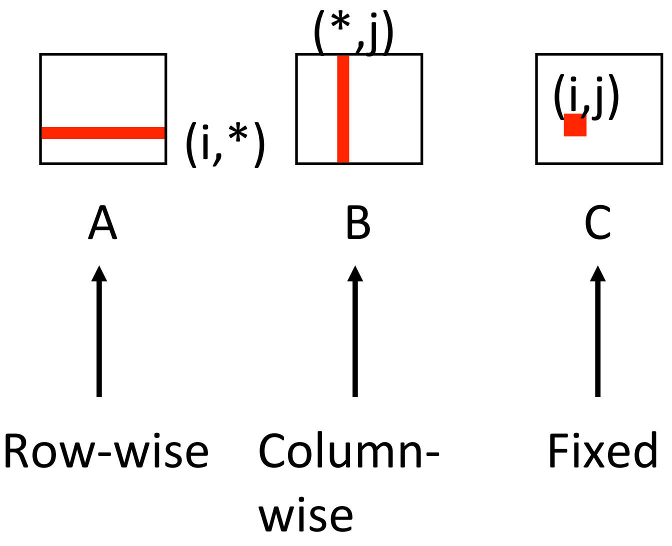
  -
    ```
    for (i = 0; i < N; i++)
      sum += a[0][i];
    ```
  - accesses successive elements
  - if block size (B) > sizeof(a_{ij}) bytes, exploit spatial locality
    - miss rate = sizeof(a_{ij}) / B
- **Stepping through rows in one column:**

  -
    ```
    for (i = 0; i < n; i++)
      sum += a[i][0];
    ```
  - accesses distant elements
  - no spatial locality!
    - miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}                          matmult/mm.c
```

Inner loop:



(i,*)

(*,j)

(i,j)

A

B

C

Row-wise

Column-wise

Fixed

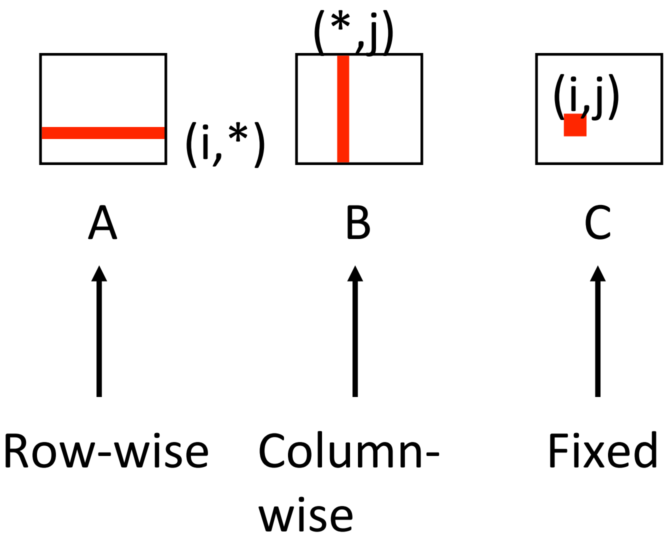
Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++)  {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}                     matmult/mm.c
```

Inner loop:



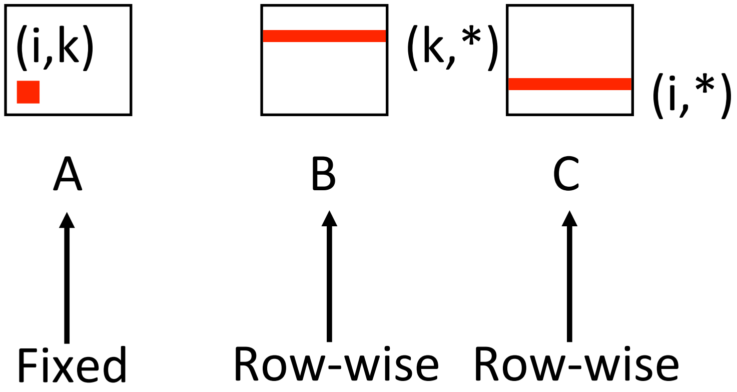
Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* kij */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}                                 matmult/mm.c
```

Inner loop:



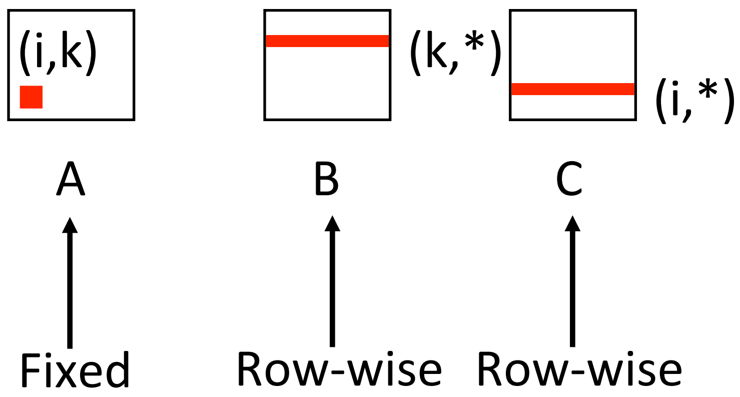
Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}                              matmult/mm.c
```

Inner loop:



(i,k)

(k,*)

(i,*)

A

B

C

Fixed

Row-wise

Row-wise

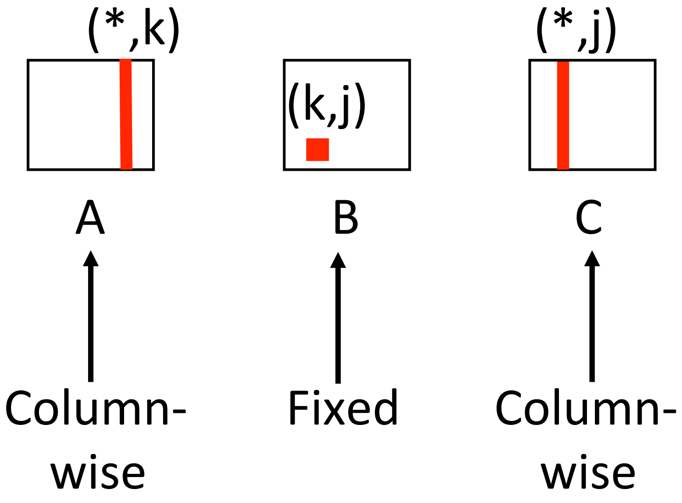
Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}                                  matmult/mm.c
```

Inner loop:



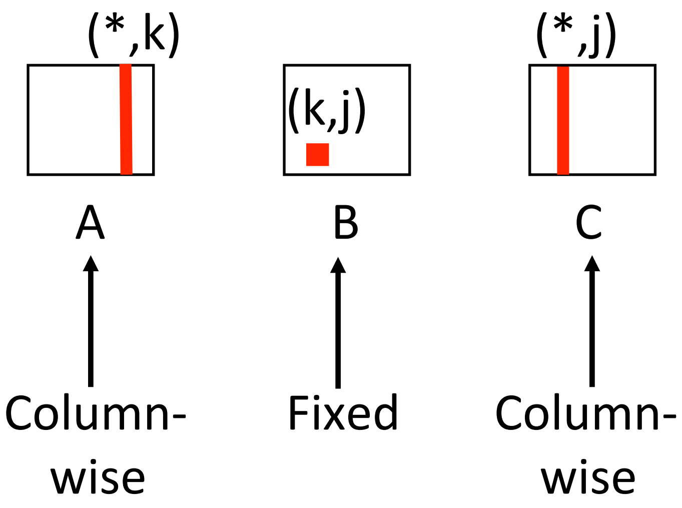
Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
                           matmult/mm.c
```

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = **1.25**

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):

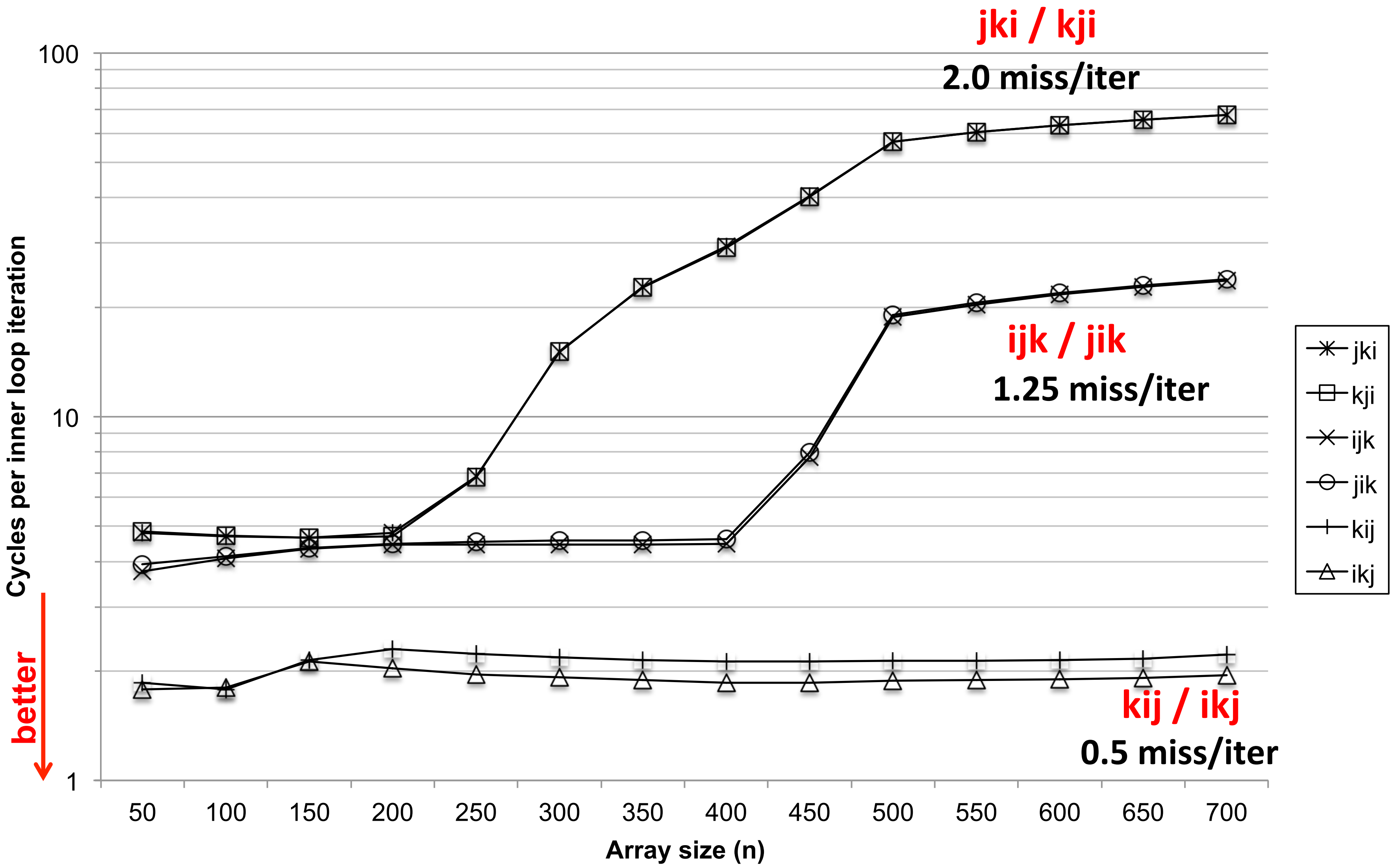
- 2 loads, 1 store
- misses/iter = **0.5**

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):

- 2 loads, 1 store
- misses/iter = **2.0**

Core i7 Matrix Multiply Performance


jki / kji
2.0 miss/iter
ijk / jik
1.25 miss/iter
kij / ikj
0.5 miss/iter
Cycles per inner loop iteration
better
Array size (n)
jki
kji
ijk
jik
kij
ikj