AEE 427: Aircraft Performance and Dynamics
General semester exam equation sheet

Equations

$p = \rho RT$	$a = \sqrt{\gamma RT}$	$M \equiv \frac{V}{a}$
$C_L \equiv \frac{L}{qS}$	$C_D \equiv \frac{D}{qS}$	$C_M \equiv \frac{M}{qSc}$
$Re \equiv \frac{\rho V c}{\mu}$	$AR \equiv b^2/S$	$q \equiv \frac{1}{2}\rho V^2$
$C_D = C_{D,0} + \frac{C_L^2}{\pi e AR}$	$T_R = \frac{W}{L/D} = \frac{W}{C_L/C_D}$	$R/C = \frac{P_A - P_R}{W}$
$C_{L_\alpha} = a = \frac{a_0}{1+\frac{57.3a_0}{\pi ARe_1}} = \frac{c_{l_\alpha}}{1+\frac{57.3c_{l_\alpha}}{\pi ARe_1}}$	$C_{L_\alpha} = a = \frac{\Delta C_L}{\Delta \alpha}$	$C_d = c_d + \frac{C_L^2}{\pi e AR}$
$P_R = T_R V$	$P_A = T_A V$	$C_D = C_{D,0} + \frac{C_L^2}{\pi e AR}$
$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$	$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$	$a = \frac{T_2 - T_1}{h2 - h_1}$
$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$	$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{-g}{aR}}$	$C_p T_1 + \frac{1}{2}V_1^2 = C_p T_2 + \frac{1}{2}V_2^2$
$V_{LO} = 1.2 V_{stall}$	$V_T = 1.3 V_{stall}$	$\frac{p_2}{p_1} = e^{\frac{-g_0}{RT}(h_2 - h_1)}$
$\frac{T_A}{T_{A,sl}} = \frac{\rho}{\rho_{sl}}$	$\frac{P_R}{P_{R,sl}} = \left(\frac{\rho_{sl}}{\rho}\right)^{1/2}$	

$V^2 = \frac{2\gamma p}{\rho(\gamma-1)}\left[\left(\frac{\Delta p}{p}+1\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$	$V_c = V_i + \Delta V_p$	$V = \sqrt{\frac{2\Delta p}{\rho_{sl}}}$

$R = \frac{V_\infty^2}{g\sqrt{n^2-1}}$	$R = \frac{V_\infty^2}{g(n-1)}$	$R = \frac{V_\infty^2}{g(n+1)}$
$\omega = \frac{g\sqrt{n^2-1}}{V_\infty}$	$\omega = \frac{g(n-1)}{V_\infty}$	$\omega = \frac{g(n+1)}{V_\infty}$
	$\tan\theta = \frac{1}{L/D}$	

Quadratic formula: $\lambda_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$\left(\frac{C_L^{1/2}}{C_D}\right)_{max} = \frac{(\frac{1}{3}C_{D,0}\pi eAR)^{1/4}}{\frac{4}{3}C_{D,0}}$	$\left(\frac{C_L^{3/2}}{C_D}\right)_{max} = \frac{(3C_{D,0}\pi eAR)^{3/4}}{4C_{D,0}}$
$\left(\frac{C_L}{C_D}\right)_{max} = \frac{(C_{D,0}\pi eAR)^{1/2}}{2C_{D,0}}$	$D_{TO} = \frac{1}{2}\rho_\infty V^2 S\left(C_{D,0} + \phi\frac{C_L^2}{\pi eAR}\right)$
$R = \frac{\eta}{c}\frac{C_L}{C_D}\ln\frac{W_0}{W_1}$	$E = \frac{\eta}{c}\frac{C_L^{3/2}}{C_D}(2\rho_\infty S)^{1/2}(W_1^{-1/2} - W_0^{-1/2})$
$R = 2\sqrt{\frac{2}{\rho_\infty S}}\frac{1}{c_t}\frac{C_L^{1/2}}{C_D}(W_0^{1/2} - W_1^{1/2})$	$E = \frac{1}{c_t}\frac{C_L}{C_D}\ln\frac{W_0}{W_1}$
$s_{LO} = \frac{1.44W^2}{g\rho_\infty SC_{L,max}\{T-[D+\mu_r(W-L)]_{0.7V_{LO}}\}}$	$s_L = \frac{1.69W^2}{g\rho_\infty SC_{L,max}[D+\mu_r(W-L)]_{0.7V_T}}$

$$V_{max} = \left[\frac{\left(\frac{T_A}{W}\right)_{max}\left(\frac{W}{S}\right) + \left(\frac{W}{S}\right)\sqrt{\left(\frac{T_A}{W}\right)_{max}^2 - \frac{2C_{D,0}}{\pi eAR}}}{\rho_\infty C_{D,0}}\right]^{1/2}$$

$X - mg\sin\theta = m(\dot{u} + qw - rv)$	$L = I_x\dot{p} - I_{xz}\dot{r} + qr(I_z - I_y) - I_{xz}pq$
$Y + mg\cos\theta\sin\phi = m(\dot{v} + ru - pw)$	$M = I_y\dot{q} + rp(I_x - I_z) + I_{xz}(p^2 - r^2)$
$Z + mg\cos\theta\cos\phi = m(\dot{w} + pv - qu)$	$N = -I_{xz}\dot{p} + I_z\dot{r} + pq(I_y - I_x) + I_{xz}qr$

$p = \dot{\phi} - \dot{\psi}\sin\theta$	$\dot{\theta} = q\cos\phi - r\sin\phi$
$q = \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi$	$\dot{\phi} = p + q\sin\phi\tan\theta + r\cos\phi\tan\theta$
$r = \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi$	$\dot{\psi} = (q\sin\phi + r\cos\phi)\sec\theta$

$$C_{m_{cg}} = C_{m_0} + C_{m_\alpha}\alpha + C_{m_{\delta_e}}\delta_e$$

$$C_{m_0} = C_{m_{0_w}} + C_{m_{0_f}} + \eta V_H C_{L_{\alpha_t}}(\epsilon_0 + i_w - i_t))$$

$$C_{m_\alpha} = C_{L_{\alpha_w}}\left(\frac{x_{cg}}{\bar{c}} - \frac{x_{ac}}{\bar{c}}\right) + C_{m_{\alpha_f}} - \eta V_H C_{L_{\alpha_t}}(1 - \frac{d\epsilon}{d\alpha})$$

$C_{m_{\delta_e}} = -\eta V_H C_{L_{\alpha_t}} \tau$	$C_{m_{\dot{\alpha}}} = -2\eta C_{L_{\alpha_t}} V_H \frac{l_t}{\bar{c}} \frac{d\epsilon}{d\alpha}$	$C_{n_r} = -2\eta_v V_v \frac{l_v}{b} C_{L_{\alpha v}}$
$C_{n_{\delta_r}} = -\eta V_v C_{L_{\alpha v}} \tau$	$C_{l_{\delta_a}} = \frac{2C_{L_{\alpha w}} \tau}{Sb} \int_{y_1}^{y_2} cy dy$	$C_{y_r} = -2\frac{l_v}{b}(C_{y_\beta})_{tail}$
$C_{y_\beta} = -\eta \frac{S_v}{S} C_{L_{\alpha v}} (1 + \frac{d\sigma}{d\beta})$	$C_{m_q} = -2\eta C_{L_{\alpha_t}} V_H \frac{l_t}{\bar{c}}$	$C_{x_u} = -[C_{D_u} + 2C_{D,0}] + C_{T_u}$

$\dot{x} + \frac{1}{\tau}x = 0$	$x(t) = C_1 e^{-t/\tau}$
$\ddot{x} + 2\zeta\omega_n \dot{x} + \omega_n^2 x = F(t)$	$\omega = \omega_n \sqrt{1-\zeta^2}$
$t_{half} = \frac{0.693}{\lvert\eta\rvert}$	$N_{half} = 0.110 \frac{\lvert\omega\rvert}{\lvert\eta\rvert}$

Homogeneous solutions:

$\lvert\zeta\rvert > 1$	$x(t) = C_1 e^{-\omega_n\left(\zeta+\sqrt{\zeta^2-1}\right)t} + C_2 e^{-\omega_n\left(\zeta-\sqrt{\zeta^2-1}\right)t}$
$\lvert\zeta\rvert = 1$	$x(t) = (C_1 + C_2 t)e^{-\zeta\omega_n t}$
$\lvert\zeta\rvert < 1$	$x(t) = C_3 e^{-t/\tau}\sin(\omega t) + C_4 e^{-t/\tau}\cos\omega t$

Constants

$R = 287$ J/kg-K $= 1716$ ft-lb/slug-°R
$\gamma = 1.4$
$T_{sl} = 288.16$ K $= 518.69$°R
$p_{sl} = 101325$ N/m^2 $= 2116.2$ lb/ft^2
$\rho_{sl} = 1.2250$ kg/m^3 $= 0.002377$ slug/ft^3

Conversions

550 ft-lb/s = 746 W = 1hp
50°C = 50 +273.15 = 323.15 K
50°F = 50 +459.67 = 509.67°R
1 nautical mile = 1.15 mi = 1.852 km