LECTURE 2

Curvature for curves, atlases

(1) There was a pretty glaring mistake in the reparametrization by arc length. Given $\gamma : \mathbb{R} \to \mathbb{R}^n$, consider the arclength function $l : t \mapsto \int_0^t |\dot{\gamma}| dt$. Since γ is an immersion, this defines a diffeomoprhism from $\mathbb{R}$ to an open subset of $\mathbb{R}$. Let f be the inverse of this map. Then $\gamma \circ f$ is the reparametrization by arc length. Can you catch the mistake in the lecture? It was a cavalier and regrettable misapplication of the chain rule.

(2) The word "chart" is a term still used to refer to maps used by navigators—pilots and sailors, for instance. Of course, a book fall of charts is called an atlas, hence the terminology.

(3) In the homework, to study isometries on $\mathbb{R}^n$, it may be helpful to try and prove that any isometry must preserve straight lines. The solution I had in mind for inner products preservation may lead you to some technology we'll only encounter later in class, but I'm excited to see your solutions (if you have any).