LECTURE 3

Partitions of unity, submersions

(1) In the Lemma, we want f to be strictly positive on the interior of K. That's what the proof actually achieves, too.

(2) Again, don't get bogged down by the point-set topology. Just love that we have the result; we'll be using partitions of unity, and you will too (in your homework).

(3) Somebody asked if every submanifold Z of U is the pre-image of some submersion to Euclidean space. I thought about this a little bit, and certainly a sufficient condition is for the submanifold to have trivial normal bundle in U. (No worries if you don't know what this means.) Then near Z, the different normal directions define a smooth function from U to Euclidean space whose dimension is the codimension of Z, and the preimage of 0 is Z. (Far away from Z itself, you can modify the smooth function however you want to make sure you don't hit 0 again.) The converse is true, too—this is easiest to see once you choose a Riemannian metric, but essentially, small enough neighborhoods of 0 in the target of the submersion define normal directions in a coherent way along all of Z, trivializing the normal bundle. So the normal bundle's triviality is both sufficient and necessary.