Homework 3

Thursday, September 17, 2015 9:30 AM

Due in class Thursday 9/24.

1. a) Consider the linear transformation $f: \mathbb{R}^3 \to \mathbb{R}^3$ given by

(*) $$f(x,y,z) = (3y, -x+y-2z, x-z).$$

Express f as a 3×3 matrix.

b) What is $f^{-1}(x,y,z)$?

2. a) Let $f: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation with

$$f(1,0,0) = (1,2,-1)$$
$$f(0,1,0) = (1,0,2)$$
$$f(0,0,1) = (1,1,3).$$

Give a 3×3 matrix for f. What is $f(2,-1,3)$?

b) Let $g: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation with

$$g(1,2,-1) = (1,0,0)$$
$$g(1,0,2) = (0,1,0)$$
$$g(1,1,3) = (0,0,1)$$

Give a 3×3 matrix for g. What is $g(4,5,6)$?

c) Let $h: \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation with

$$h(1,2,-1) = (1,2,3)$$
$$h(1,0,2) = (4,5,6)$$
$$h(1,1,3) = (7,8,9).$$

Give a 3×3 matrix for h. What is $h(4,5,6)$?

3. By hand (not using a computer), find all polynomials

$$p(x) = a + bx + cx^2 + dx^3$$

that satisfy $p(1) = 2$, $p'(3) = 1$ and $p''(2) = 0$.

4. Which of the following are vector spaces? In each case, explain why/why not.

And if it is a vector space, give the dimension and a basis.

a) the set $\mathbb{R}$ of real numbers

b) the set of solutions (x_1, x_2) to the equations

$$5x_1 + 2x_2 = 0$$
$$3x_1 - 2x_2 = 2$$

c) the set of solutions (x_1, x_2) to the equation

$$x_1 x_2 = 0$$

d) the span of the vectors

$(2,3,4)$, $(-1,-1,-4)$ and $(0,1,-4)$

e) the set of **anti-symmetric** 4×4 matrices
(recall: a matrix A is antisymmetric if $A^T = -A$)

5. Determine which of the following sets of vectors are linearly independent. For those sets that are linearly dependent, write one of the vectors as a linear combination of the others.

a) $\{(1,2,3), (2,1,0), (1,5,9)\}$

b) $\{(1,2,3), (0,4,5), (0,0,6), (1,1,1)\}$

c) $\{(3,2,1), (1,0,0), (2,1,0)\}$

d) $\{(2,2,2,2), (2,2,0,2), (2,0,2,2)\}$

e) $$\left\{ \begin{pmatrix}1\\2\\0\\4\\0\\3\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\4\\1\\3\\0\end{pmatrix}, \begin{pmatrix}0\\2\\1\\4\\0\\3\\0\end{pmatrix}, \begin{pmatrix}0\\2\\0\\4\\0\\3\\1\end{pmatrix} \right\}$$

6. Determine the dimension of the space spanned by the set

$$S = \left\{ \begin{pmatrix}1\\2\\-1\\3\end{pmatrix}, \begin{pmatrix}1\\0\\0\\2\end{pmatrix}, \begin{pmatrix}2\\8\\-4\\8\end{pmatrix}, \begin{pmatrix}1\\1\\1\\1\end{pmatrix}, \begin{pmatrix}3\\3\\0\\6\end{pmatrix} \right\}$$

$$\left\{ \begin{pmatrix} -1 \\ 3 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \end{pmatrix}, \begin{pmatrix} -4 \\ 8 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 6 \end{pmatrix} \right\}$$

7. Determine whether or not the set

$$B = \{(2,3,2),\ (1,1,-1)\}$$

is a basis for the space spanned by the set

$$A = \{(1,2,3),\ (5,8,7),\ (3,4,1)\}.$$

8. For the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 1 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \end{pmatrix}$$

determine A^{300}.

First do the calculation in Matlab. Then do the same calculation by hand, justifying why the answer is correct.

9. a) Give a basis for the set of 2×2 matrices.

b) In terms of the same basis, give the matrix for the operator that maps a matrix A to

$$\begin{pmatrix} 1 & 2 \\ 3 & 6 \end{pmatrix} A .$$

c) Give a **completely different** basis for 2×2 matrices than the one you used in a. No element of the new basis should be proportional to an element of the old basis.

Specify the 4×4 matrix that changes from the old basis from part a to the new basis. Specify the matrix that changes back. Make sure your notation is clear.

d) Using the basis change matrices from part c,

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recompute the answers to part b
As always, show your work.

10 a) Consider the space of polynomials in x of degree at most 5. What is the dimension of this space? Give a simple basis for it.

b) Do the same for polynomials of degree at most 6.

c) Using the above bases, give the matrix that represents multiplication by $2+3x$.

d) Look up on Wikipedia Chebyshev polynomials of the first kind. Repeat parts a, b, c using these polynomials for your bases.