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Fall 2015 CMSC250/250H PRACTICE Midterm I

0301 (8am: Amr)	0302 (9am: Amr)
0303 (1pm: Parsa)	0304 (3pm: 1121, Parsa)
250H (11am: 2117, Peter Sutor)	

Do not open this exam until you are told. Read these instructions:

1. **This is a closed book exam.** No calculators, notes, or other aids are allowed. If you have a question during the exam, please raise your hand.
2. You must turn in your exam **immediately** when time is called at the end.
3. In order to be eligible for as much partial credit as possible, show all of your work for each problem, write legibly, and clearly indicate your answers. **Credit will not be given for illegible answers.**
4. After the last page there is paper for scratch work. If you need extra scratch paper after you have filled these areas up, please raise your hand. Scratch paper must be turned in with your exam, with your name and ID number written on it, but scratch paper will not be graded. You may also use the scratch paper as extra space for answers, but you must cross reference it.
5. You may not give or receive any unauthorized assistance on this examination.
6. **Write your name legibly on the top of each page of this exam.** Credit will not be given for lost/missing pages.

1. Define the following propositions:

   - b= "I must wear a black shirt"
   - r= "I must wear a red shirt"

   ...and the follow predicates:

   - W(x)= "Shirt x is white"
   - L(x)= "Shirt x is in the laundry"

   In the problems that follow, assume the domain of discourse is my shirts.

   Write the following sentences in formal logic using the above propositions and predicates. You may use the connectives $\{\wedge, \vee, \rightarrow, \neg\}$ and the quantifiers $\{\exists, \forall\}$. You may use the equals sign $\{=\}$ to denote that two shirts are the same.

   (a) I have no white shirts in the laundry.

   (b) There is exactly one white shirt in the laundry.

   (c) If all of my white shirts are in the laundry, then I must wear a black shirt or a red shirt (or both).

   (d) I don't have to wear a black shirt only if there is a white shirt that is not in the laundry.

2. Simplify the following expression. Show every step of your derivation, one step per line.

$$(p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (\neg p \vee q) \wedge (r \vee q)$$

3. Simplify the following expression so that no negation appears to the left of a quantifier, and no negation appears outside of a parenthesis containing multiple predicates. In other words, move the negations "as far in" to the expression as possible. Write one step of your simplification per line. You do NOT need to state the law used to obtain each line.

$$\neg(\forall x, \exists y)[p(x) \wedge (\neg q(x) \vee r(x))]$$

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4. (5 points) Using only AND, OR, and NOT gates that take up two inputs, draw a circuit that outputs z from input wires x and y:

$$z = (x \wedge \neg y) \vee (y \wedge \neg x)$$

5. Using disjunction normal form, write a logical expression for s, which is defined by the following truth table. Do not simplify.

p	q	r	s
T	T	T	F
T	T	F	F
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	F
F	F	F	T

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6. (a) Write the base-6 number $(205)_6$ in binary (base-2) form. Show your work.

(b) Write the binary number 100101001011100 in hexadecimal form.

(c) The following number is written using an 8-bit **two's complement** representation. Write the number in base 10. Include a "+" or "−" sign in front of your answer. Show your work.

11110101

7. (10 points) (a) State the definition of "odd" as given in class and in the book.

(b) Suppose that a and b are odd. Prove that $a+3b$ is even. You must use the basic definitions of even and odd (as in part a). You may NOT use the fact that the sum of an even and odd number is odd.

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8. (a) Prove that if

$$a \equiv 3 \mod 8,$$

then a is odd.

(b) What is the additive inverse of $2 \mod 6$?

(c) Prove that 2 has no multiplicative inverse in mod 6 arithmetic.

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9. (15 points) (a) Prove that $\sqrt{3}$ is irrational.

(b) Prove there exists two irrational numbers, a and b, such that $\frac{a+b}{2}$ is rational. You may use your result from part (a).

(c) Give a constructive proof of the following statement: **Between any two irrational numbers,** a **and** b, **there is another irrational number.** You may use the fact that the sum of a rational number and irrational number is irrational.

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