Weak Induction

1. Use Mathematical Induction to prove:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

2. Use Mathematical Induction to prove:

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

3. Use Mathematical Induction to prove:

$$\prod_{i=2}^{n} (1 - \frac{1}{i}) = \frac{1}{n}$$

4. Use Mathematical Induction to show that for $n \geq 1$

$$\sum_{i=1}^{n} \frac{1}{\sqrt{i}} > 2(\sqrt{n+1} - 1)$$

5. Use mathematical Induction to prove that a set with n elements has $n(n-1)/2$ subsets containing exactly two elements.

6. Use mathematical Induction to prove that a set with n elements has $n(n-1)(n-2)/6$ subsets containing exactly three elements.

7. Use mathematical induction to show that for propositions p_i $(1 \leq i \leq n)$

$$\neg(p_1 \vee p_2 \vee \cdots \vee p_n) \equiv \neg p_1 \wedge \neg p_2 \wedge \cdots \wedge \neg p_n \ .$$

Strong Induction

1. (a) Determine which amounts of postage can be formed using just 3-cent and 10-cent stamps.
   (b) Prove your answer using Weak Mathematical Induction.
   (c) Prove your answer using Strong Mathematical Induction.

2. Consider the following recurrence where $a_0 = 1$.

$$a_n \;=\; a_{n-1} + 2a_{n-2} + 4a_{n-3} + 8a_{n-4} + \cdots + 2^{n-1}a_0 \;.$$

   (a) Rewrite the recurrence with the right side as a summation.
   (b) Evaluate a_n for $n = 0, 1, 2, 3, 4, 5$.
   (c) Write a simple formula for a_n.
   (d) Prove that your formula for a_n is correct using Weak Mathematical Induction.
   (e) Prove that your formula is correct using Strong Mathematical Induction (ignoring the base case which has already been proved).

3. Prove that a simple polygon with n sides, where n is an integer with $n \geq 3$, can be triangulated into $n - 2$ triangles. [Use the lemma from the next problem.]

4. Prove that every simple polygon with at least four sides has an interior diagonal.

Constructive Induction

1. Assume that you guess that $\sum_{i=1}^{n} i^2$ is a cubic polynomial. Use Constructive Induction to derive the exact polynomial.

2. Use Constructive Induction to find an upper bound on F_n in the recurrence

$$F_n = F_{n-1} + 3F_{n-2}$$

   where $F_0 = 1$ and $F_1 = 2$.