Section 12.1, #39. Find the vector of length 4 in the direction of $\mathbf{u} = \langle -1, -1 \rangle$.

**Solution.** We first find the vector of length 1 in the direction of $\mathbf{u}$ via computing $\mathbf{u}/\|\mathbf{u}\|$. We have $\|\mathbf{u}\| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$. Therefore $\mathbf{u}/\|\mathbf{u}\| = \langle -1/\sqrt{2}, -1/\sqrt{2} \rangle$. To find the vector of length 4, then, we scalar multiply this result by 4, obtaining $4\mathbf{u}/\|\mathbf{u}\| = \langle -4/\sqrt{2}, -4/\sqrt{2} \rangle$.

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Section 12.2, #36. Find a vector parametrization for the line that passes through $(1, 1, 1)$, parallel to the line through $(2, 0, -1)$ and $(4, 1, 3)$.

**Solution.** The line through $(2, 0, -1)$ and $(4, 1, 3)$ is parallel to the vector $(4, 1, 3) - (2, 0, -1) = (2, 1, 4)$. Therefore the vector parametrization of the line in question is $\mathbf{r}(t) = \langle 1, 1, 1 \rangle + t\langle 2, 1, 4 \rangle$.

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Section 12.3, #48. Find $\|2\mathbf{e} - 3\mathbf{f}\|$ assuming that $\mathbf{e}$ and $\mathbf{f}$ are unit vectors such that $\|\mathbf{e} + \mathbf{f}\| = \sqrt{3/2}$.

**Solution.** We repeatedly make use of the fact that $\|\mathbf{v}\|^2 = \mathbf{v} \cdot \mathbf{v}$. From the given $\|\mathbf{e} + \mathbf{f}\| = \sqrt{3/2}$, we thus obtain $(\mathbf{e}+\mathbf{f}) \cdot (\mathbf{e}+\mathbf{f}) = 3/2$, so that using the distributive property of dot products of vectors, we have $\mathbf{e} \cdot \mathbf{e} + 2\mathbf{e} \cdot \mathbf{f} + \mathbf{f} \cdot \mathbf{f} = 3/2$. We are also given that $\mathbf{e}$ and $\mathbf{f}$ are unit vectors, i.e. $\mathbf{e} \cdot \mathbf{e} = 1$ and $\mathbf{f} \cdot \mathbf{f} = 1$. Therefore $\mathbf{e} \cdot \mathbf{f} = -1/4$ (verify the algebra!). To find $\|2\mathbf{e} - 3\mathbf{f}\|$, we first find $\|2\mathbf{e} - 3\mathbf{f}\|^2 = (2\mathbf{e} - 3\mathbf{f}) \cdot (2\mathbf{e} - 3\mathbf{f}) = 4 - 12\mathbf{e} \cdot \mathbf{f} + 9 = 13 - 12(-1/4) = 13 + 3 = 16$. So $\|2\mathbf{e} - 3\mathbf{f}\| = 4$ (note that $\|.\|$ is always nonnegative).