HOMEWORK 2 SOLUTIONS

Section 11.1 #38

We need to find parametric equations for the curve $y = \cos(x)$ translated so that a maximum occurs at $(3,5)$. Recall that to translate a parametric curve $c(t) = (x(t), y(t))$ horizontally a units and vertically b units, we use $c(t) = (a + x(t), b + y(t))$. Start with parametric equations for cosine: $c(t) = (t, \cos(t))$. It's easy to read off the maximum of cosine which occurs at $(0,1)$. We'd like to move this to the point $(3,5)$, so we translate the whole curve 3 units to the right and 4 units up. The resulting parametric curve is:

$$x(t) = 3 + t \text{ and } y(t) = 4 + \cos(t).$$

Section 12.4 #24

To find $v \times w$, it will suffice to find its magnitude and direction. Recall that $\|v \times w\| = \|v\|\|w\| \sin(\theta)$. We're told v and w have length 3 and that the angle between them is $\pi/6$, so $\|v \times w\| = 3 \cdot 3 \cdot \frac{1}{2} = 9/2$. Now for the direction. We know that $v \times w$ must be orthogonal to both v and w. Since v and w each lie in the xz-plane, $v \times w$ must lie along the y-axis. The question is whether it is in the positive or negative direction of the y-axis. This is answered by the right-hand rule, which tells us the cross product is in the direction of the negative y-axis. The unit vector in this direction is $\langle 0, -1, 0 \rangle$, so we have

$$v \times w = (9/2)\langle 0, -1, 0 \rangle = \langle 0, -9/2, 0 \rangle.$$

Section 12.5 #26

We happily observe that the origin lies on both lines, so the vectors $\langle 1,2,3 \rangle$ and $\langle 3,1,8 \rangle$ (which are the direction vectors of the two lines) both lie in the plane. Thus, we can find the normal vector to the plane by computing their cross product.

$$\begin{aligned} \langle 1,2,3 \rangle \times \langle 3,1,8 \rangle &= \langle 2 \cdot 8 - 3 \cdot 1, -(1 \cdot 8 - 3 \cdot 3), 1 \cdot 1 - 2 \cdot 3 \rangle \\ &= \langle 13, 1, -5 \rangle. \end{aligned}$$

Since the origin is a point in the plane, the equation of the plane is

$$\langle 13, 1, -5 \rangle \cdot \langle x, y, z \rangle = 0.$$

Section 13.1 #36

We would like to parameterize the ellipse $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$ in the xy-plane, translated to have center $(9, -4, 0)$. We'll find a parameterization when the ellipse is centered at the origin, then move it to its proper spot. Let $x = 2\cos(t)$ and $y = 3\sin(t)$. Since the ellipse must lie in the xy-plane, we must have $z = 0$. Thus, $r(t) = \langle 2\cos(t), 3\sin(t), 0 \rangle$ parameterizes the ellipse in the xy-plane centered at the origin. Translating the center gives

$$r(t) = \langle 9 + 2\cos(t), -4 + 3\sin(t), 0 \rangle.$$