Section 14.5, #43.

**Solution.** Let $f(x, y, z) = xz + 2x^2y + y^2z^3$. Then our surface is given by $f(x, y, z) = 11$. $\nabla f = \langle z + 4xy, 2x^2 + 2yz^3, x + 3y^2z^2\rangle$, so $\nabla f(2, 1, 1) = \langle 9, 10, 5\rangle$. This is our normal vector to the equation of the tangent plane at $P$ (to the surface). Therefore the equation is $9(x - 2) + 10(y - 1) + 5(z - 1) = 0$.

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Section 14.5, #50.

**Solution.** We need to find a function $f(x, y, z)$ such that $f_x = x$, $f_y = y^2$, and $f_z = z^3$. Integrating the equation $f_x = x$ with respect to $x$, $f(x, y, z) = \frac{1}{2}x^2 + h(y, z)$ for some function $h$ that depends only on $y, z$. $h_y = y^2$, so we can take $h = \frac{1}{3}y^3 + g(z)$, and finally $g_z = z^3$ so we can take $g = \frac{1}{4}z^4$. So $f(x, y, z) = \frac{1}{2}x^2 + \frac{1}{3}y^3 + \frac{1}{4}z^4$ is one function that does the job.

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Section 14.6, #27.

**Solution.** Differentiating the equation $e^{xy} + \sin(xy) + y = 0$ with respect to $y$, we have $e^{xy} \cdot x + \cos(xz) \cdot \frac{\partial(xz)}{\partial y} + 1 = 0$. Now by product rule, $\frac{\partial(xz)}{\partial y} = \frac{\partial x}{\partial y} \cdot z + \frac{\partial z}{\partial y} \cdot x = \frac{\partial z}{\partial y} \cdot x$ because $x$ does not depend on $y$ (our assumption). So $\frac{\partial z}{\partial y} = \frac{-1 - e^{xy} \cdot x}{x \cos(xz)}$.