Rice's theorem

Rice's Theorem

Let L be a language over Turing machine descriptions. Suppose L satisfies the following properties:

1. (Nontrivial) There are TMs M_{YES} and M_{NO}, where $M_{YES} \in L$ and $M_{NO} \notin L$
2. (Semantic) For all TMs M_1 and M_2 such that $L(M_1) = L(M_2)$, $M_1 \in L$ if and only if $M_2 \in L$

Then L is undecidable.

A Huge Hammer for Undecidability

Examples and Non-Examples

Semantic Properties *P(M)*

- M accepts 0
- for all w, M(w) accepts iff M(w^R) accepts
- L(M) = {0}
- L(M) is empty
- L(M) is regular
- M accepts exactly 154 strings

Not Semantic!

- M halts and rejects 0
- M tries to move its head off the left end of the tape, on input 0
- M never moves its head left on input 0
- M has exactly 154 states
- M halts on all inputs

L = {M | *P*(M) is true} is undecidable