Sequential Data Structures, Part 1

Lists in Racket

Please take a handout &
Sit in row H or forward

But first, let's review **recursion**

Suppose we want a Racket function `halve-count`.

Input: a strictly positive integer n.

Output: The number of times we must divide n by 2 until we reach 1. (i.e., an integer approximation to $\log_2$ n.)

base case

$$\texttt{halve-count(n)} = \begin{cases} 0 & \text{if } \mathtt{n} = 1 \\ 1 + \texttt{halve-count}(\lfloor \frac{\mathtt{n}}{2} \rfloor) & \text{otherwise} \end{cases}$$

recursive step

But first, let's review **analysis**

Let's analyze this function, mathematically!

1. What's our *cost metric*, i.e., what are we counting?
2. What mathematical tool should we use?

```
(define (halve-count n)
  (if (= n 1)
      0
      (+ 1 (halve-count (quotient n 2)))))
```

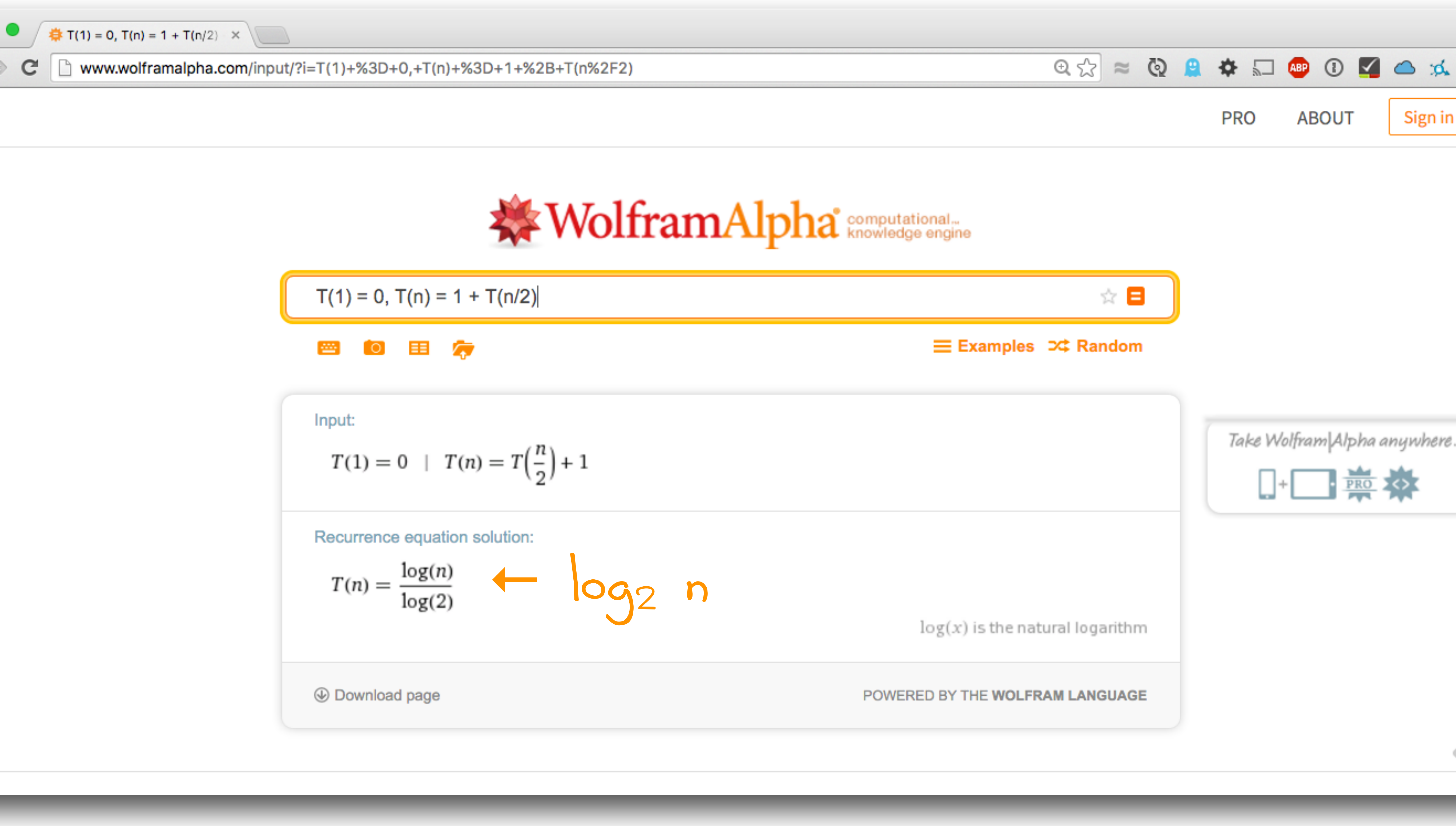
base case

recursive step

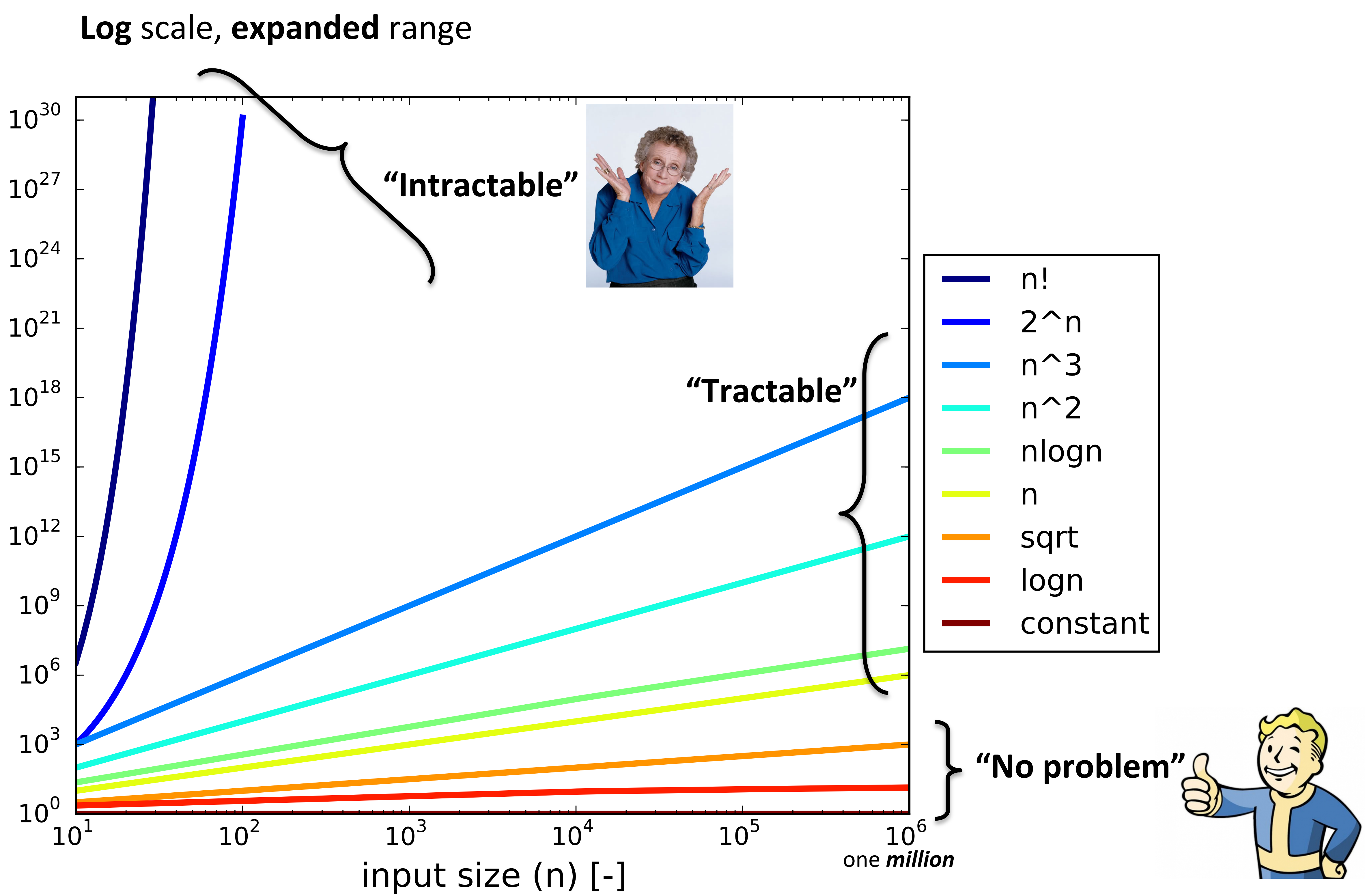
`T(1) = 0` *adds*
`T(n) = 1 + T(n/2)` *adds*

Ask for help solving, if needed

In CS 60, we only need to know how to set up recurrence relations, not solve them.



Big O helps us talk about problem classes



How do we learn a new data structure?

What kinds of things do we want to know?

Order matters: Sequence of genetic mutations determines how cancer behaves

Date: February 11, 2015

Source: University of Cambridge

Summary: The order in which genetic mutations are acquired determines how an individual cancer behaves, according to new research.

WEDNESDA

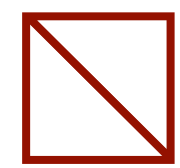
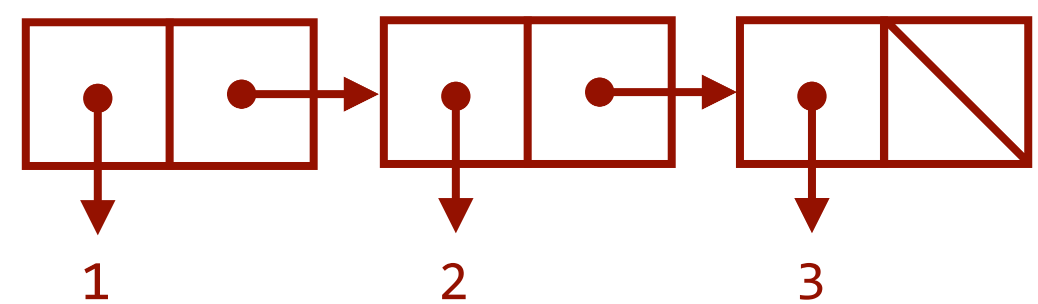
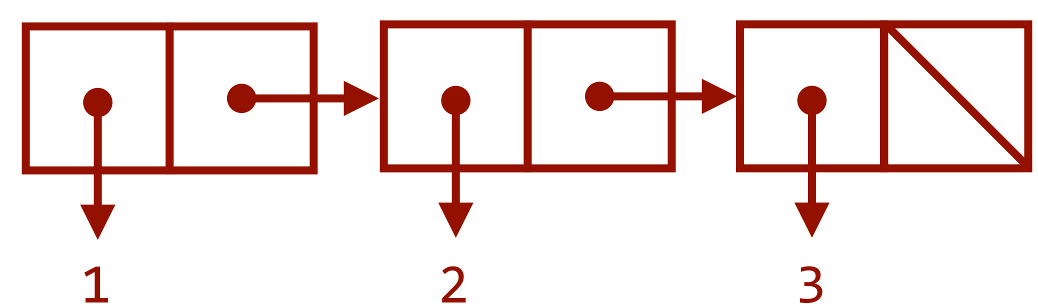
	LUNCH
Soup	Creamy Chicken Soup
	Roasted Garden Vegetable Soup V
	Curried Carrot Soup V
	Breadsticks V
Creations	Made to Order Salad
Exhibition	Beef Pho Noodle Bowl
Oven	The Hawaiian
	Housemade Pepperoni Pizza
	Cheese Pizza V
	Wild Mushroom & Pesto Flatbread Pizza
	Hot Italian Sausage Pizza
Grill	Burger Bar

A Step-By-Step Guide To Making A BuzzFeed Post

So you've joined the BuzzFeed Community and you want to make your own post, but don't know where to begin. Don't worry, we've got you covered!

posted on Oct. 14, 2014, at 8:37 a.m.

Creating lists in Racket

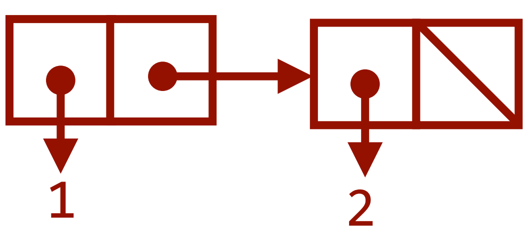
syntax what we *write*	**printed representation** what Racket *prints*	**semantics** what it *means*
empty *make an empty list*	> empty '()	
(list <value1> ... <valueN>) *make a list with N values*	> (list 1 2 3) '(1 2 3)	
(cons <value> <list>) *add an element to the front of a list*	> (cons 1 (list 2 3)) '(1 2 3)	

Creating lists: let's practice

write down the answers as either a drawing or a Racket expression

1. `(list 3 2 1)` ← *draw the picture*

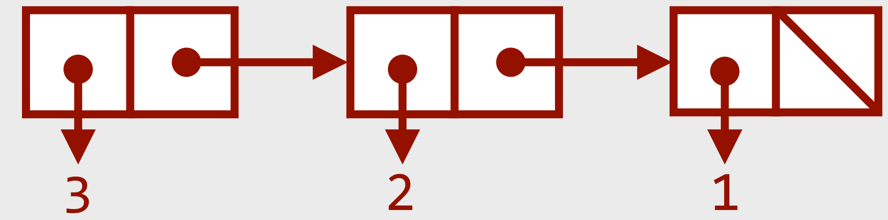
2. `(cons 3 (list 2 1))` ← *draw the picture*

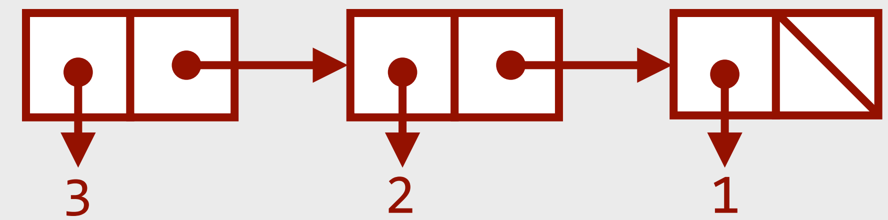
3. 

← *write the expression*

4. `'(1)` ← *write the expression that makes Racket display this*

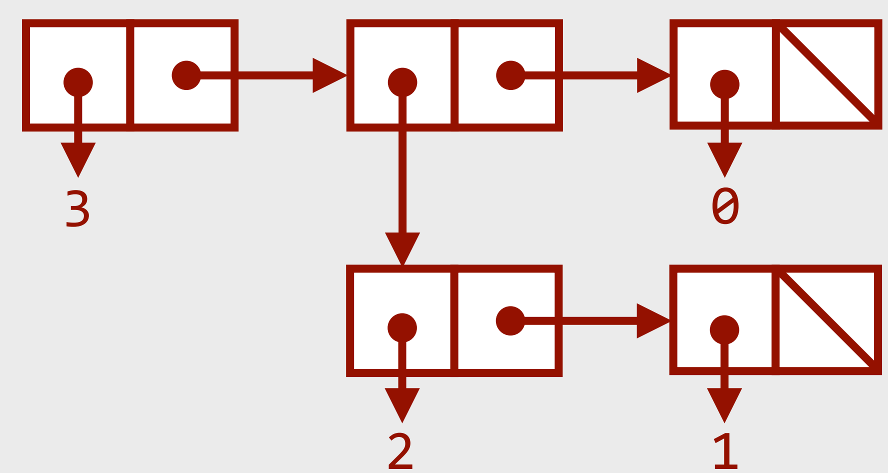
5. `(list 3 (list 2 1) 0)` ← *draw the picture*





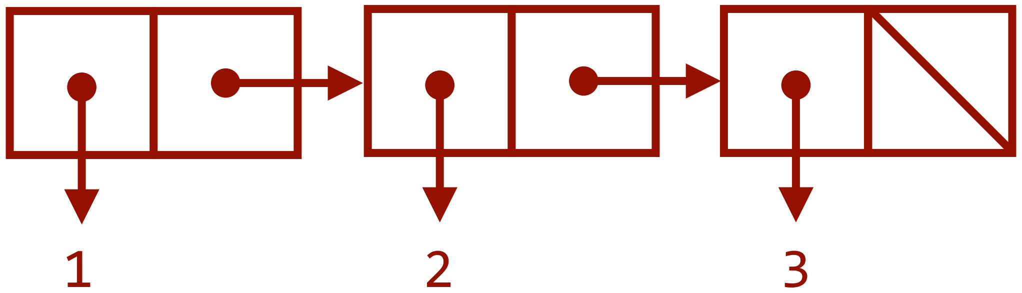
`(list 1 2)`

`(list 1)`



Aside: we don't actually *need* `list`!

`list` is "syntactic sugar" for one or more calls to `cons`




```
(list 1 2 3)
```

is the same as

```
(cons 1 (cons 2 (cons 3 empty)))
```

More syntactic sugar for lists

You can use Racket's display notation to construct a value

```
'()
```

is the same as

```
empty
```

```
'(1 2 3)
```

is the same as

```
(list 1 2 3)
```

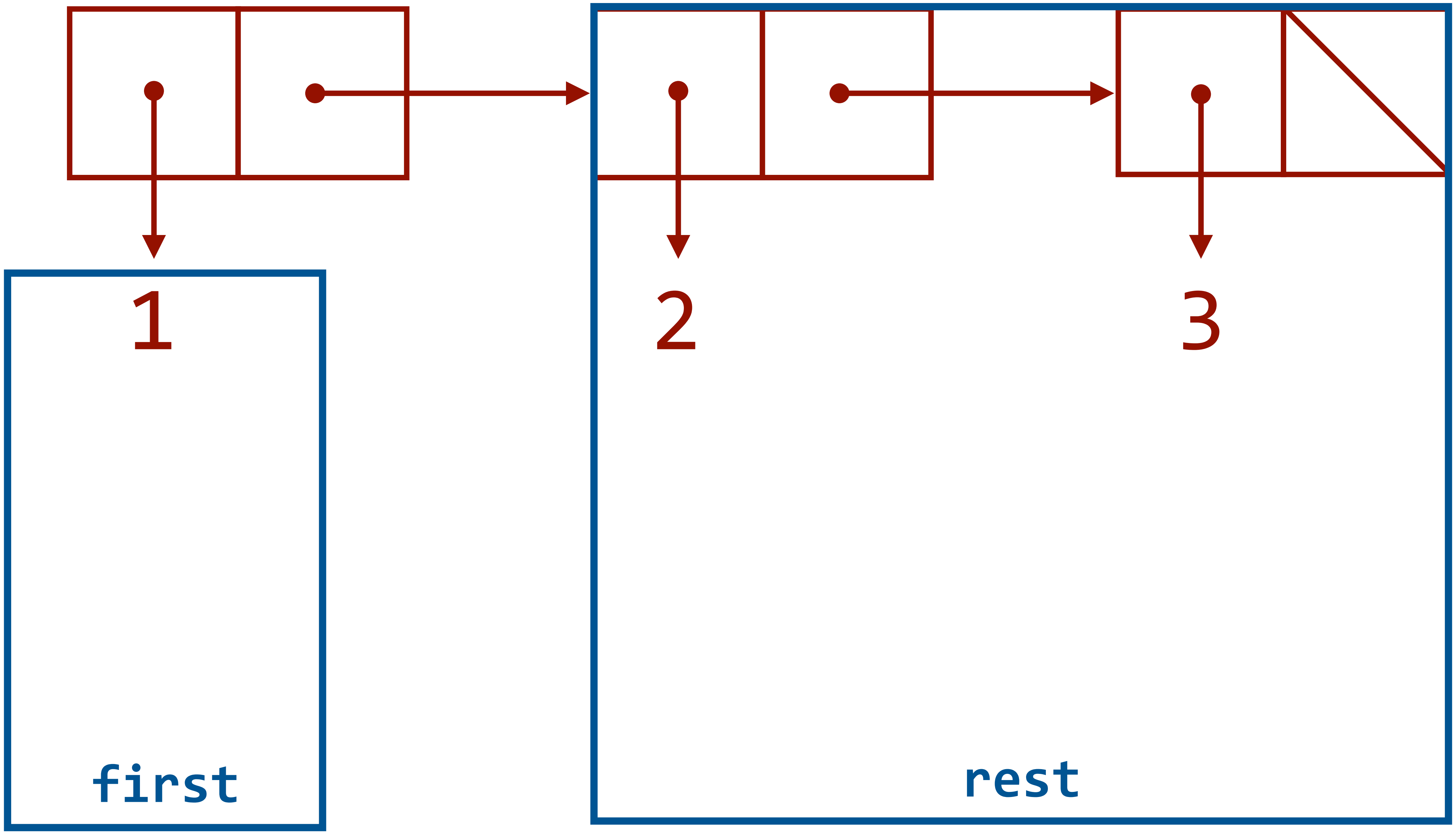
```
'(3 (1 2) 0)
```

is the same as

```
(list 3 (list 1 2) 0)
```

But: be careful!
(more in a bit)

Accessing Racket lists



Accessing lists: let's practice

Assume the variable L has the value `'(1 2 3)`. Fill in the table.

result	expression that uses L to compute result
`1`	`(first L)`
`'(2 3)`	
`2`	
`'(3)`	

Accessing lists: let’s practice

Assume the variable `L` has the value `'(1 2 3)`. Fill in the table.

result	expression that uses L to compute result
`1`	`(first L)`
`'(2 3)`	`(rest L)`
`2`	`(first (rest L))`
`'(3)`	`(rest (rest L))`

Watch out!

Don't do these things (and if you accidentally do, know how to recognize them)

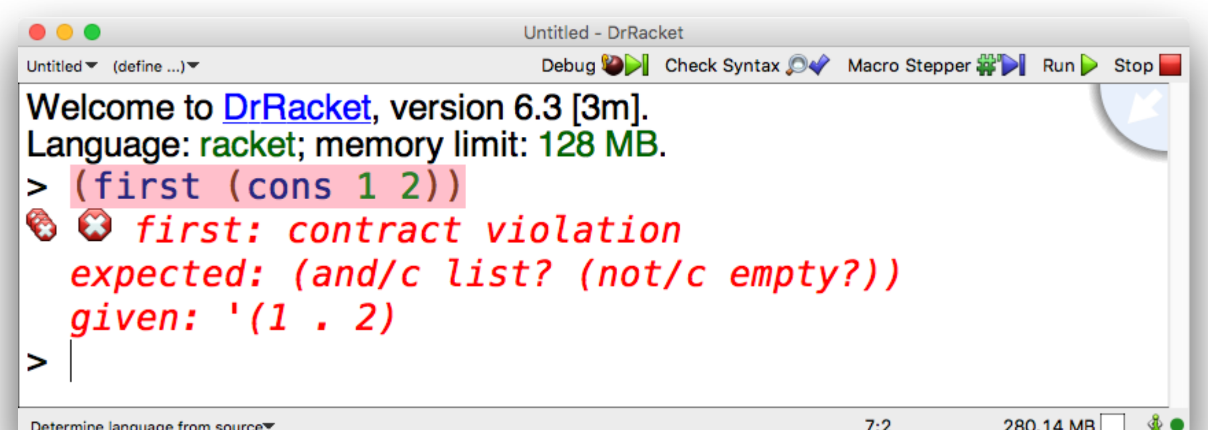
```
> (cons 1 2)
'(1 . 2)
```

not a list!

This expression builds a *pair.*

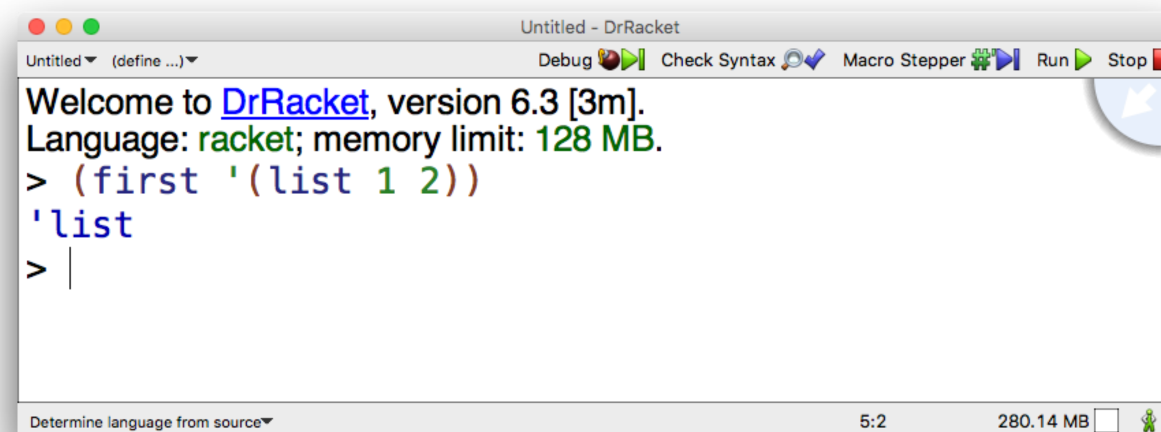
A pair is **not** a list.

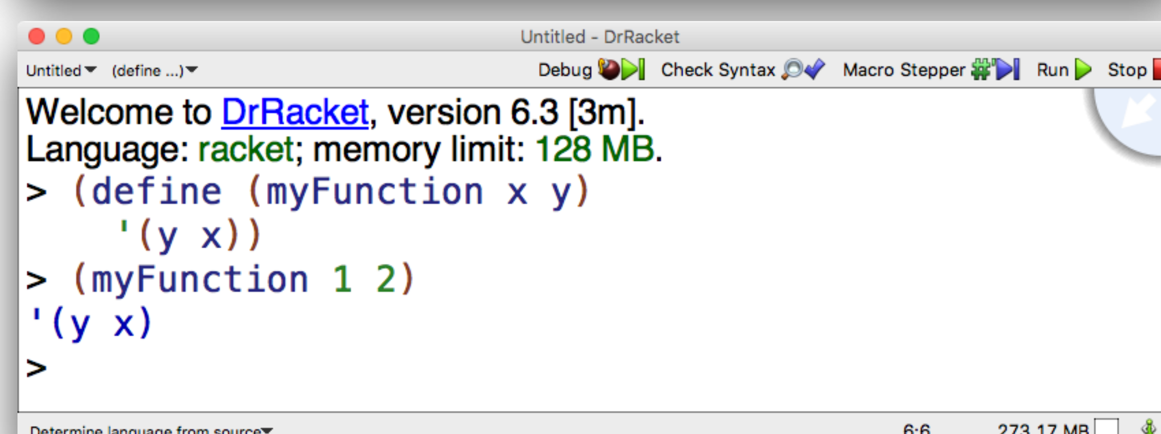
You can't call `first` on it.
You can't call `rest` on it.




```
> '(list 1 2)
'(list 1 2)
```

This expression builds a list whose first element is `'list`!





For more info, see: docs.racket-lang.org/guide/Pairs__Lists__and_Racket_Syntax.html

upload.wikimedia.org/wikipedia/commons/8/8c/Key_break.jpg

Racket lists: the basics

We'll use these built-in operations the *most*, when working with Racket lists

Constructors

to make an empty list: **`empty`**

to make a list with *n* elements: **`list`**

to add an element to the beginning of a list: **`cons`**

Selectors

to test if a list is empty: **`empty?`**

to get the first element of a list: **`first`**

to get a sublist, starting with the second element: **`rest`**

For more operations, see: https://docs.racket-lang.org/reference/pairs.html

Inductive Data Types & Recursive Operations

(Putting Together & Taking Apart)

```
empty
cons

empty?
first
rest
```

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)



                              )
```

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)



                              )


; tests
(check-equal? (len '()) 0)
(check-equal? (len '(1 2 3)) 3)
(check-equal? (len '((1 2 3))) 1)
```

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)
                                                base case

                             )          recursive step

; tests
(check-equal? (len '()) 0)
(check-equal? (len '(1 2 3)) 3)
(check-equal? (len '((1 2 3))) 1)
```

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)
  (if (empty? L)                             base case


                        ))                   recursive step


; tests
(check-equal? (len '()) 0)
(check-equal? (len '(1 2 3)) 3)
(check-equal? (len '((1 2 3))) 1)
```

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)
  (if (empty? L)
      0                                             base case
                                       ))       recursive step

; tests
(check-equal? (len '()) 0)
(check-equal? (len '(1 2 3)) 3)
(check-equal? (len '((1 2 3))) 1)
```

Recursive functions over lists

```
;; len
;;   inputs: a list, L
;;   outputs: the number of elements in the list
(define (len L)
  (if (empty? L)
      0                              base case
      (+ 1 (len (rest L)))))         recursive step

; tests
(check-equal? (len '()) 0)
(check-equal? (len '(1 2 3)) 3)
(check-equal? (len '((1 2 3))) 1)
```

Let's practice: `sum`

```
;; sum
;;   inputs: a list of integers, L
;;   outputs: the sum of the integers in L
(define (sum L)





; tests
(check-equal? (sum '()) 0)
(check-equal? (sum '(1)) 1)
(check-equal? (sum '(1 2 1)) 4)
(check-equal? (sum '(1 2 3 4)) 10)
```

Let's practice: `sum`

```
;; sum
;;   inputs: a list of integers, L
;;   outputs: the sum of the integers in L
(define (sum L)
  (if (empty? L)
      0
      (+ (first L) (sum (rest L)))))

; tests
(check-equal? (sum '()) 0)
(check-equal? (sum '(1)) 1)
(check-equal? (sum '(1 2 1)) 4)
(check-equal? (sum '(1 2 3 4)) 10)
```

Let's practice: analyzing `sum`

what happens as the size of L grows very large?

```
(define (sum L)
  (if (empty? L)
      0
      (+ (first L) (sum (rest L)))))
```

- What mathematical tool should we use?
- What's the first question we should ask ourselves?
- What do you think is the answer?

Let's practice: analyzing `sum`

what happens as the size of L grows very large?

```
(define (sum L)
  (if (empty? L)
      0                                    base case
      (+ (first L) (sum (rest L)))))       recursive step
```

T(0) = 1 *step*

T(n) = 1 + T(n-1) *steps*

Recurrence equation solution:

$$t(n) = n + 1$$

Let's practice: remove

```
;; remove
;;   inputs: an integer i
;;           a list of integers, L
;;   outputs: a new list with 1st occurrence
;;            of i removed
(define (remove e L)




; tests
(check-equal? (remove 1 '()) '())
(check-equal? (remove 1 '(1)) '())
(check-equal? (remove 1 '(1 1)) '(1))
(check-equal? (remove 4 '(1 2 3)) '(1 2 3))
```

Let's practice: `remove`

```
;; remove
;;   inputs: an integer i
;;           a list of integers, L
;;   outputs: a new list with 1st occurrence
;;            of i removed
(define (remove e L)
  (cond [(empty? L) '()]
        [(= (first L) e) (rest L)]
        [else (cons (first L) (remove e (rest L)))]))

; tests
(check-equal? (remove 1 '()) '())
(check-equal? (remove 1 '(1)) '())
(check-equal? (remove 1 '(1 1)) '(1))
(check-equal? (remove 4 '(1 2 3)) '(1 2 3))
```