**Prove $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.**

To prove

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C),$$

we will show that $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$ and $A \cup (B \cap C) \supset (A \cup B) \cap (A \cup C)$.

First, let $x \in A \cup (B \cap C)$. Then by defintion, either $x \in A$ or $x \in B \cap C$. If $x \in A$, then $x \in A \cup B$ and $x \in A \cup C$, therefore, $x \in (A \cup B) \cap (A \cup C)$. In the other case, if $x \in B \cap C$, then $x \in B$ and $x \in C$ so $x \in A \cup B$ and $x \in A \cup C$, therefore, $x \in (A \cup B) \cap (A \cup C)$. In either case, $x \in A \cup (B \cap C)$ implies $x \in (A \cup B) \cap (A \cup C)$. Therefore

$$A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C).$$

Now, let $x \in (A \cup B) \cap (A \cup C)$. Then $x \in A \cup B$ and $x \in A \cup C$. Since either $x$ is in $A$ or not, we will consider two cases, either $x \in A$ or $x \notin A$. If $x \in A$ then $x \in A \cup (B \cap C)$. If $x \notin A$ since we have assumed $x \in A \cup B$ and $x \in A \cup C$, this means that $x \in B$ and $x \in C$ and thus $x \in B \cap C$ and $x \in A \cup (B \cap C)$. Therefore

$$A \cup (B \cap C) \supset (A \cup B) \cap (A \cup C).$$

Finally, since $A \cup (B \cap C) \subset (A \cup B) \cap (A \cup C)$ and $A \cup (B \cap C) \supset (A \cup B) \cap (A \cup C)$, $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.