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**Presentation**
**Math 432 – Topological Spaces**

**Claim:** Every Well Ordered set has the least upper bound property.

**Proof:**
Assume $A$ is a well ordered set with order relation $<$. That means that every non-empty subset of $A$ has a smallest element.

For A to have the least upper bound property it must be true that every subset of A that is bounded above must have a least upper bound.

Assume $A_1 \subset A$ such that A is bounded above. So that means $\exists b \in A$ such that $a \leq b$, $\forall a \in A_1$.

Let $B$ be the set containing all upper bounds of $A$. We know B is non empty since $A_1$ is bounded above, and we also know by the definition of upper bound that every element of $B$ is an element of $A$.

So that means $B$ is a nonempty subset of $A$. So $B$ has a smallest element. So that means that $A_0$ has a least upper bound.

So that means if $A_1$ is a subset of $A$, that is bounded above, $A_1$ has a least upper bound. So A has the least upper bound property. Which means that every well ordered set has the least upper bound property.