Math 432 – Topological Spaces **Homework 8**

Due April 27

For each problem below, please submit well-written, complete sentence solutions.
Prepare Questions 1-3 for Quiz on Friday 4/22

1. Given spaces X and Y, let $[X, Y]$ denote the set of homotopy classes of maps of X into Y.
   (a) Let $I = [0, 1]$. Show that for any X, the set $[X, I]$ has a single element.
   (b) Show that if Y is path connected then the set$[I, Y]$ has a single element.
2. A space X is said to be contractible if the identity map $i_X : X \to X$ is nulhomotopic.
   (a) Show the I and $\mathbb{R}$ are contractible.
   (b) Show that a contractible space is path connected.
   (c) Show that if Y is contractible, then for any X. the set $[X, Y]$ has a single element.
   (d) Show that if X is contractible and Y is path connected, then $[X, Y]$ has a single element.
3. A subset A of $\mathbb{R}^n$ is said to be star convex is for some point a_0 of A, all the line segments joining a_0 to other points in A lie in A.
   (a) Find a star convex set that is not convex.
   (b) Show that is A is star convex, A is simply connected.
4. Prove using the techniques of algebraic topology the fundamental theorem of algebra, i.e: Show that every nonconstant polynomial with coefficients in $\mathbb{C}$ has a root in $\mathbb{C}$. (Hint: you may refer to the proof in Hatcher, explain the details in your own words.)
5. Show that composition of paths satisfies the following cancellation property: If $f_0 \cdot g_0 \simeq f_1 \cdot g_1$ and $g_0 \simeq g_1$ then $f_0 \simeq f_1$.
6. Read through the associativity part of Thm 51.2 and summarize it.
7. (Grad) Show that every continuous map $h : D^2 \to D^2$ has a fixed point, that is, a point x with $h(x) = x$.