Sets

Section 1.3: Some Special Sets

There are many named special sets of numbers that we have seen and will continue to see. Take a look at the table below and fill in the missing names, descriptions and examples.

Symbol	Name	Description	A list of examples
$\mathbb{N}$ or $\mathbb{Z}^*$	natural numbers	positive integers and 0	$0, 1, 2, 3, \ldots$
$\mathbb{P}$ or $\mathbb{Z}^+$	positive integers	whole numbers, positive	$1, 2, 3, 4 \ldots$
$\mathbb{Z}$	integers	the set of all whole numbers positive, negative and 0	$\ldots -2, -1, 0, 1, 2 \ldots$
$\mathbb{Q}$	rational number	any number that can be written as the quotient of two integers	$2, 1/2, \frac{8}{7} \ldots$
$\mathbb{R}$	real numbers	any number that can be written using (possibly infinite) decimal expansion	$2, 1/2, \pi, \sqrt{2}, \ldots$

We use "rational number" and not "fraction" since $\frac{\pi}{2}$ is a "fraction" but is not rational

Note: Some people call the set of positive integers the natural numbers.

If a number(or other object) is an element (member) of a set, we use the symbol $\in$ to denote this. For example, $3/2 \in \mathbb{Q}$. We can use the symbol $\notin$ to represent when the number (or object) is not a member of the set.

We'll prove this later

For each number and set pair below, use the symbol $\in$ or $\notin$ to make the statement true.

$2 \in \mathbb{R}$, $\quad \pi \notin \mathbb{Q}$, $\quad \sqrt{2} \notin \mathbb{Q}$, $\quad -5 \notin \mathbb{N}$,

$7 \in \mathbb{P}$, $\quad \pi \in \mathbb{R}$, $\quad \sqrt{2} \in \mathbb{R}$, $\quad 5/2 \notin \mathbb{N}$

When we list out the elements of a set, we use braces: $\{\}$. For examples, the set of positive even integers less than or equal to 8 is

$$\{2, 4, 6, 8\}.$$

Sometimes, we get lazy and don't want to write out every element of a set, so we write "...". We can either do this in the middle, or if its an infinite set we can do it at the end. Some facts about sets - order doesn't matter and repeats don't change the set. $\{1, 2, 3\} = \{3, 2, 1\} = \{1, 1, 2, 3, 3, 3, 2\}$

For each of the sets below, find the number of elements in the set.

Set	Number of Elements	
$\{2, 4, 6, 8, \ldots, 64\}$	32	(multiples of 2 between 2 & 64)
$\{2, 4, 8, \ldots, 64\}$	6	$\{2^1, 2^2, 2^3, \ldots, 2^6\}$
$\{2, 5, 7, 9, 15, \ldots\}$	∞	
$\{-3, 8, \pi, 15, 24\}$	5	

Time for a game! Think of a set with approximately 20 elements. Try to think of one that is different from your group mates. (Scattergories Style) Describe your set below in as few words as possible. The person with a unique set described (exactly) in as few words as possible wins.

examples

primes below 60

face cards

integers between 60 and 70

Idea when discussing a set of numbers, we usually say a set first (integers/primes/real numbers) and then some rule

We often want to describe sets whose elements follow some pattern. To do this we use the { : } notation. Here's are two examples:

$$\{x : x \in \mathbb{R} \text{ and } 2 \leq x < 4\}$$

$$\{n \in \mathbb{N} : n \text{ is even}\}$$

that's how this notation works

The first one reads as "The set of x, such that x is in $\mathbb{R}$ and x is greater than or equal to 2 and less than 4" or "The set of x such that x is a real number and x is between 2 and 4, including 2".

The second one reads as "The set of all n in $\mathbb{N}$ such that n is even".

Note that before the colon (:) we either have just the variable or the variable and what set it belongs to. This is pretty common. The colon (:) is read as "such that". After the colon is the conditions or rules for the set. They may be connected by "and" or "or" statements. Another way to use the notation is to specify a rule and then a set. For example, to write the sets of all perfect squares, you could write:

this one uses this notation

$$\{n^2 : n \in \mathbb{N}\}$$

Don't use [0,6) it always means "all real numbers between 0 & 6"

Below are several sets. For each row, fill in the blanks.

Name	{ : }	list of elements in the set or another way to write it	number of elements
A	$\{n \in \mathbb{N} : n < 6\}$	$\{0,1,2,3,4,5\}$	6
B	$\{x \in \mathbb{R} \mid 1 < x \leq 15\}$	$(1, 15]$ (interval notations for real numbers) we use (,) for open intervals and [,] for closed ones	∞
C	$\{(-1)^n : n \in \mathbb{N}\}$	$\{-1, 1\}$	2
D	$\{n \in \mathbb{Z} : \lvert n \rvert \leq 3\}$	$\{-3, -2, -1, 0, 1, 2, 3\}$	7
E	$\{r \in \mathbb{Q} : r^2 = 2\}$	$\{\} = \emptyset$ (empty set)	0
F	$\{x \in \mathbb{R} : x < 15\}$	$(-\infty, 15)$	∞

Set E has a special notation: $\emptyset$

We say that one set is a subset of another if every element of the first set is also an element of the second set. For example, since every natural number is also a real number, we can say that the natural numbers are a subset of the real numbers. The symbol for subset is $\subseteq$. This is similar to $\leq$ in that we can use it even if the sets are equal to each other. If we want to explicitly say that the sets are not equal we can use $\subset$instead, this is called a proper subset. So we could say:

$$\mathbb{N} \subseteq \mathbb{R} \quad \text{or} \quad \mathbb{N} \subset \mathbb{R}$$

For each set on the first page, write which ones are subsets of which other ones. (You should be able to make a long string of inclusions).

$$\mathbb{N} \subseteq \mathbb{P} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}$$

For each set on the ~~third~~ second page (A-F), write which set are subsets are which other ones.

Note: $\emptyset \subseteq$ any set, since every element (there's none) in $\emptyset$ are also in the set (trivially) we say this is "vacuously true"

So $E \subseteq A$ $E \subseteq B \subseteq C$ $E \subseteq D$ $E \subseteq F$

Also $A \subseteq F$, $C \subseteq D \subseteq F$,

Consider the set $S = \{a, b, c\}$. Notice it's elements are letters - that's allowed. In fact the elements of a set can be whatever you want them to be. List out as many subsets of S as possible.

$\{\} = \emptyset$ $\{a, b\}$

$\{a\}$ $\{a, b, c\}$

$\{b\}$ $\{b, c\}$

$\{c\}$ $\{a, c\}$

The set of all subsets of S is called the power set of S, denoted $\mathcal{P}(S)$. For the specific S above, $\mathcal{P}(S)$ should have 8 elements, each of which is a set. (Remember the elements of a set can be whatever they want - in this case their sets themselves.)

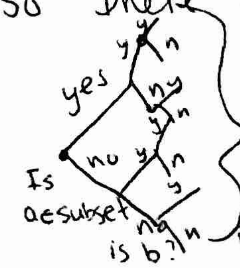
*If T is a finite set with n elements, how many elements does $\mathcal{P}(T)$ have?

$\mathcal{P}(T)$ has 2^n elements.

each of the n elements of T is either in the subset or out (2 choices) so there are

$\underbrace{2 \cdot 2 \cdots 2}_{\text{one for each element of T}} = 2^n$ choices total

e.



8 choices total.

Now we will look at one more strange looking set. Let's say we are creating a new language and our alphabet only consist of the letters $\{a, b, c, d, e\}$. How many words can we make?

We can make words like

a, aa, aaa, aaaa, ...

So there are ∞'ly many.

How many one letter words are there?

5 a, b, c, d, e.

How many two letter words are there?

There are 5 choices for first letter & 5 for second

so $5 \cdot 5 = 25$.

*How many n letter words are there?

5 choices for each of n letters so $\underbrace{5 \cdot 5 \cdots 5}_{n \text{ times}} = 5^n$.

This is related to a set we will see throughout the book. We define an **alphabet** to be a finite nonempty set, called Σ, whose members are symbols called **letters**. Then we can get a **word** by taking a finite string of letters. The length of the word is the number of letters in it. The set of all words with letters from Σ is called Σ^* (sigma-star). There is one special word in Σ^* which has zero letters. It's called the **empty word**(or null word or null string) and denoted λ. Any subset of Σ^* is called a **language** over Σ.

Is the English language a language according to this definition? Why or why not?

Yes. The English language is a subset of the set of all Words on our alphabet $\{a, b, c, \ldots, y, z\}$ thus it fits this definition of language.