ACM 11, SPRING 2016, HOMEWORK 2

Due: Tuesday, May 3

Submit by 23:59:59 PDT, as a zip file `LastnameFirstnameHW2.zip` to acm11spring2016@gmail.com, with a subject line "Lastname Firstname HW2."

Problem 1: Fair Die (10 points)

Download the file `DiceData.mat` from the piazza page[1] and load it into MATLAB using the `load` command. There are two vectors, `die1` and `die2`, which are the results of 10^4 rolls of two dice. One die is a fair die, meaning that the integers $1, \ldots, 6$ all have equal probability. The other die is not a fair die.

Which die is fair,`die1` or `die2`? Explain what lead you to your conclusion, and what MATLAB commands you used in a script file `dice.m`.

Problem 2: Random Triangles (20 points)

Suppose that a stick of wood (modeled as unit interval $[0, 1]$) is broken into 3 pieces at random. The breakpoints are modeled as a sample of size two from the uniform distribution on $[0, 1]$. What is the probability that the three pieces form the three edges of a triangle? Write a script `triangles.m`, where you compute empirical probability for this problem using $N = 10^5$ samples. Your script should display the computed empirical probability in the end. Your program should be vectorized; in particular, there should be no `if/switch` conditions or `for/while` loops. You can check the obtained empirical result by solving this problem analytically.

Problem 3: Jurassic Park (70 points)

You are in Jurassic park, surrounded by three raptors that will eat you (sorry) in a moment. Let $h(t) \in \mathbb{R}^2$ is your position, and $r_1(t), r_2(t), r_3(t) \in \mathbb{R}^2$ are the positions of the raptors. Initially (at time $t = 0$), you are at the center of a $d = 20$ m equilateral triangle and the raptors are at the vertices:

$$h(0) = (0, 0), \quad r_1(0) = (-10, -a), \quad r_2(0) = (10, -a), \quad r_3(0) = (0, 2a),$$

where $a = d\sqrt{3}/6$. You chose a direction $e \in \mathbb{R}^2$, $\|e\| = 1$ and and run in this direction at a constant speed $v_h = 6$ m/s. Thus,

$$h(t) = v_h t e.$$

Assume that raptors are not good at predicting your location, and at any time t, they simply run directly at you. So, the raptor's motion can be modeled by the following ODE:

$$\dot{r}_i = v_i \frac{h(t) - r_i(t)}{\|h(t) - r_i(t)\|},$$

where v_i is the speed of the i^{th} raptor. Assume that raptors 1 and 2 are healthy and $v_1 = v_2 = 25$ m/s and raptor 3 is injured and $v_3 = 20$ m/s.

[1] `https://piazza.com/caltech/spring2016/acm11/resources`

(1) (50 points) Write a MATLAB function `timeToMeal.m` that takes the angle α of your direction (so that $e = (\cos\alpha, \sin\alpha)$) and calculates the time `tDevour` until you is caught by the raptors. Your function should accept angles in degrees; internally, you may convert to radians using `deg2rad` if you wish. Assume that you are caught when a raptor is within 0.1 meters (use events function to implement this). To solve the ODEs, use solver `ode45`. How long will take raptors to catch you if you run at an angle of $\alpha = 30^\circ$ above the horizontal? For grading, we will also test your function on an arbitrary angle.

(2) (20 points) Write a MATLAB script `angles.m` that find the best angle α_1 and the worst angle α_2 at which you should run to maximize and minimize `tDevour`.