# CS 589, Principles of Database Systems, Notes 2-1



**Lecture 2-1**
**Spring 2016**
**Dependencies and Inference**





### Model Theoretic View of Databases

A database schema is a theory or specification

The database state is a model or instance that satisfies the specification

- When is this view useful?
  - discussing constraints and implication
  - comparing representation capabilities of two database schemata
  - representing incomplete information: DB as partial model

## "Specification" Part

relation name

`assigned( PILOT FLIGHT DATE TIME)`

relation scheme

attribute

Each attribute A has a dom(A)

So the relation scheme defines attributes and allowable values for a conforming relation instance



Principles of Database Systems

## Relation Instance

relation instance

assigned(

| PILOT | FLIGHT | DATE | TIME) |
|---|---|---|---|
| Cushing | 83 | 9 Aug | 10:15a |
| Cushing | 116 | 10 Aug | 1:25p |
| Clark | 281 | 8 Aug | 5:50a |
| Clark | 301 | 12 Aug | 6:35p |
| Clark | 83 | 11 Aug | 10:15a |
| Chin | 83 | 13 Aug | 10:15a |
| Chin | 116 | 12 Aug | 1:25p |
| Copley | 281 | 9 Aug | 5:50a |
| Copley | 281 | 13 Aug | 5:50a |
| Copley | 412 | 15 Aug | 1:25p |

Expect a relation instance to satisfy requirements of the schema

*Data dependencies*: further constraints that the schema puts on a satisfying instance

## Functional Dependency

| assigned( | PILOT | FLIGHT | DATE | TIME) |
|---|---|---|---|---|
| | Cushing | 83 | 9 Aug | 10:15a |
| | Clark | 83 | 11 Aug | 10:15a |
| | Chin | 83 | 13 Aug | 10:15a |
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A generalization of keys: when tuples agree on certain attributes, must agree on others

FLIGHT → TIME (L → T)
PILOT DATE TIME → FLIGHT (PDT → L)
FLIGHT DATE → PILOT (LD → P)





## Functional Dependency Definition

A relation instance r(R) satisfies the *functional dependency* (FD) X → Y (X and Y subsets of R), if

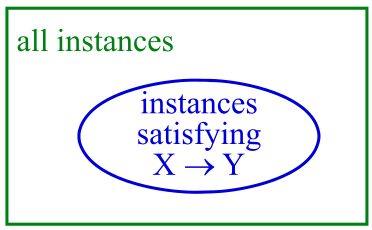
```
for every two tuples t,s in r,
if t[X] = s[X],
then t[Y] = s[Y]
```

"functionally determines"

## FDs as Specification

Usually, we don't ask which FDs a relation instance satisfies.

Rather, specify FDs that we expect all instances of to satisfy as part of the relation schema



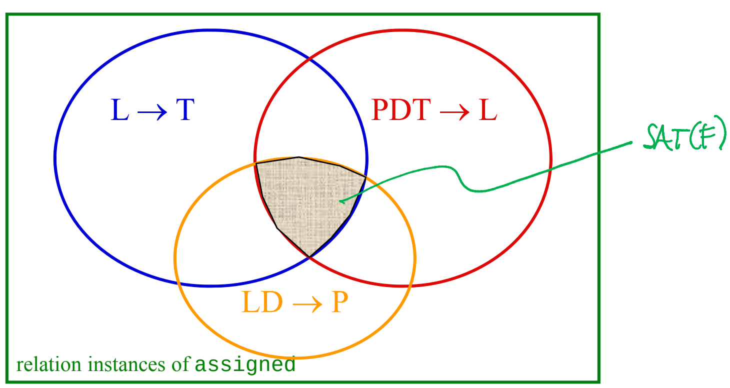





## Sets of FDs

A relation instance r satisfies a set of FDs F if it satisfies every FD in F

F = {L → T, PDT → L, LD → P}



## Implication

If an instance r satisfies FDs F, there may be another FD $X \to Y$ it necessarily satisfies

$\models$

Say that F *implies* $X \to Y$ ($F \models X \to Y$)

```
Consider L → T
Let r be any relation instance
  satisfying it
take t,s in r where t[PL] = s[PL]
so t[L] = s[L]
by L → T, t[T] = s[T]
thus r also satisfies PL → T
```





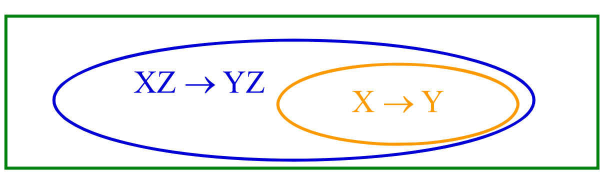
## Inference Axioms

General patterns that describe implications. For X, Y, Z, W sets of attributes

F1. Reflexivity: $X \to X$

$X \cup Z$

F2. Augmentation: if $X \to Y$, then $XZ \to YZ$

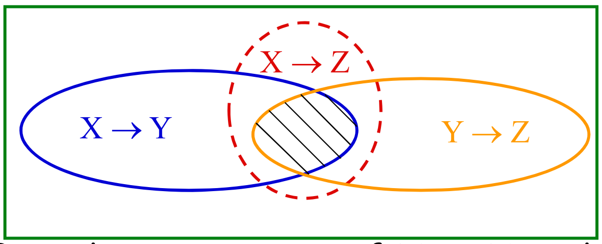



F3. Additivity: if $X \to Y$ and $X \to Z$, then $X \to YZ$

## More Axioms

F4. Projectivity: if $X \to YZ$, then $X \to Y$

F5. Transitivity: if $X \to Y$ and $Y \to Z$, then $X \to Z$




F6. Pseudotransitivity: if $X \to Y$ and $YZ \to W$, then $XZ \to W$





## Can Do Proofs

F = {L → T, PDT → L, LD → P}

1. L → T (in F)
2. LD → TD (augmentation from 1.)
3. LD → P (in F)
4. LD → PTD (additivity from 2. and 3.)

## Derivation

Say that F *derives* $X \to Y$ if there is a proof of $X \to Y$ from FDs in F using inference axioms F1 - F6.

$F \vdash X \to Y$

Is implies the same as derives?

$$\underbrace{F \models}_{\text{semantic}} X \to Y \text{ if and only if } \underbrace{F \vdash}_{\text{syntactic}} X \to Y$$

soundness

completeness





## Soundness

Prove that each inference axiom is correct. That is, it holds for any relation instance.

F5. If $X \to Y$ and $Y \to Z$, then $X \to Z$

Take any tuples t,s in r, where r satisfies $X \to Y$ and $Y \to Z$.

Suppose t[X] = s[X]. Then from $X \to Y$, must have t[Y] = s[Y].

But by $Y \to Z$, then have t[Z] = s[Z].

So r satisfies $X \to Z$.

## Definitions

*Closure of F*, $F^+$, is F plus all FDs that can be derived from F by F1. - F6.

So $F \vdash X \rightarrow Y$ means $X \rightarrow Y$ in $F^+$

*Closure of X*, $X^+$, is largest Z such that $X \rightarrow Z$ is in $F^+$

```
{L → T, PDT → L, LD → P}
What is (LD)+?
What is T+?
```

LD → LD

LD → TD

LD → PLD

LD → TPLD

LD+

T → T = T+





## Completeness

If it's true, you can prove it

or, equivalently

If you can't prove it, it's not true.

contrapositive

### Instance to show not true

F is FDs on relation scheme R

Take any $X \rightarrow Y$ over R not in $F^+$.

Consider $X^+$ and let $X^- = R - X^+$.

| | $X^+$ atts | $X^-$ atts |
|---|---|---|
| t | a1 a2 … an | b1 b2 … bm |
| s | a1 a2 … an | c1 c2 … cn |

$X \subseteq X^+$

bi $\neq$ ci

This relation instance violates $X \rightarrow Y$.

- $t[X] = s[X]$, but claim $t[Y] \neq s[Y]$
- Suppose $t[Y] = s[Y]$. Then $Y \subseteq X^+$
- Since $X \rightarrow X^+$ in $F^+$, so is $X \rightarrow Y$ by projectivity, a contradiction





### The instance satisfies $F^+$

Take any $W \rightarrow Z$ in $F^+$

Case 1. $W \not\subset X^+$.

- Then $t[W] \neq s[W]$, no problem

Case 2. $W \subseteq X^+$.

- Then $t[W] = s[W]$
- Have $X \rightarrow X^+$ in $F^+$, $X^+ \rightarrow W$ by reflexivity and projectivity, and $W \rightarrow Z$ in $F^+$.
- That gives $X \rightarrow Z$ by transitivity applied twice
- Additivity on $X \rightarrow X^+$ and $X \rightarrow Z$ gives $X \rightarrow X^+Z$

Case 2, continued

We have proved $X \rightarrow X^{+}Z$ is in $F^{+}$

But wait! $X^{+}$ is supposed to be the maximum set of attributes that X determines.

Must be that $Z \subseteq X^{+}$.

So $t[Z] = s[Z]$

Thus our instance satisfies $W \rightarrow Z$.





### Armstrong Relation for F

An "exact model" for F:

- Satisfies $F^{+}$
- Violates every FD not in F ($F^{-}$)

Use the 2-tuple instances in the last proof, combined into one big instance

Example $R = ABC$, $F = \{A \rightarrow B, B \rightarrow C\}$

Only need FDs in $F^{-}$ with one attribute on right side

$F^- = \{C \to A, C \to B, B \to A, BC \to A\}$

| A | B | C |
|---|---|---|
| a1 | b1 | c1 |
| a2 | b2 | c1 |
| a3 | b3 | c2 |
| a4 | b4 | c2 |
| a5 | b5 | c3 |
| a6 | b5 | c3 |
| a7 | b6 | c4 |
| a8 | b6 | c4 |





## Join Dependency

Note the redundancy in the assigned table: repeat time for each flight.

Can split up

**asgn1(**

| PILOT | FLT | DATE) |
|---|---|---|
| Cushing | 83 | 9 Aug |
| Clark | 83 | 11 Aug |
| Chin | 83 | 13 Aug |
| Cushing | 116 | 10 Aug |
| Chin | 116 | 12 Aug |
| Clark | 281 | 8 Aug |
| Copley | 281 | 9 Aug |
| Copley | 281 | 13 Aug |
| Clark | 301 | 12 Aug |
| Copley | 412 | 15 Aug |

**asgn2(**

| FLT | TIME) |
|---|---|
| 83 | 10:15a |
| 116 | 1:25p |
| 281 | 5:50a |
| 301 | 6:35p |
| 412 | 1:25p |

## Can Recover with a Join

assigned = asgn1 $\bowtie$ asgn2

assigned satisfies a *join dependency* (JD)

$\bowtie$[PLD, LT]

assigned = $\pi_{PLD}$(assigned) $\bowtie$ $\pi_{LT}$(assigned)

A "lossless" join





## What about $\bowtie$[PL, PDT]?

```
asgn3(PILOT   FLT) asgn4(PILOT   DATE  TIME)
     Cushing  83         Cushing  9 Aug 10:15a
     Cushing 116         Cushing 10 Aug  1:25p
       . . .                 . . .

asgn3⋈asgn4(PILOT   FLT  DATE   TIME)
           Cushing  83   9 Aug  10:15a
           Cushing  83  10 Aug   1:25p
               . . .
```

loss of information
false tuples

### Join Dependency, General Form

r(R), with R1, R2, …, Rn subsets of R
r satisfies $\bowtie$[R1, R2, …, Rn] if
$r = \pi_{R1}(r) \bowtie \pi_{R2}(r) \bowtie \ldots \bowtie \pi_{Rn}(r)$

notice that always have
$r \subseteq \pi_{R1}(r) \bowtie \pi_{R2}(r) \bowtie \ldots \bowtie \pi_{Rn}(r)$

*A multi-valued dependency (MVD)* is a special case where n = 2.





### Implications of FDs and JDs

There are some axioms

```
X → Y on R, Z = R - XY
FJ1. X → Y implies ⋈[XY, XZ]

Example
L → T, so ⋈[LT, LPD]
```

However, there is no finite, complete set of axioms for just FDs and JDs (or JDs alone).

### Are We Stuck?

No

- There are complete axiom sets for FDs and multi-valued dependencies.
- There are complete axiom sets for classes of constraints that include FDs and JDs.
- There is an *inference procedure* for FD and JD implication.
  - not based on axioms





### Need to Use Tableaux

A tableau is a "template" for a relation

Has *rows* of variables instead of tuples of values

| U( | P | L | D | T) |
|---|---|---|---|---|
| v | *p1* | *f1* | *d1* | *m1* |
| w | *p2* | *f1* | *d2* | *m2* |

Treat tableau rows like tuples

w(L) = *f1*

### Valuation

A *valuation* ρ for tableau U maps its variables to domain values.

ρ(v) must be in dom(A) if v in A column

ρ(*p1*)=Chin ρ(*f1*)=86 ρ(*d1*)=1 June ρ(*m1*)=4p
ρ(*p2*)=Chao ρ(*d2*)=2 June ρ(*m2*)=4p





### Extend Valuation to Rows and Tableaux

ρ(w) = apply ρ to each variable in w

ρ(w) **(***A***)** = ρ(w**(***A***)**)

ρ(*p1 f1 d1 m1*) = <Chin 86 1June 4p>

ρ(U) = {ρ(w) | w in U}

ρ(**U**)

| | P | L | D | T |
|---|---|---|---|---|
| ρ(v) | Chin | 86 | 1June | 4p |
| ρ(w) | Chao | 86 | 2June | 4p |

## How About This Valuation?

$\rho(p1)$=Chin $\rho(f1)$=86 $\rho(d1)$=1 June $\rho(m1)$=4p
$\rho(p2)$=Chao $\rho(d2)$=2 June $\rho(m2)$=5p

| $\rho$(U)( | P | L | D | T ) |
|---|---|---|---|---|
| $\rho$(v) | Chin | 86 | 1June | 4p |
| $\rho$(w) | Chao | 86 | 2June | 5p |

violates [illegible] L → T





## Want to Enforce L → T

Need to make sure that $\rho$(m1) = $\rho$(m2).

Can we "fix" the tableau?

| | U(P | L | D | T) |
|---|---|---|---|---|
| v | *p1* | *f1* | *d1* | *m1* |
| w | *p2* | *f1* | *d2* | ***m1*** |

Have "applied" L → T to tableau U.

## The Chase

Apply FDs and JDs to a tableau, to reason about an example relation (or sub-relation)

Tableau rule for FD $X \rightarrow Y$

For rows v and w

If v[X] = w[X], make v[Y] = w[Y] by equating variables in Y

[Book gives rules for replacing variables]

*FD chase* of tableau U for F: Apply FD rules for FDs in F to U until no change. (*Why must this terminate?*)





## Example

{AB → E, AG → C, BE → D, E → G, DG → H}

| A | B | C | D | E | G | H |
|---|---|---|---|---|---|---|
| *a1* | *a2* | *a3* | *a4* | *a5* | *a6* | *a7* |
| *a1* | *b2* | *b3* | *b4* | *a5* | *b6* | *b7* |

Apply $E \rightarrow G$

| A | B | C | D | E | G | H |
|---|---|---|---|---|---|---|
| *a1* | *a2* | *a3* | *a4* | *a5* | *a6* | *a7* |
| *a1* | *b2* | *b3* | *b4* | *a5* | ***a6*** | *b7* |

$AG^+ = ACEG$

Apply $AG \rightarrow C$

| A | B | C | D | E | G | H |
|---|---|---|---|---|---|---|
| a1 | a2 | a3 | a4 | a5 | a6 | a7 |
| a1 | b2 | **a3** | b4 | a5 | a6 | b7 |

$F \models$
$AE \rightarrow C$ $\quad F \not\models AE \rightarrow H$





## Testing FD Implication with the Chase

<u>Method in book is ambiguous</u>

Want to test $F \models X \rightarrow Y$ on scheme
R = A1 A2 … An

1. Set up a tableau $U_X$ with rows v, w:
   v(Ai) = ai
   w(Ai) = ai if Ai in X
   = bi otherwise
2. Chase $U_X$ with FDs in F
3. If v[Y] = w[Y], then $F \models X \rightarrow Y$
   (if not, we have a counterexample)

## Example

F = {AB → C, B → D, CD → E, CE → GH, G → A} ⊨ BG → C? ✓

| $U_{BG}$ | A | B | C | D | E | G | H |
|---|---|---|---|---|---|---|---|
| v | *a1* | *a2* | *a3* | *a4* | *a5* | *a6* | *a7* |
| w | ~~b1~~ a1 | a2 | ~~b3~~ a3 | ~~b4~~ a4 | ~~b5~~ a5 | a6 | ~~b7~~ a7 |





## You Try It

F = {AB → E, AG → C, BE → D, E → G, DG → H}

| A | B | C | D | E | G | H |
|---|---|---|---|---|---|---|
| *a1* | *a2* | *a3* | *a4* | *a5* | *a6* | *a7* |
| b1 | a2 | b3 | ~~b4~~ a4 | a5 | ~~b6~~ a6 | ~~b7~~ a7 |

Does F ⊨ BE → A? X

Does F ⊨ BE → H? (can use same chase) ✓