Implications of FDs and IDs

Haven't discussed how infinite relations affect implication

- Examples have all been finite sets of tuples
- Hasn't mattered for combinations of dependencies seen so far

It does make a difference when dealing with FDs and IDs together

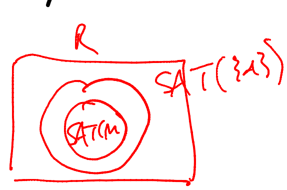
SAT

Go back to implication as containment of sets of instances

SAT(M, R) = all instances over scheme R that satisfy all dependencies in M

R will usually be understood, so we write SAT(M)

Have: M implies dependency d if and only if SAT(M) $\subseteq$ SAT({d})



Finite Implication

Let FSAT(M) = all <u>finite</u> instances that satisfy all dependencies in M

Definition: M *finitely implies* dependency d if FSAT(M) $\subseteq$ FSAT({d}).

Can have "finitely implies" without (regular) "implies"

But not vice versa

Principles of Databaess

First, a Fact

If r is finite and r satisfies $A \rightarrow B$, then the number of distinct A-values in r is greater or equal to the number of distinct B-values.

|r[A]| ≥ |r[B]|

A	B
a1	b1
a2	b2
a3	b1

Finite Implication without regular Implication

Relation r(AB)

$M = \{A \rightarrow B,\ r[A] \subseteq r[B]\}$

$d = r[B] \subseteq r[A]$

Part 1. Assume r is finite

Since $A \rightarrow B$, then $|r[A]| \geq |r[B]|$

Since $r[A] \subseteq r[B]$, r finite $|r[A]| \leq |r[B]|$

So, $|r[A]| = |r[B]|$ and the inclusion must be an equality: $r[A] = r[B]$

Thus, $r[B] \subseteq r[A]$

Part 2. Allow r to be infinite

Here is a relation instance that satisfies $\{A \rightarrow B,\ r[A] \subseteq r[B]\}$, but not $r[B] \subseteq r[A]$

0 6 0 ∉

A	B
1	0
2	1
3	2
…	
i	i-1
…	

What Gives?

It is hard to express "r is finite" in normal logic

Thus generally hard to define FSAT[M] and reason about finite implication

Finite implication is often undecidable

Null Values

Used to represent missing information

At least two kinds

<u>dne</u> — does not exist
<u>unk</u> — value exists but unknown

PILOT	FLIGHT	DATE	TIME
Cushing	615	5Oct	**unk**
dne	704	6Oct	5:50a
Cook	**dne**	**dne**	**dne**

“Does Not Exist” Null

Like adding an additional value to domain of an attribute

So it enlarges the set of possible relation instances

Issue: dne = dne?

Maybe no: LD → P

PILOT	FLIGHT	DATE	TIME
Cook	**dne**	**dne**	**dne**
Cadiz	**dne**	**dne**	**dne**

Maybe yes: FIRST MI LAST → AGE

FIRST	MI	LAST	AGE
Alex	**dne**	Cook	37
Alex	**dne**	Cook	39

Unknown Null

More like variables

Can view an instance with **unk** nulls as a set of possible fully-defined instances

PILOT	FLIGHT	DATE	TIME
Cushing	615	5Oct	**unk**
Cook	704	6Oct	5:50a
Cushing	**unk**	6Oct	**unk**

Some Possible Instances

PILOT	FLIGHT	DATE	TIME
Cushing	615	5Oct	5:00p
Cook	704	6Oct	5:50a
Cushing	872	6Oct	11:00p

PILOT	FLIGHT	DATE	TIME
Cushing	615	5Oct	6:00p
Cook	704	6Oct	5:50a
Cushing	615	6Oct	~~6:00p~~ 7:00p

F $\not\to$ T

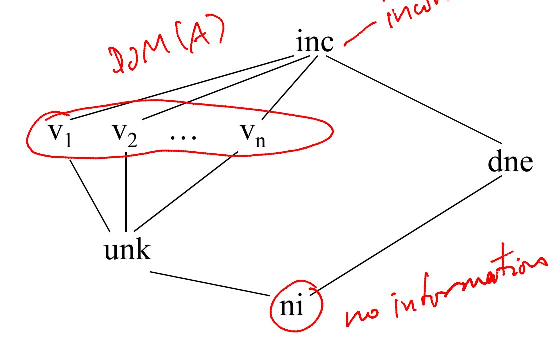
Also, “No-Information” and “Inconsistent”

"at least as"

x ⊑ y: y more informative than x

ni ⊑ unk ⊑ v_i ⊑ inc   unk ⊆ unk

ni ⊑ dne ⊑ inc



Extend Order to Tuples

Use ⊑ on each component

```
<Fred, unk, 6> ⊑ <Fred, 10.25, 6>
<Fred, unk, 6> ⊑ <Fred, unk, 6>
<ni, dne, 6> ⊑ <unk, inc, 6>
<ni, dne, 6> ⊑ <inc, inc, inc>
```

<Fred, 10.25, unk>
<Fred, 9.25, unk>

Extending to Relations

Concentrate on unk null

ord1(<u>Person Price Amt</u>) $\stackrel{?}{\sqsubseteq}$ ord4 (Pe Pr Am)

Person	Price	Amt
Fred	unk	6
Fred	10.25	unk
Fritz	unk	unk

Pe	Pr	Am
Fred	10.25	6
Fritz	unk	7

ord2(<u>Person Price Amt</u>)

Person	Price	Amt
Fred	11.50	6
unk	10.25	7

ord2 $\stackrel{?}{\subseteq}$ ord3

ord3(<u>Person Price Amt</u>)

Person	Price	Amt
Fred	11.50	6
Fred	10.25	7
Fritz	12.25	7

What Does a Tuple Mean?

For partial relations r, q, what should r ⊑ q require?

For tuple t in r
- exactly one s in q with t ⊑ s?
- at least one s in q with t ⊑ s?

For tuples t1, t2 in r, can there be a single s in q with t1 ⊑ s and t2 ⊑ s?

Can there be s in q where there is no t in r such that t ⊑ s?

Book's Definition

"Fill in the blanks"

every tuple in q has a corresponding tuple in r

$r \sqsubseteq q$ means there is a total, onto map θ from r to q where for every tuple t in r

$$t \sqsubseteq \theta(t)$$

Examples

ord1 (Person Price Amt) $\sqsubseteq$ ord3 (Person Price Amt)

Person	Price	Amt		Person	Price	Amt
Fred	unk	6	→	Fred	11.50	6
Fred	10.25	unk	→	Fred	10.25	7
Fritz	unk	unk	→	Fritz	12.25	7

θ

ord1 (Person Price Amt) $\sqsubseteq$ ord4 (Person Price Amt)

Person	Price	Amt		Person	Price	Amt
Fred	unk	6	→	Fred	10.25	6
Fred	10.25	unk	→	Fritz	12.25	unk
Fritz	unk	unk				

θ

But Consider

ord2(Person Price Amt) $\not\sqsubseteq$ ord3(Person Price Amt)

ord2 Person	Price	Amt	θ	ord3 Person	Price	Amt
Fred	11.50	6	$\longrightarrow$	Fred	11.50	6
unk	10.25	7	$\longrightarrow$	Fred	10.25	7
		?	$\longrightarrow$	Fritz	12.25	7

θ

Possible Worlds

View a partial relation as specifying a set of possible worlds

POSS(r) = {q | r $\sqsubseteq$ q and q is fully defined}

Can include constraints C

POSS(r, C) = {q | q in POSS(r) and q in SAT(C)}

What Happens if We Have Constraints?

Recall

PILOT	FLIGHT	DATE	TIME
Cushing	615	5Oct	**unk**
Cook	704	6Oct	5:50a
Cushing	**unk**	6Oct	**unk**

Not All Satisfy the FDs

$F = \{L \rightarrow T, PDT \rightarrow L, LD \rightarrow P\}$

PILOT	FLIGHT	DATE	TIME
Cushing	615	5Oct	5:00p
Cook	704	6Oct	5:50a
Cushing	872	6Oct	11:00p

✓

PILOT	FLIGHT	DATE	TIME
Cushing	615	5Oct	5:00p
Cook	704	6Oct	5:50a
Cushing	615	6Oct	6:00p

X

Use FD-Chase on Instances with Unknowns

r

PILOT	FLIGHT	DATE	TIME
Cushing	704	5Oct	**unk**
Cook	704	6Oct	5:50a
Cushing	**unk**	6Oct	**unk**

r'

PILOT	FLIGHT	DATE	TIME
Cushing	704	5Oct	5:50a
Cook	704	6Oct	5:50a
Cushing	**unk**	6Oct	**unk**

POSS(r, F) = POSS(r', F)

≠ 704

Querying Relations with Unknowns

Let r be a relation with unknowns, and
Q a query on full relations

Would like Q' on partial relations
Q'(r) represents {Q(q) | q in POSS(r)}

When r is fully defined, would like
Q'(r) and Q(r) to agree. (<u>Faithful</u>)

Relational Algebra

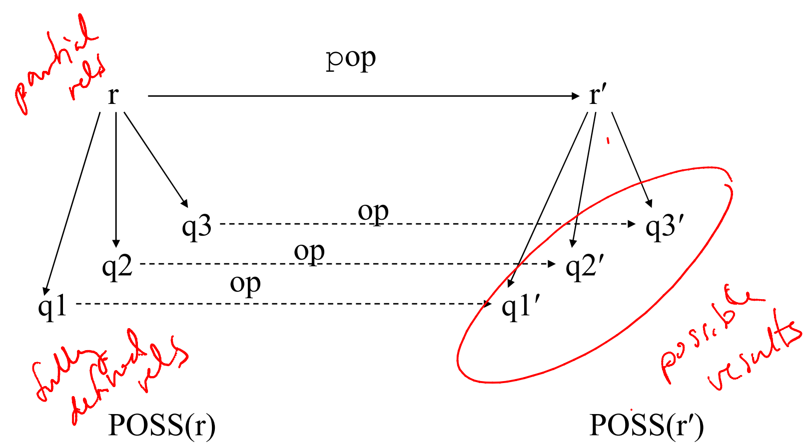
One approach: Multi-relation

Let pop be partial-relation version of operator op

pπ and π

POSS(pop(r)) = {op(q) | q in POSS(r)}

Diagram



Sometimes Exists

r

PILOT	FLIGHT	DATE	TIME
Cushing	615	5Oct	**unk**
Cook	704	6Oct	5:50a
Cushing	**unk**	6Oct	**unk**

$p\pi_{DT}(r)$

DATE	TIME
5Oct	**unk**
6Oct	5:50a
6Oct	**unk**

Not Always

$\sigma_{L=872}(r)$

r1 ∈ poss(r)

$r \sqsubseteq r1$

PILOT	FLIGHT	DATE	TIME
Cushing	615	5Oct	5:00p
Cook	704	6Oct	5:50a
Cushing	872	6Oct	11:00p

ans(r1)

PILOT	FLIGHT	DATE	TIME
Cushing	872	6Oct	11:00p

Not Always

$\sigma_{L=872}(r)$

$r \sqsubseteq r2$

$r_2 \in Poss(r)$

PILOT	FLIGHT	DATE	TIME
Cushing	615	5Oct	5:00p
Cook	704	6Oct	5:50a
Cushing	615	6Oct	5:00p

$p\sigma_{L=872}(r)$

ans(r2)

PILOT	FLIGHT	DATE	TIME
∅			

$ans(r1) \supseteq r'$

ans(r2) ⊇? can't have

Selection Conditions

t.B = 6

A B	
<Fred 6>	true
<Fred 4>	false
<Fred unk>	maybe

3-valued logic

or	T	F	M
T	T	T	T
F	T	F	M
M	T	M	M

Example

	A	B
t1	<Fred	unk>
t2	<Fritz	unk>

	t.A = Fritz	**or**	t.B = 6	
t1	F	**or**	M	= M
t2	T	**or**	M	= T

But

t1 <Fred unk>

	t.B < 6	**or**	t.B ≥ 6	
t1	M	**or**	M	= M

But it would be T for any value for unk