CS589 Principles of DB Systems
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Unit 4: Recursive Query Processing

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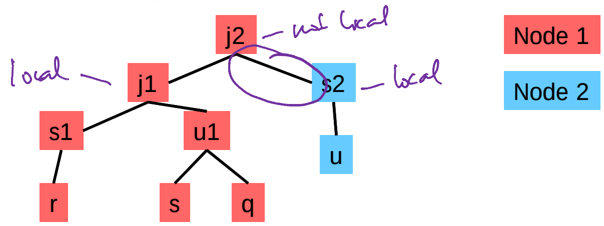
Goals for this Unit

- Study recursive query processing using Datalog
  - There are other data languages with recursion: QBE, G-Whiz, SQL:1999, LDL
- Models of a Datalog program
- Evaluation methods
- Negation and recursion
- (If time) Datalog and Streams

Example Recursive Program

Three predicates, that talk about a distributed algebra expression:

- `Op1(Id, In, Loc)`: unary operation `Id` with input `In` is performed at location `Loc`
- `Op2(Id, In1, In2, Loc)`: binary operation `Id` with inputs `In1` and `In2` is performed at location `Loc`
- `Local(Id, Loc)`: expression with root `Id` can be evaluated completely at location `Loc`



Clauses

operation input location

```
Local1(r, node1).
Local1(s, node1).
Local1(q, node1).
Local1(u, node2).
Local(Id, Loc) :- Op1(Id, In, Loc), Local(In, Loc).
Local(Id, Loc) :- Op2(Id, In1, In2, Loc),
                      Local(In1, Loc), Local(In2, Loc).
          Local(Id, Loc) :- Local1(Id, Loc).
Op1(s1, r, node1).
Op1(s2, u, node2).

Op2(j1, s1, u1, node1).
Op2(u1, s, q, node1).
Op2(j2, j1, s2, node1).
```

Facts versus Rules

It will be useful if we can separate predicates into

- Extensional: defined by facts
- Intensional: defined by rules

Notice that in our program the `Local` predicate has both facts and rules.

How could we modify the program to get this separation?

Conventions

- Assume predicates are extensional or intensional
- If I write just the intensional rules, we will assume that predicates without rules are extensional predicates.

Can think of them as the database

extensional predicates

```
Local(Id, Loc) :- Local0(Id, Loc).
Local(Id, Loc) :- Op1(Id, In, Loc), Local(In, Loc).
Local(Id, Loc) :- Op2(Id, In1, In2, Loc),
                  Local(In1, Loc), Local(In2, Loc).
```

Computed Predicates

Bancilhon and Ramakrishnan allow for computed predicates

- `Sum(X, Y, Z)`: true if X + Y = Z as integers
- Can think of `Sum` as an infinite extensional predicate:

```
Sum(1,1,2), Sum(1,2,3), Sum(1,3,4), …
Sum(2,1,3), Sum(2,2,4), Sum(2,3,5), …
Sum(3,1,4), …
…
```


Safety and Computed Predicates

Uses of computed predicates have required binding patterns so we get finite answers

- `Sum(1,7,8)` ok
- `Sum(2,7,10)` ok
- `Sum(2,7,Z)` ok
- `Sum(X,7,9)` ok
- `Sum(2,Y,9)` ok
- `Sum(X,7,Z)`
- `Sum(X,Y,Z)` not ok
- Note that other literals in a rule body can provide these bindings
- Each computed predicate has a *natural interpretation*

Example with Computed Predicate

`Local(Id, Loc, Num)`: expression with root `Id` has `Num` elements and can be evaluated completely at location `Loc`

```
Local(Id, Loc, 1) :- Local0(Id, Loc).

Local(Id, Loc, K) :- Op1(Id, In, Loc),
  Local(In, Loc, M), Sum(M, 1, K).

Local(Id, Loc, K) :- Op2(Id, In1, In2, Loc),
  Local(In1, Loc, M), Local(In2, Loc, N),
  Sum(M, N, J), Sum(J, 1, K).

:- Local(j1, L, C).
```

Find values of L + C where this is true

Ground Instance of a Clause

Given a Datalog clause C, we get a *ground instance* of C by replacing all variables with constants from the program.

Clause:

```
Local(Id, Loc, K) :- Op1(Id, In, Loc),
  Local(In, Loc, M), Sum(M, 1, K).
```

Ground instances:

```
Local(s2, node2, 2) :- Op1(s2, u, node2),
  Local(u, node2, 1), Sum(1, 1, 2).
Local(s2, node2, 2) :- Op1(s2, r, node2),
  Local(r, node2, 1), Sum(1, 1, 2).
```

Model of a Datalog Program

A *database (*aka *interpretation)* M for a program P is a set of facts over the predicates in P.

A database M is a *model* for program P if

1. It assigns each computable predicate its natural interpretation
2. For any ground instance of a clause
   `H :- L1, L2, …, Ln.`
   if {`L1, L2, …, Ln`} $\subseteq$ M then H $\in$ M.

Note that all facts of P must be in M.

A Program Can Have Many Models

Program

```
      u  node2                       u   node2
Local(Id, Loc, 1) :- Local0(Id, Loc)
      s2  node2  2        s2   u   node2
Local(Id, Loc, K) :- Op1(Id, In, Loc),
   Local(In, Loc, M), Sum(M, 1, K).
        u  node2  2      1     2
Local(Id, Loc, K) :- Op2(Id, In1, In2, Loc),
   Local(In1, Loc, M), Local(In2, Loc, N),
   Sum(M, N, J), Sum(J, 1, K).
Local0(r, node1).
Local0(s, node1).
Local0(q, node1).
Local0(u, node2).
Op1(s1, r, node1).
Op1(s2, u, node2).
Op2(j1, s1, u1, node1).
Op2(u1, s, q, node1).
Op2(j2, j1, s2, node1).
```

Model 1

```
Local(r, node1, 1).
Local(r, node2, 1).
Local(s, node1, 1).
Local(q, node1, 1).
Local(u, node2, 1).
Local(s1, node1, 2).
Local(u1, node1, 3).
Local(j1, node1, 6).
Local(s2, node2, 2).
Local0(r, node1).
Local0(r, node2).
Local0(s, node1).
Local0(q, node1).
Local0(u, node2).
Op1(s1, r, node1).
Op1(s2, u, node2).
Op2(j1, s1, u1, node1).
Op2(u1, s, q, node1).
Op2(j2, j1, s2, node1).
```

A Second Model

Program

```
Local(Id, Loc, 1) :- Local0(Id, Loc)
Local(Id, Loc, K) :- Op1(Id, In, Loc),
   Local(In, Loc, M), Sum(M, 1, K).
Local(Id, Loc, K) :- Op2(Id, In1, In2, Loc),
   Local(In1, Loc, M), Local(In2, Loc, N),
   Sum(M, N, J), Sum(J, 1, K).
Local0(r, node1).
Local0(s, node1).
Local0(q, node1).
Local0(u, node2).
Op1(s1, r, node1).
Op1(s2, u, node2).
Op2(j1, s1, u1, node1).
Op2(u1, s, q, node1).
Op2(j2, j1, s2, node1).
```

Model 2

```
Local(r, node1, 1).
Local(s, node1, 1).
Local(q, node1, 1).
Local(u, node2, 1).
Local(s1, node1, 2).
Local(u1, node1, 3).
Local(j1, node1, 6).
Local(s2, node2, 2).
Local(j2, node2, 9).
Local0(r, node1).
Local0(s, node1).
Local0(q, node1).
Local0(u, node2).
Op1(s1, r, node1).
Op1(s2, u, node2).
Op2(j1, s1, u1, node1).
Op2(u1, s, q, node1).
Op2(j2, j1, s2, node1).
```


Closure Under Intersection

Lemma: If M1 and M2 are both models for Datalog program P, then so is M1 $\cap$ M2.

- Proof: Let M = M1 $\cap$ M2. Both M1 and M2 give any computed predicate its natural interpretation, so part 1. of model definition is covered.
- How could M fail to be a model of P?
- Must have a ground instance of a clause:
- `H :- L1, L2, …, Ln.`
- Where {`L1, L2, …, Ln`} $\subseteq$ M but H $\notin$ M.
- So H must be missing from M1 or M2 (or both).

Example Intersection

M2

```
Local(r, node1, 1).
Local(s, node1, 1).
Local(q, node1, 1).
Local(u, node2, 1).
Local(s1, node1, 2).
Local(u1, node1, 3).
Local(j1, node1, 6).
Local(s2, node2, 2).
Local(j2, node2, 9).
Local0(r, node1).
Local0(s, node1).
Local0(q, node1).
Local0(u, node2).
Op1(s1, r, node1).
Op1(s2, u, node2).
Op2(j1, s1, u1, node1).
Op2(u1, s, q, node1).
Op2(j2, j1, s2, node1).
```

~~M1~~ M1∩M2

```
Local(r, node1, 1).
Local(r, node2, 1).
Local(s, node1, 1).
Local(q, node1, 1).
Local(u, node2, 1).
Local(s1, node1, 2).
Local(u1, node1, 3).
Local(j1, node1, 6).
Local(s2, node2, 2).
Local0(r, node1).
Local0(r, node2).
Local0(s, node1).
Local0(q, node1).
Local0(u, node2).
Op1(s1, r, node1).
Op1(s2, u, node2).
Op2(j1, s1, u1, node1).
Op2(u1, s, q, node1).
Op2(j2, j1, s2, node1).
```


Minimal Model

A model M for program P is *minimal* if no proper subset of M is also a model.

Every Datalog program P (without negation) has a <u>unique</u> minimal model.

Consider two different minimal models N1 and N2 for P. Then N = N1 ∩ N2 is also a model, and is a proper subset of at least one of them.

Minimum model of P: Unique minimal model.

Minimum Model and Queries

When we answer queries against Datalog program P, we want to answer them against the minimal model. (*Closed-World Assumption*)

```
:- Local(j1, L, C).
```

We want all `Local` facts in the minimum model where the first component is `j1`.

Evaluation Methods

- Oblivious: Compute the minimum model, then look up query answers.
- Directed: Use information from the query to reduce the portion of the minimum model computed to answer the query.

Naïve Evaluation of P

A "bottom-up" or "right-to-left" method.

Have a current set of facts f (a partial model).
Use clauses in P in a right-to-left manner to derive additional facts that must also be in a minimum model for P.

Example Right-to-Left Derivation

Consider:

```
            a6  node2  3                  a6   p5   node2
Local(Id, Loc, K) :- Op1(Id, In, Loc),
             p5  node2   2      2      3
   Local(In, Loc, M), Sum(M, 1, K).
```

Suppose we have in f:

```
Op1(a6, p5, node2).  Local(p5, node2, 2).
Op1(s4, u3, node1).  Local(u2, node2, 3).
Op1(a7, r, node1).   Local(r, node1, 1).
Op1(s9, a7, node1).  …
…
```

Can derive: Local(a6, node2, 3),
Local(a7, node1, 2)

(not Local(s9, node1, 3) yet)

Immediate Consequence Operator

If P is a Datalog program, the *immediate consequence operator* $I_P(f)$ computes all facts derivable from the set of facts f by a single application of the clauses in P.

Note: If f is a subset of the minimum model of P, then so is $I_P(f)$, $I_P(I_P(f))$, $I_P(I_P(I_P(f)))$, ...

So start the process with f = ∅, because we know we have a subset of the minimum model.

Inflationary semantics of program P: Start with empty set of facts, apply I_P until no change.

Naïve Evaluation

```
Naïve(P, q)
   f_old = ∅
   f = I_P(f_old)
   while f ≠ f_old do
      f_old = f
      f = I_P(f)
   return match(q, f)
```

$I_p(f) \supseteq f$

I_p rederives everything in f

Naïve always halts on programs that don't use computed predicates, with f as the minimum model.

