An example on Predicate Logic inference for the class.

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1 The setting

We will give a very simplistic setting with just two implications that attempt to describe when somebody becomes fit. Table 1 details all propositional logic rules that we can use on ground predicates (or ground statements corresponding to combinations of those ground predicates) and table 2 outlines the "lifted" versions of modus ponens and modus tollens that predicate logic uses.

Modus Ponens	Modus Tollens	Disjunctive addition	Conjunctive addition	Conjunctive Simplification
p $p \Rightarrow q$ $\therefore q$	$\neg q$ $p \Rightarrow q$ $\therefore \neg p$	p $\therefore p \vee q$	p, q $\therefore p \wedge q$	$p \wedge q$ $\therefore p, q$
Disjunctive syllogism	Hypothetical syllogism	Unit Resolution	Resolution	
$p \vee q$ $\neg p$ $\therefore q$	$p \Rightarrow q$ $q \Rightarrow r$ $\therefore p \Rightarrow r$	$p \vee q$ $\neg q$ $\therefore p$	$p \vee q$ $\neg q \vee z$ $p \vee z$	

Table 1: Propositional Logic inference rules we can use.

Universal Modus Ponens	**Universal Modus Tollens**
$(\forall x \in D) P(x) \Rightarrow Q(x)$	$(\forall x \in D) P(x) \Rightarrow Q(x)$
$P(A)$ for some A $\in D$	$\neg Q(A)$ for some A $\in D$
$Q(A)$	$\neg P(A)$

Table 2: Predicate Logic inference rules we can use.

2 Predicates

Table 3 describes the predicates that we will use.

Predicate	**Meaning**
$Eats(p, m)$	Person p eats in mode m ($Healthily$ or $Unhealthily$)
$Vegetarian(p)$	Person p is a vegetarian.
$Exercises(p)$	Person p exercises.
$Fit(p)$	Person p is fit.

Table 3: The predicates we will use when translating English language statements to Predicate Logic statements.

3 Knowledge base

3.1 In English

(i) Everybody who eats healthily and exercises is fit.

(ii) If somebody is a vegetarian, this means that they eat healthily.

3.2 In Predicate Logic

(The students should be doing this.)

(i) $(\forall p) Eats(p, Healthily) \wedge Exercises(p) \Rightarrow Fit(p)$

(ii) $(\forall p) Vegetarian(p) \Rightarrow Eats(p, Healthily)$

4 The input and the derivation goal

Given the statement "Mary is a vegetarian who exercises", we want to derive that she is fit.

5 The proof

5.1 Formulation of input and derivation goal

We translate the input into predicate logic to get:

$$Vegetarian(Mary) \wedge Exercises(Mary) \tag{1}$$

Our goal is to derive $Fit(Mary)$.

5.2 Derivation steps

By conjunctive simplification on (1), we can derive

$$Vegetarian(Mary) \tag{2}$$

and

$$Exercises(Mary). \tag{3}$$

By Universal Modus Ponens between (ii) and (2), we can derive

$$Eats(Mary, Healthily) \tag{4}$$

By conjunctive addition between (4) and (3), we can derive:

$$Eats(Mary, Healthily) \wedge Exercises(Mary) \tag{5}$$

Finally, by Universal Modus Ponens between (i) and (5) we can derive $Fit(Mary)$, which is our desired conclusion. This concludes our proof.